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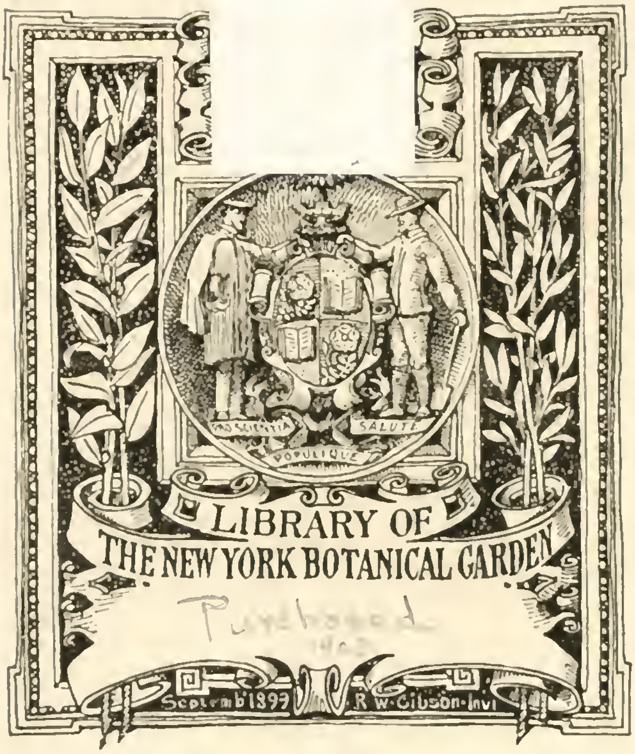
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BIOMETRIKA

A JOURNAL FOR THE STATISTICAL STUDY OF BIOLOGICAL PROBLEMS

EDITED
IN CONSULTATION WITH FRANCIS GALTON
BY
W. F. R. WELDON KARL PEARSON
AND
C. B. DAVENPORT

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VOLUME II

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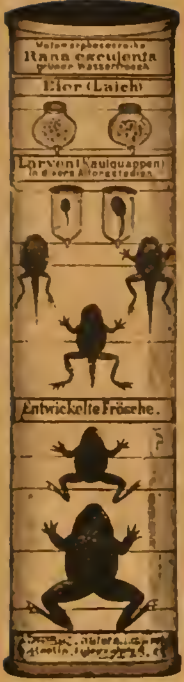

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VOLUME II NOVEMBER, 1902 No. 1

BIOMETRIKA.

ON THE SYSTEMATIC FITTING OF CURVES TO OBSERVATIONS AND MEASUREMENTS.

PART II.*

BY KARL PEARSON, F.R.S., University College, London.

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(9) *Illustration V.* In the previous illustration we have fitted the best curve not to the ordinates, but to the logarithms of the ordinates. This method was forced upon us by the complexity of Makeham's formula. It will clearly give good results in many cases where it might be difficult to calculate the moments of the ordinates of the curve, but in which the moments of the logarithms of the ordinates follow quite easily. For example in curves of the type $y = e^{f(x)}$ we shall often introduce considerable simplicity into our work without loss of practical efficiency by fitting to the data a curve $Y = f(x)$, where $Y = \log y$. In particular

---

* Part I. was published in Vol. I. p. 265.

if $f(x)$ be an algebraical expression in integer powers of x, we reduce the fitting of curves of this type to the theory of parabola-fitting with which we shall be occupied later.

A very interesting case of such work arises in dealing with frequency distributions, which we suppose to be normal or approximately normal, but of which only a portion of the distribution can be known or observed. For example, the marks of candidates in a competitive examination, wherein candidates below a certain grade have been rejected by a preliminary examination, or are cast out without placing. Or again, the statures of the soldiers in a regiment with a minimum admissible height.

Clearly in such cases as these we have to fit a curve

$$y = y_0 e^{-(ax^2+bx)},$$

given a certain number only of values of y and x.

The method of least squares or that of moments would enable us theoretically to determine y_0, a and b and so to find the constants of the best fitting normal distribution. But with the curve in its above form the equations, especially in the case of least squares, become unmanageable. If, however, we write $y = e^Y$, we find the problem reduces to fitting

$$Y = a'x^2 + b'x + c',$$

where Y is known for a certain range of values of x.

As far as I know the first attempt to determine the constants of a normal curve when only a portion of the distribution is known was made by Mr Francis Galton in his memoir on the speed distribution of American Trotting Horses*. The American record contains only horses which can trot a mile in less than a given number of seconds. Hence assuming the distribution to be normal we obtain only a portion of the frequency distribution, i.e. the number of horses that can trot a mile in each number of seconds less than this maximum.

Taking a normal curve

$$y = y_0 e^{-\frac{(x-h)^2}{2\sigma^2}},$$

Mr Galton has determined the position of the mode, i.e. the value of h, only by inspection of the plotted figures. It seemed worth while to compare his results with what we should get by fitting curves

$$Y = a'x^2 + b'x + c'$$

to the logarithms of his frequency data, using the method of fitting parabolas of the second order discussed on p. 14 below†.

It seems well to briefly indicate the process used. The curve for the year 1893 was determined by me, those for 1892, 1894 and 1895 I owe entirely to the energy of Mr Leslie Bramley-Moore.

---

* *R. S. Proc.* Vol. 62, p. 310.

† The curve being parabolic the methods of moments and of least squares are now sensibly identical in result, although not alike in their stages.

Taking Mr Galton's polygon for the data of 1892 nineteen ordinates were obtained for speeds at equal intervals of a second, 29—28, 28—27 ... 11—10 from the observations on 1324 trotters. On an arbitrary scale these nineteen ordinates are given in Column (1). In Column (2) are their logarithms to three figures. Column (3) gives the first moment m_1 about the middle of the range $2l = 19$. Column (4) the second moment m_2 about the same point. From m_0, m_1, m_2 and l, λ_0, λ_1 and λ_2 were found and hence e_0, e_1 and e_2 (see p. 14).

(1) y	(2) Y	x	(3) $S(xY)$	(4) $S(x^2Y)$
92·8	1·968	−9	−17·712	159·408
100·4	2·002	−8	−16·016	128·128
95·0	1·978	−7	−13·846	96·922
71·2	1·852	−6	−11·112	66·672
67·6	1·830	−5	− 9·150	45·750
61·3	1·787	−4	− 7·148	28·592
61·4	1·788	−3	− 5·364	16·092
44·8	1·651	−2	− 3·302	6·604
44·5	1·648	−1	− 1·648	1·648
45·8	1·661	0	−85·298	
38·4	1·584	+1	1·584	1·584
27·8	1·444	+2	2·888	5·776
19·8	1·297	+3	3·891	11·673
10·7	1·029	+4	4·116	16·464
15·8	1·199	+5	5·995	29·975
7·9	·898	+6	5·388	32·328
5·0	·699	+7	4·893	34·251
2·1	·322	+8	2·576	20·608
5·6	·748	+9	6·732	60·588
	$m_0 = 27·385$		+38·063	$m_2 = 763·063$
			−85·298	
			$m_1 = -47·235$	

Thus $$y_0 = \frac{m_0}{2l} = 1·441{,}316, \quad \lambda_1 = \frac{m_1}{m_0 l} = -·181{,}564,$$

$$\lambda_2 = \frac{m_2}{m_0 l^2} = ·308{,}7454.$$

Whence $$e_0 = 1·092{,}205, \qquad e_1 = -·544{,}692, \qquad e_2 = -·276{,}6143.$$

These give us for the required parabola:

$$Y = 1·441{,}316 \left\{1·092{,}205 - ·544{,}692\left(\frac{x}{l}\right) - ·276{,}614\left(\frac{x}{l}\right)^2\right\}.$$

This may be thrown into the form:

$$Y = 1·441{,}316 \left\{1·360{,}583 - ·276{,}614\left(\frac{x+\bar{x}}{l}\right)^2\right\},$$

where $$\bar{x} = -9·3575.$$

Now l is 9·5; hence the centre of the normal curve is at ·1425 beyond the start of the range which is 29 secs., i.e. 28·8575 secs. is the modal speed of the group of trotters.

If the normal curve be

$$y = z_0 e^{-\frac{(x+\bar{x})^2}{2\sigma^2}},$$

$$Y = \log y = \log z_0 - \frac{(x+\bar{x})^2}{2\sigma^2} \log e.$$

Whence we find

$$\log z_0 = 1{\cdot}441{,}316 \times 1{\cdot}360{,}583,$$

$$2\sigma^2 = \frac{l^2 \log e}{1{\cdot}441{,}316 \times {\cdot}276{,}614},$$

and ultimately

$$z_0 = 9{\cdot}14176, \quad \sigma = 7{\cdot}011{,}073.$$

Thus the required normal curve is

$$y = 9{\cdot}14176\, e^{-\frac{1}{2}x^2/(7{\cdot}011{,}073)^2}.$$

The "probable error" corresponding to the above value of σ is 4·7289, which enables us to compare our results directly with Mr Galton's numbers.

Like Mr Galton we have omitted from consideration the group of horses with speeds between 29 and 30 secs., for it includes a large number of doubtful trotters, whose speed is allowed by grace to fall within the 30-second limit.

The following are the actual determinations for four years of the 'centre' and variability of American trotters. They are compared with Mr Galton's determinations by inspection of the 'centre' and his calculation from this of the probable error by quartiles.

Year	Constant	Moments	Galton
1892	Centre	28·8575	29
	S. D.	7·0111	—
	P. E.	4·7289	4·25
1893	Centre	28·0120	26
	S. D.	8·8345	—
	P. E.	5·9588	5·0
1894	Centre	25·6256	27
	S. D.	6·2946	—
	P. E.	4·2456	4·5
1895	Centre	28·0119	27
	S. D.	8·5459	—
	P. E.	5·7641	5·0

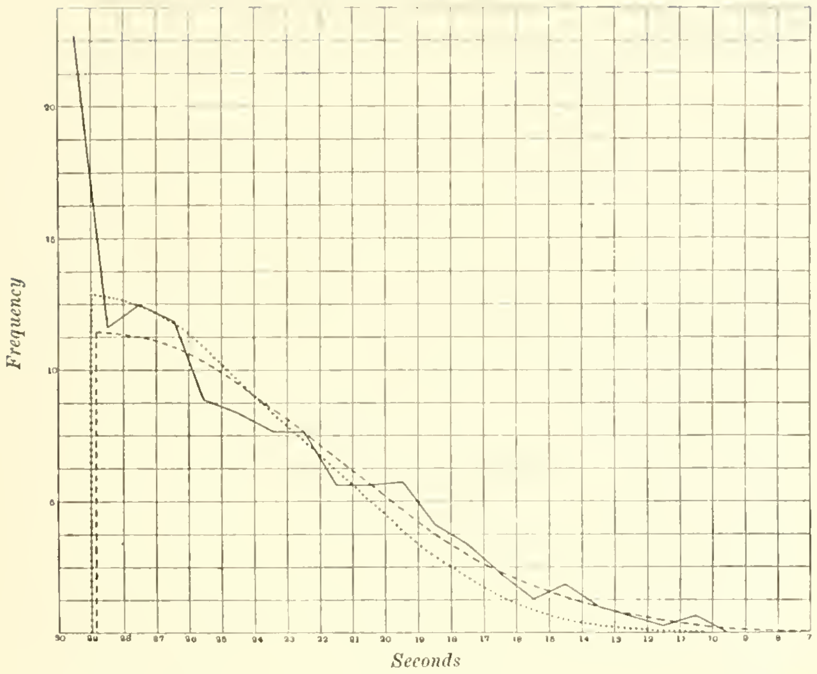


FIG. 5. Distribution of Speed of American Trotters, 1892. 1324 Observations.

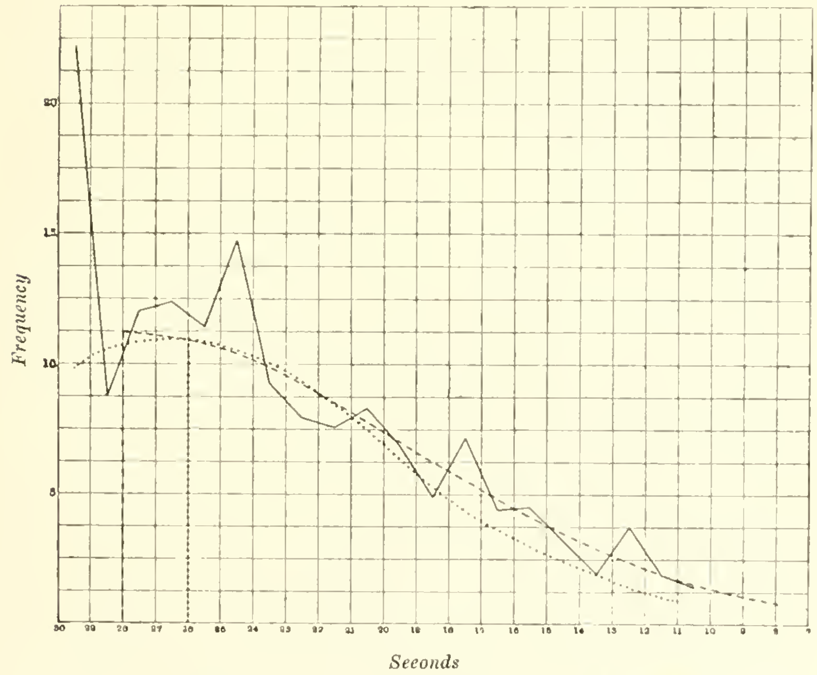


FIG. 6. Distribution of Speed of American Trotters, 1893. 982 Observations.

Moments - - - - Inspection and Quartile ......

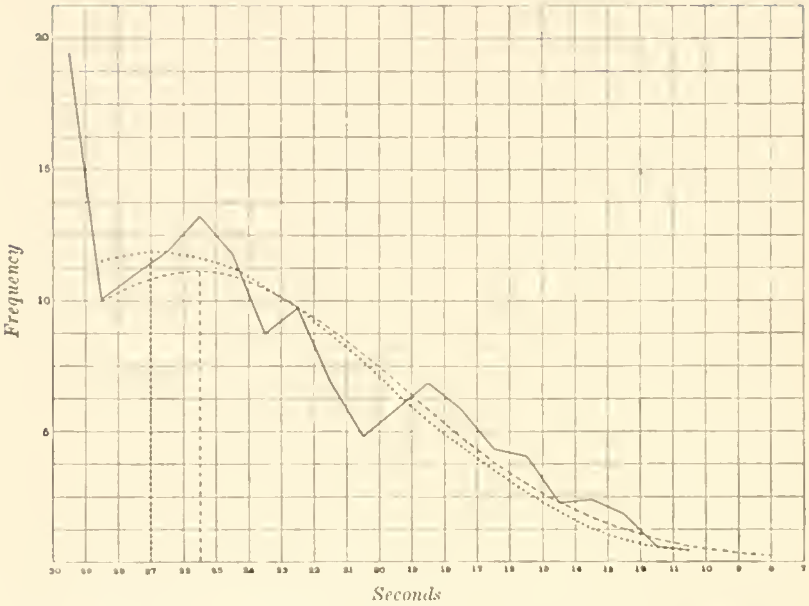


Fig. 7. Distribution of Speed of American Trotters, 1894. 1204 Observations.

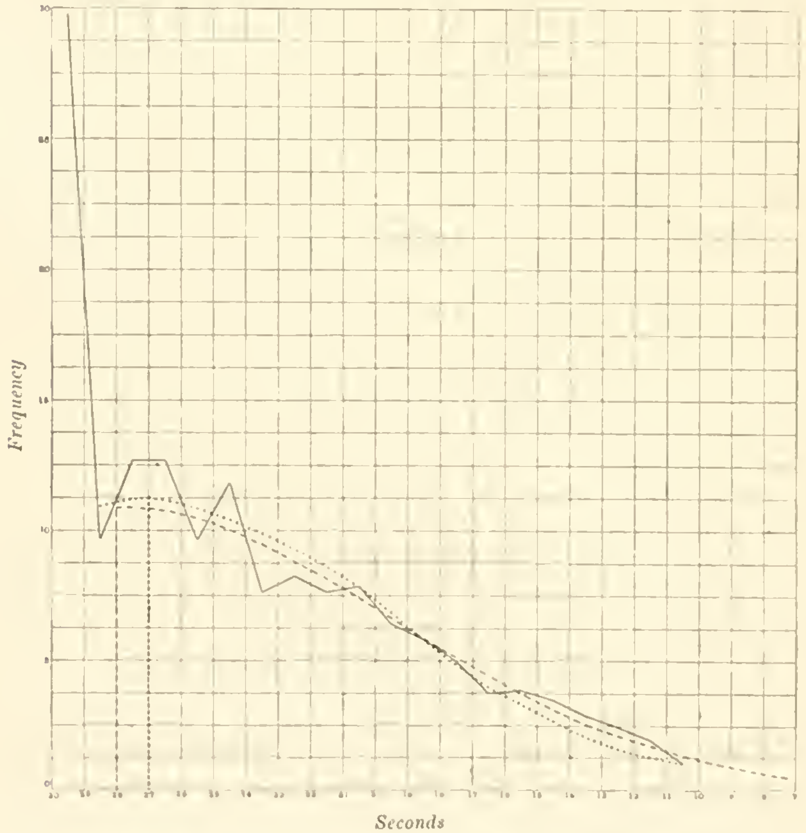
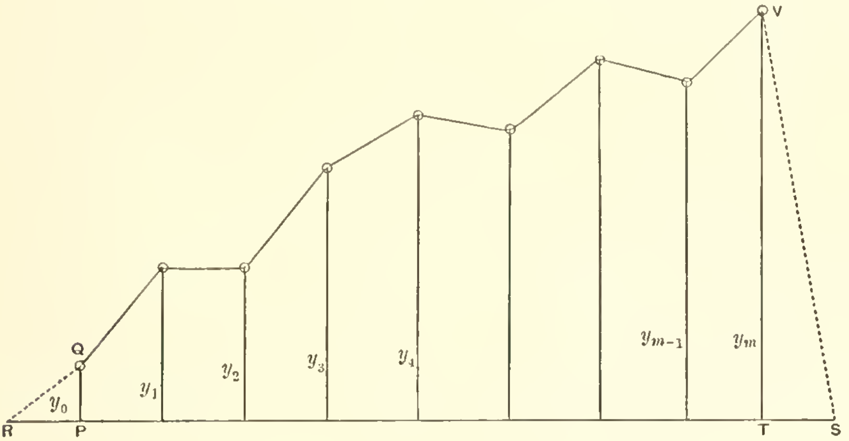


Fig. 8. Distribution of Speed of American Trotters, 1895. 1124 Observations.
Moments - - - - Inspection and Quartile ......

A glance at the accompanying Figs. 5, 6, 7 and 8 will show that the fit has been much improved by adopting the systematic method of moments. At the same time using a Brunsviga for all multiplications and divisions and a Comptometer for additions the labour is not very severe. Of course it is not contended that this accuracy is necessary in the present case; Mr Galton's approximations are probably close enough for the ends he had in view. We have only used his data to illustrate a method, which may be of service for special cases, where the best available determinations of the constants are needed.

(10) In calculating the moments in the previous illustration we have simply concentrated along the mid-ordinates. This was close enough for the purpose of illustration. When the ordinates of a curve which requires fitting are true ordinates, say for example, measurements obtained by observation, their irregularity is often such that it appears idle to use complex quadrature formulæ. Such formulæ give very good results, if the ordinates are those of a mathematical formula, or if we have a fairly smooth system of points. But a very frequent case is a case like that of the accompanying figure, in which a quadrature formula for the moments seems idle and yet we are scarcely justified in concentrating along the ordinates to find the moments, if the base unit be not indefinitely small. Here it seems reasonable to take the area and moments of the system of trapezia as fairly representing the area and moments of the curve to be fitted. It would be idle to use a formula like Simpson's, for example; because the changes in curvature in the curve, which would be allowed for by Simpson's method of striking a parabola through three successive points, have clearly no existence in the general sweep of the observations and are due only to irregularities of observation.



Accordingly we want an expression for the moments of a system of trapezia. Let the ordinates be $y_0, y_1, y_2 \dots y_m$ and the corresponding abscissæ $x_0, x_1, x_2 \dots x_m$.

We shall limit our discussion to equal base elements b, which can always be taken as unity.

Let
$$v_n' = \overset{m}{\underset{0}{S}}(x^n y).$$

Then the following expression for the nth moment M_n' of the trapezoidal area about the axis of y is easily deduced:

$$\begin{aligned} M_n' = v_n' b + \frac{n(n-1)}{12} v'_{n-2} b^2 + \frac{n(n-1)(n-2)(n-3)}{360} v'_{n-4} b^4 \\ + \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{20160} v'_{n-6} b^6 + \dots \\ - y_m \left\{ \frac{x_m{}^n}{\underline{|2}} b + \frac{n}{\underline{|3}} x_m{}^{n-1} b^2 + \frac{n(n-1)}{\underline{|4}} x_m{}^{n-2} b^3 + \dots \right\} \\ - y_0 \left\{ \frac{x_0{}^n}{\underline{|2}} b - \frac{n}{\underline{|3}} x_0{}^{n-1} b^2 + \frac{n(n-1)}{\underline{|4}} x_0{}^{n-2} b^3 - \dots \right\}. \end{aligned}$$

Here the "corrective terms" in y_m and y_0 are nothing more than the subtraction of the nth moments of the triangles PQR and STV from the nth moment of the whole figure on base RS, which is represented by the remainder of the expression. They can be thrown into the simple forms

$$- y_m \frac{(x_m + b)^{n+2} - (n+2) x_m{}^{n+1} b - x_m{}^{n+2}}{(n+1)(n+2)},$$

$$- (-1)^n y_0 \frac{(-x_0 + b)^{n+2} - (n+2)(-x_0)^{n+1} b - (-x_0)^{n+2}}{(n+1)(n+2)}.$$

Now let $PT = 2l$ and let us take moments about the middle of PT. Let M_n be the nth moment about this point, and x being measured from it, let

$$v_n = \overset{m}{\underset{0}{S}}(x^n y).$$

We have
$$x_m = -x_0 = l,$$

and accordingly, if l be measured in b as unit,

$$\begin{aligned} M_n = v_n + \frac{n(n-1)}{12} v_{n-2} + \frac{n(n-1)(n-2)(n-3)}{360} v_{n-4} \\ + \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{20160} v_{n-6} + \dots\dots \\ - \frac{(l+1)^{n+2} - l^{n+2} - (n+2) l^{n+1}}{(n+1)(n+2)} \{y_m + (-1)^n y_0\}. \end{aligned}$$

The corrective term is thus very simple: it may be written

$$L_n (y_m + (-1)^n y_0),$$

where L_n may be calculated once for all for $n = 0$ to 5 and for values of $l = 3$ to 20 or for all odd numbers of ordinates from 7 to 41. These values are given in the accompanying Table, which will enable the reader to readily find the corrective term, bearing in mind that, if the moment is even, L_n must be multiplied by $y_m + y_0$, and that if it is odd it must be multiplied by $y_m - y_0$.

Table of Corrective Terms for Moments of Trapezia.

$$L_n = \frac{(l+1)^{n+2} - (n+2)l^{n+1} - l^{n+2}}{(n+1)(n+2)}.$$

l	L_0	L_1	L_2	L_3	L_4	L_5
3	·5	1·667	5·583	18·800	63·633	216·524
4	·5	2·167	9·417	41·050	179·500	787·357
5	·5	2·667	14·250	76·300	409·367	2200·857
6	·5	3·167	20·083	127·550	811·233	5167·024
7	·5	3·667	26·917	197·800	1455·100	10715·857
8	·5	4·167	34·750	290·050	2422·967	20257·357
9	·5	4·667	43·583	407·300	3808·833	35641·524
10	·5	5·167	53·417	552·550	5718·700	59218·357
11	·5	5·667	64·250	728·800	8270·567	93897·857
12	·5	6·167	76·083	939·050	11594·433	143210·024
13	·5	6·667	88·917	1186·300	15832·300	211364·857
14	·5	7·167	102·750	1473·550	21138·167	303312·357
15	·5	7·667	117·583	1803·800	27678·033	424802·524
16	·5	8·167	133·417	2180·050	35629·900	582445·357
17	·5	8·667	150·250	2605·300	45183·767	783770·857
18	·5	9·167	168·083	3082·550	56541·633	1037289·024
19	·5	9·667	186·917	3614·800	69917·500	1352549·857
20	·5	10·167	206·750	4205·050	85537·367	1740203·357

(11) *On the fitting of Parabolic Curves to given Data.*

Let us consider this problem first from the standpoint of the method of least squares.

Let the parabola be of the $(n-1)$th order and represented by

$$y = c_0 + c_1 x + c_2 x^2 + \ldots + c_{n-1} x^{n-1}.$$

Let us write for brevity

$$v_r' = S(x^r y),$$

$$\lambda_r' = S(x^r),$$

where S denotes a summation extending to the values of x and y for every observation. Then proceeding to make

$$S(y - c_0 - c_1 x - c_2 x^2 - \ldots - c_{n-1} x^{n-1})^2$$

a minimum for values of $c_0, c_1, c_2 \dots c_{n-1}$ we easily find the type equations

$$\begin{aligned}
\nu_0' &= c_0\lambda_0' + c_1\lambda_1' + c_2\lambda_2' + \dots + c_{n-1}\lambda'_{n-1}\\
\nu_1' &= c_0\lambda_1' + c_1\lambda_2' + c_2\lambda_3' + \dots + c_{n-1}\lambda'_{n}\\
\nu_2' &= c_0\lambda_2' + c_1\lambda_3' + c_2\lambda_4' + \dots + c_{n-1}\lambda'_{n+1}\\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\
\nu'_{n-1} &= c_0\lambda'_{n-1} + c_1\lambda'_{n} + c_2\lambda'_{n+1} + \dots + c_{n-1}\lambda'_{2n-1}.
\end{aligned}$$

These are n equations to find the n constants.

Now in any curve-fitting with n constants to be found we must determine the area and moments $\nu_0', \nu_1', \nu_2' \dots \nu'_{n-1}$; but we find that the method of least squares involves also the discovery of the $2n$ quantities $\lambda_0', \lambda_1', \lambda_2' \dots \lambda'_{2n-1}$.

If as is very usually the case the observations are taken at equal intervals and there are m of them, we can by choosing this interval as our unit and a proper origin, write

$$\lambda_r' = \Sigma(x^r) = 1^r + 2^r + 3^r + \dots + m^r,$$

or λ_r' is the sum of the rth powers of the first m natural numbers. Thus the values of λ_r' for successive values of m can be tabled once and for all. This is done for $m = 1$ to 20 and $r = 1$ to 7 in the accompanying Table.

Table of the Sums of the first seven Powers of the Natural Numbers.*

Up to	$S(x)$	$S(x^2)$	$S(x^3)$	$S(x^4)$	$S(x^5)$	$S(x^6)$	$S(x^7)$
1	1	1	1	1	1	1	1
2	3	5	9	17	33	65	129
3	6	14	36	98	276	794	2316
4	10	30	100	354	1300	4890	18700
5	15	55	225	979	4425	20515	96825
6	21	91	441	2275	12201	67171	376761
7	28	140	784	4676	29008	184820	1200304
8	36	204	1296	8772	61776	446964	3297456
9	45	285	2025	15333	120825	978405	8080425
10	55	385	3025	25333	220825	1978405	18080425
11	66	506	4356	39974	381876	3749966	37567596
12	78	650	6084	60710	630708	6735950	73399404
13	91	819	8281	89271	1002001	11562759	136147921
14	105	1015	11025	127687	1539825	19092295	241561425
15	120	1240	14400	178312	2299200	30482920	412420800
16	136	1496	18496	243848	3347776	47260136	680856256
17	153	1785	23409	327369	4767633	71397705	1091194929
18	171	2109	29241	432345	6657201	105409929	1703414961
19	190	2470	36100	562666	9133300	152455810	2597286700
20	210	2870	44100	722666	12333300	216455810	3877286700

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* Mr W. Palin Elderton has calculated an extended table of this kind, giving the sums of the 7th powers of all the natural numbers up to 100. I hope it may be possible eventually to reproduce and distribute this much enlarged table.

It will be seen at once that λ_7' is very large for any number of observations greater than 10. Even if we go as far as λ_7', we shall, however, be only fitting a parabola of the *third* order, and the four type equations to be solved will be found as a rule even in this case to be rather unmanageable. For parabolas of the fourth, fifth and sixth orders, the labour becomes very severe.

It is clear however that if we evaluated the determinant

$$\begin{vmatrix} \lambda_0', & \lambda_1', & \lambda_2', & \dots \lambda'_{n-1} \\ \lambda_1', & \lambda_2', & \lambda_3', & \dots \lambda'_{n} \\ \lambda_2', & \lambda_3', & \lambda_4', & \dots \lambda'_{n+1} \\ \dots & \dots & \dots & \dots \\ \lambda'_{n-1}, & \lambda'_{n}, & \lambda'_{n+1}, & \dots \lambda'_{2n-1} \end{vmatrix}$$

and its minors for values of $n = 1, 2, \dots 5, 6$, and of m from 1 up to 20, we should have a set of constants which would enable us readily to find $c_0, c_1, \dots$ as soon as the $\nu_0', \nu_1', \dots \nu'_{n-1}$ were given. The arithmetical work to be *once* done would be considerable, but it might be worth doing, supposing the method of moments were not available with a simpler solution.

It may be noted that a considerable simplification of the least square type equations can be introduced if there be *an odd number* of observations. Let us take the origin at the middle observation, then clearly

$$\lambda_1' = \lambda_3' = \lambda_5' = \dots = 0,$$

or all the odd x sums vanish. Let us use undashed letters to denote moments about the centre of the range, then we find our system of type equations breaks up into two

$$\begin{aligned} \nu_0 &= c_0\lambda_0 + c_2\lambda_2 + c_4\lambda_4 + \\ \nu_2 &= c_0\lambda_2 + c_2\lambda_4 + c_4\lambda_6 + \\ \nu_4 &= c_0\lambda_4 + c_2\lambda_6 + c_4\lambda_8 + \\ &\dots\dots \end{aligned} \qquad \text{and} \qquad \begin{aligned} \nu_1 &= c_1\lambda_2 + c_3\lambda_4 + c_5\lambda_6 + \\ \nu_3 &= c_1\lambda_4 + c_3\lambda_6 + c_5\lambda_8 + \\ \nu_5 &= c_1\lambda_6 + c_3\lambda_8 + c_5\lambda_{10} + \\ &\dots\dots \end{aligned}$$

Our Table will now enable us to find λ_6 for any number of observations up to 41; and for parabolas of the third order only, we have simply to solve two sets of linear equations, each of which contains only two unknowns. Thus the work becomes extremely simple. This is in fact how the cubical parabola was fitted by the method of least squares to the observations in § 3 (Vol. I. p. 280).

On the other hand even with this choice of origin we require λ_8, λ_{10} and λ_{12} to fit parabolas of the fourth, fifth and sixth orders; and the sum of the 10th or 12th powers of the natural numbers* and simultaneous equations with three or four variables and coefficients of very diverse magnitudes are rather troublesome matters to deal with.

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* Of course λ_r may be calculated from the Bernoulli number formula on p. 286, *Biometrika*, Vol. I.

Of course if the λ determinant and its minors were once worked out, say for the first six parabolas and for the customary range of values of m, we should have no more labour, but meanwhile it seems to me that the method of least squares must for practical purposes be laid on one side even for parabolic curves except in the simple cases of those of the first, second and third orders. But if the method of least squares be of small practical use even in this one of the simplest cases of curve-fitting, it may be questioned whether it is not better to adopt the uniform process of moments throughout.

(12) Let us now apply the method of moments to the parabolic curve

$$y = e_0 + e_1 x + e_2 x^2 + \dots + e_{n-1} x^{n-1}$$

for which the expansion by Maclaurin's Theorem is exact. Let $2l$ be the range for which this curve is to be fitted to the observations, and let us take the origin at the mid-point of the range.

Let m_0 be the area and $m_1, m_2 \dots m_{n-1}$ the first n moments of the observation polygon about the axis of y, i.e. the perpendicular to the range at its mid-point. Let us write $m_0 = 2l \times y_0$, so that y_0 is the mean value of the ordinate. Then the curve to be fitted may be written in the form

$$y = y_0 \left(e_0 + e_1 \frac{x}{l} + e_2 \left(\frac{x}{l}\right)^2 + \dots + e_{n-1} \left(\frac{x}{l}\right)^{n-1} \right).$$

Multiply by $\left(\frac{x}{l}\right)^{2r}$ and integrate from $x = l$ to $x = -l$:

$$m_{2r}/l^{2r} = 2y_0 l \left\{ \frac{e_0}{2r+1} + \frac{e_2}{2r+3} + \dots + \frac{1-(-1)^n}{2} \frac{e_{n-1}}{2r+n} \right\}.$$

If we multiply by $(x/l)^{2r+1}$ and integrate,

$$m_{2r+1}/l^{2r+1} = 2y_0 l \left\{ \frac{e_1}{2r+3} + \frac{e_3}{2r+5} + \dots + \frac{1+(-1)^n}{2} \frac{e_{n-1}}{2r+n+1} \right\}.$$

It is obvious that the even e's will be given in terms of the even moments and the odd e's in terms of the odd moments by two independent series of equations.

Let us write $\lambda_s = m_s/(m_0 l^s)$, thus $\lambda_0 = 1$.

Then

$$\begin{aligned}
\lambda_0 &= e_0 + \tfrac{1}{3} e_2 + \tfrac{1}{5} e_4 + \dots \\
\lambda_2 &= \tfrac{1}{3} e_0 + \tfrac{1}{5} e_2 + \tfrac{1}{7} e_4 + \dots \\
\lambda_4 &= \tfrac{1}{5} e_0 + \tfrac{1}{7} e_2 + \tfrac{1}{9} e_4 + \dots \\
&\dots\dots\dots\dots\dots\dots \\
&\dots\dots\dots\dots\dots\dots \\
\lambda_1 &= \tfrac{1}{3} e_1 + \tfrac{1}{5} e_3 + \tfrac{1}{7} e_5 + \dots \\
\lambda_3 &= \tfrac{1}{5} e_1 + \tfrac{1}{7} e_3 + \tfrac{1}{9} e_5 + \dots \\
\lambda_5 &= \tfrac{1}{7} e_1 + \tfrac{1}{9} e_3 + \tfrac{1}{11} e_5 + \dots \\
&\dots\dots\dots\dots\dots\dots \\
&\dots\dots\dots\dots\dots\dots
\end{aligned}$$

Hence it is clear that any e can be expressed at once in terms of the moments and of one or other of the determinants

$$\begin{vmatrix} 1, & 1/3, & 1/5, & 1/7, & \dots \\ 1/3, & 1/5, & 1/7, & 1/9, & \dots \\ 1/5, & 1/7, & 1/9, & 1/11, & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \end{vmatrix}, \qquad \begin{vmatrix} 1/3, & 1/5, & 1/7, & 1/9, & \dots \\ 1/5, & 1/7, & 1/9, & 1/11, & \dots \\ 1/7, & 1/9, & 1/11, & 1/13, & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \end{vmatrix}$$

and their respective minors.

It is thus quite easy to express the general result of working with $e_0, e_1 \dots e_{n-1}$. But as a matter of practical application, it would involve far too much troublesome arithmetic to calculate moments beyond the fifth or sixth. We have accordingly only to calculate once and for all the numerical coefficients of the λ's in the values of the e's for the first few cases and these will serve for all future applications.

Case (i). *To fit a straight line to a series of observations.*

Let the line be $$y = y_0\left(e_0 + e_1\frac{x}{l}\right).$$

Then $$\lambda_0 = e_0, \quad \lambda_1 = \tfrac{1}{3}e_1.$$

Thus the equation of the line is

$$y = y_0\left(\lambda_0 + 3\lambda_1\frac{x}{l}\right).$$

Geometrical Construction. Let the broken line AB be the observations and $A'B'$ the best straight line. Then $A'B'EF$ and $ADBEF$ must have the same first moments and the same area. Let CK be the vertical through the centroid of the observations, i.e. obtained by taking their mean FK. Now the trapezium may be considered as made up of two triangles $A'EF$ and $A'B'E$, the centroids of which lie in the vertical lines G_1H_1 and G_2H_2 trisecting FE. Hence the area $A'B'EF$ acting in CK must be equivalent to the areas $A'EF$ and $A'B'E$, or $l \times A'F$ and $l \times B'E$, acting in G_1H_1 and G_2H_2. Now $A'F + B'E$ is known, for it equals $2y_0$, the double of the mean ordinate of the observations.

Take $01 = 2y_0$ and from any point O, draw $O0$ to meet G_1H_1 and CK in t and u. Draw uv parallel to $O1$ to meet G_2H_2 in v, and then draw $O2$ parallel to tv to meet 01 in 2. We shall then have $02 = A'F$ and $21 = B'E$, the required lengths, which fully determine the line $A'B'$.

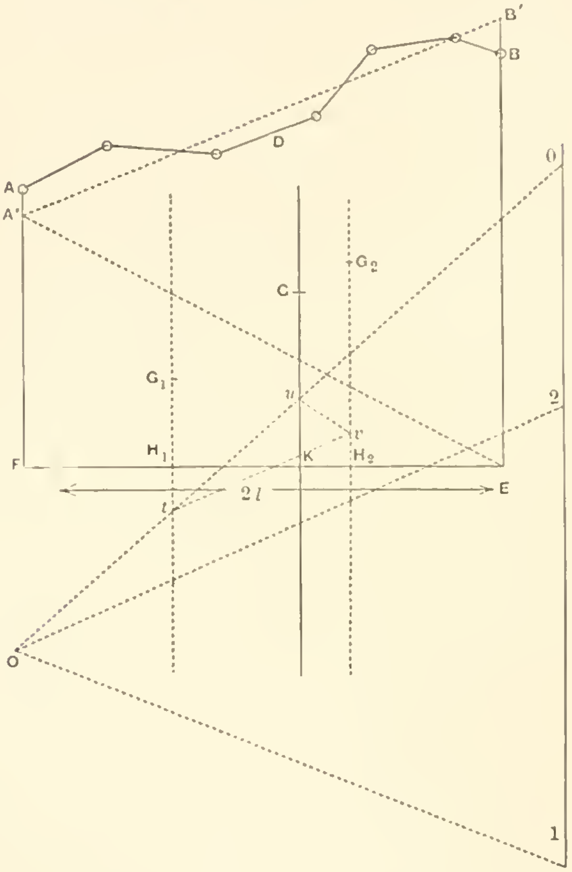
The construction given is the familiar graphical one for finding the components in the lines G_1H_1 and G_2H_2 parallel to CK of a force $2y_0$ acting in CK. The principle of moments would also give a solution. Thus take moments about H_2:

$$A'F \times \frac{2l}{3} = 2y_0 \times H_2K,$$

whence $$A'F = \frac{3y_0}{l} \times H_2K,$$

similarly $$B'E = \frac{3y_0}{l} \times H_1K,$$

which determine the intercepts since H_1K and H_2K are known.



Case (ii). *To fit a parabola of the second order to a series of observations.*

Let the parabola be $$y = y_0 \left\{ e_0 + e_1 \frac{x}{l} + e_2 \left(\frac{x}{l}\right)^2 \right\}.$$

Our equations are now

$$\lambda_0 = e_0 + \tfrac{1}{3} e_2,$$
$$\lambda_2 = \tfrac{1}{3} e_0 + \tfrac{1}{5} e_2,$$
$$\lambda_1 = \tfrac{1}{3} e_1.$$

Hence
$$e_0 = \tfrac{3}{4}(3\lambda_0 - 5\lambda_2),$$
$$e_1 = 3\lambda_1,$$
$$e_2 = \tfrac{15}{4}(3\lambda_2 - \lambda_0).$$

Thus e_0, e_1, e_2 are at once found, when λ_0, λ_1 and λ_2 are known. Probably the best way to construct this parabola is to draw it graphically through the three points

$$\begin{aligned} x_1 &= -l, & y_1 &= y_0(e_0 - e_1 + e_2), \\ x_2 &= 0, & y_2 &= y_0 e_0, \\ x_3 &= l, & y_3 &= y_0(e_0 + e_1 + e_2). \end{aligned}$$

Case (iii). *To fit a parabola of the third order to a series of observations.*

Let the curve be

$$y = y_0 \left\{ e_0 + e_1 \frac{x}{l} + e_2 \left(\frac{x}{l}\right)^2 + e_3 \left(\frac{x}{l}\right)^3 \right\}.$$

The equations to find the e's will now be

$$\begin{aligned} \lambda_0 &= e_0 + \tfrac{1}{3} e_2, & \lambda_1 &= \tfrac{1}{3} e_1 + \tfrac{1}{5} e_3, \\ \lambda_2 &= \tfrac{1}{3} e_0 + \tfrac{1}{5} e_2, & \lambda_3 &= \tfrac{1}{5} e_1 + \tfrac{1}{7} e_3. \end{aligned}$$

Hence

$$\begin{aligned} e_0 &= \tfrac{3}{4}(3\lambda_0 - 5\lambda_2), & e_1 &= \tfrac{15}{4}(5\lambda_1 - 7\lambda_3), \\ e_2 &= \tfrac{15}{4}(3\lambda_2 - \lambda_0), & e_3 &= \tfrac{35}{4}(-3\lambda_1 + 5\lambda_3). \end{aligned}$$

Case (iv). *To fit a parabola of the fourth order to a series of observations.*

Let the curve be

$$y = y_0 \left\{ e_0 + e_1 \frac{x}{l} + e_2 \left(\frac{x}{l}\right)^2 + e_3 \left(\frac{x}{l}\right)^3 + e_4 \left(\frac{x}{l}\right)^4 \right\}.$$

Then

$$\begin{aligned} \lambda_0 &= e_0 + \tfrac{1}{3} e_2 + \tfrac{1}{5} e_4, & \lambda_1 &= \tfrac{1}{3} e_1 + \tfrac{1}{5} e_3, \\ \lambda_2 &= \tfrac{1}{3} e_0 + \tfrac{1}{5} e_2 + \tfrac{1}{7} e_4, & \lambda_3 &= \tfrac{1}{5} e_1 + \tfrac{1}{7} e_3, \\ \lambda_4 &= \tfrac{1}{5} e_0 + \tfrac{1}{7} e_2 + \tfrac{1}{9} e_4. \end{aligned}$$

Hence we find

$$\begin{aligned} e_0 &= \tfrac{15}{64}\ (15\lambda_0 - 70\lambda_2 + 63\lambda_4), & e_1 &= \tfrac{15}{4}(5\lambda_1 - 7\lambda_3), \\ e_2 &= \tfrac{105}{32}(-5\lambda_0 + 42\lambda_2 - 45\lambda_4), & e_3 &= \tfrac{35}{4}(-3\lambda_1 + 5\lambda_3), \\ e_4 &= \tfrac{315}{64}\ (3\lambda_0 - 30\lambda_2 + 35\lambda_4). \end{aligned}$$

Case (v). *To fit a parabola of the fifth order to a series of observations.*

Let the curve be

$$y = y_0 \left\{ e_0 + e_1 \frac{x}{l} + e_2 \left(\frac{x}{l}\right)^2 + e_3 \left(\frac{x}{l}\right)^3 + e_4 \left(\frac{x}{l}\right)^4 + e_5 \left(\frac{x}{l}\right)^5 \right\}.$$

Then

$$\begin{aligned} \lambda_0 &= e_0 + \tfrac{1}{3} e_2 + \tfrac{1}{5} e_4, & \lambda_1 &= \tfrac{1}{3} e_1 + \tfrac{1}{5} e_3 + \tfrac{1}{7} e_5, \\ \lambda_2 &= \tfrac{1}{3} e_0 + \tfrac{1}{5} e_2 + \tfrac{1}{7} e_4, & \lambda_3 &= \tfrac{1}{5} e_1 + \tfrac{1}{7} e_3 + \tfrac{1}{9} e_5, \\ \lambda_4 &= \tfrac{1}{5} e_0 + \tfrac{1}{7} e_2 + \tfrac{1}{9} e_4, & \lambda_5 &= \tfrac{1}{7} e_1 + \tfrac{1}{9} e_3 + \tfrac{1}{11} e_5. \end{aligned}$$

Hence we have

$$e_0 = \tfrac{15}{64}\,(15\lambda_0 - 70\lambda_2 + 63\lambda_4), \qquad e_1 = \tfrac{105}{64}\,(35\lambda_1 - 126\lambda_3 + 99\lambda_5),$$
$$e_2 = \tfrac{105}{32}(-5\lambda_0 + 42\lambda_2 - 45\lambda_4), \qquad e_3 = \tfrac{315}{32}(-21\lambda_1 + 90\lambda_3 - 77\lambda_5),$$
$$e_4 = \tfrac{315}{64}\,(3\lambda_0 - 30\lambda_2 + 35\lambda_4), \qquad e_5 = \tfrac{693}{64}\,(15\lambda_1 - 70\lambda_3 + 63\lambda_5).$$

Case (vi). *To fit a parabola of the sixth order to a series of observations.*

Let the curve be

$$y = y_0\left\{e_0 + e_1\frac{x}{l} + e_2\left(\frac{x}{l}\right)^2 + e_3\left(\frac{x}{l}\right)^3 + e_4\left(\frac{x}{l}\right)^4 + e_5\left(\frac{x}{l}\right)^5 + e_6\left(\frac{x}{l}\right)^6\right\}.$$

Then

$$\lambda_0 = e_0 + \tfrac{1}{3}e_2 + \tfrac{1}{5}e_4 + \tfrac{1}{7}e_6, \qquad \lambda_1 = \tfrac{1}{3}e_1 + \tfrac{1}{5}e_3 + \tfrac{1}{7}e_5,$$
$$\lambda_2 = \tfrac{1}{3}e_0 + \tfrac{1}{5}e_2 + \tfrac{1}{7}e_4 + \tfrac{1}{9}e_6, \qquad \lambda_3 = \tfrac{1}{5}e_1 + \tfrac{1}{7}e_3 + \tfrac{1}{9}e_5,$$
$$\lambda_4 = \tfrac{1}{5}e_0 + \tfrac{1}{7}e_2 + \tfrac{1}{9}e_4 + \tfrac{1}{11}e_6, \qquad \lambda_5 = \tfrac{1}{7}e_1 + \tfrac{1}{9}e_3 + \tfrac{1}{11}e_5,$$
$$\lambda_6 = \tfrac{1}{7}e_0 + \tfrac{1}{9}e_2 + \tfrac{1}{11}e_4 + \tfrac{1}{13}e_6.$$

Hence we find

$$e_0 = \tfrac{35}{256}\,(35\lambda_0 - 315\lambda_2 + 693\lambda_4 - 429\lambda_6), \qquad e_1 = \tfrac{105}{64}\,(35\lambda_1 - 126\lambda_3 + 99\lambda_5),$$
$$e_2 = \tfrac{315}{256}(-35\lambda_0 + 567\lambda_2 - 1485\lambda_4 + 1001\lambda_6), \qquad e_3 = \tfrac{315}{32}(-21\lambda_1 + 90\lambda_3 - 77\lambda_5),$$
$$e_4 = \tfrac{3465}{256}\,(7\lambda_0 - 135\lambda_2 + 385\lambda_4 - 273\lambda_6), \qquad e_5 = \tfrac{693}{64}\,(15\lambda_1 - 70\lambda_3 + 63\lambda_5),$$
$$e_6 = \tfrac{3003}{256}(-5\lambda_0 + 105\lambda_2 - 315\lambda_4 + 231\lambda_6).$$

(13) *Illustration VI.* In order to thoroughly test the manner in which successive parabolas fit more and more closely to a series of observations, I have taken as a first illustration a very unpromising series of observations given by Thiele in his *Forelaesninger over Almindelig Iagttagelseslaere* (Kjøbenhavn, 1889), p. 12. I say *unpromising* because the observations are not such as one would in practice attempt to fit with a parabolic curve; they form a frequency distribution for which my skew frequency curve of limited range gives a very good fit as we have seen above in § 6. But to fit these unpromising observations even approximately is of great interest; the process shows us much more clearly than would otherwise be the case the struggles of the successive parabolas to get their points of inflexion to the approximately correct positions, and, to speak anthropomorphically, their rather futile attempts to bend themselves into the shape of the observation curve. But a still more important principle is illustrated when we compare these parabolas with the generalised frequency curve, namely that the number of constants at our disposal is no measure of the goodness of the fit. The skew frequency curve with three constants fits much better than the parabola of the sixth order with seven constants. Thus in fitting an empirical curve to observations it is all-important to make a suitable choice of that curve; i.e. to determine whether it should be algebraical, exponential, trigonometrical, etc. There is indeed very little to justify the readiness with which in practice a parabola of one or another order is selected to describe the results of observation.

A little consideration will frequently lead to the selection of a curve with as few or even fewer constants giving a far better fit.

Thiele's observations are given in § 6. We shall attempt to fit a parabola-series to Thiele's trapezoidal polygon between $x = 6$ and $x = 20$, i.e. we shall take $2l = 14$.

The origin for moments was taken at $x = 13$, and the successive moments $m_0\mu_1'$, $m_0\mu_2'$, $m_0\mu_3'$, $m_0\mu_4'$, $m_0\mu_5'$ and $m_0\mu_6'$ calculated for the system of trapezia from the concentrated y's by the formulæ

$$\begin{aligned} \mu_1' &= \nu_1', & \mu_4' &= \nu_4' + \nu_2'c^2 + \tfrac{1}{15}c^4, \\ \mu_2' &= \nu_2' + \tfrac{1}{6}c^2, & \mu_5' &= \nu_5' + \tfrac{5}{3}\nu_3'c^2 + \tfrac{1}{3}\nu_1'c^4, \\ \mu_3' &= \nu_3' + \tfrac{1}{2}\nu_1'c^3, & \mu_6' &= \nu_6' + \tfrac{5}{2}\nu_4'c^2 + \nu_2'c^4 + \tfrac{1}{28}c^6, \end{aligned}$$

where c is the base element, or in our case unity, and

$$\nu_s' = S\{y(x-13)^s\}.$$

These formulæ are deduced in my memoir on "Skew Variation in Homogeneous Material," *Phil. Trans.* Vol. 186, A, p. 349*. See also § 10 above.

Then $\lambda_s = \dfrac{\mu_s'}{l^s}$ was calculated, and the following values obtained:

$$\begin{aligned} \lambda_1 &= -\cdot162{,}857, & \lambda_2 &= \cdot114{,}748, \\ \lambda_3 &= -\cdot033{,}778, & \lambda_4 &= \cdot030{,}712, \\ \lambda_5 &= -\cdot010{,}204, & \lambda_6 &= \cdot012{,}141. \end{aligned}$$

In addition we have $y_0 = 35\cdot7143$,

Mean $x = 11\cdot86$.

From these values the e's were calculated and the following series of parabolas obtained, x being measured from the mid-range:

(i) $y = 35\cdot7143\,\{1 - \cdot488{,}571\,(x/l)\}$,

(ii) $y = 35\cdot7143\,\{1\cdot819{,}694 - \cdot488{,}571\,(x/l) - 2\cdot459{,}082\,(x/l)^2\}$,

(iii) $y = 35\cdot7143\,\{1\cdot819{,}694 - 2\cdot166{,}885\,(x/l) - 2\cdot459{,}082\,(x/l)^2 + 2\cdot797{,}191\,(x/l)^3\}$,

(iv) $y = 35\cdot7143\,\{2\cdot086{,}513 - 2\cdot166{,}885\,(x/l) - 5\cdot127{,}275\,(x/l)^2 + 2\cdot797{,}191\,(x/l)^3 + 3\cdot112{,}892\,(x/l)^4\}$,

(v) $y = 35\cdot7143\,\{2\cdot086\,513 - 4\cdot026{,}295\,(x/l) - 5\cdot127{,}275\,(x/l)^2 + 11\cdot474{,}432\,(x/l)^3 + 3\cdot112{,}892\,(x/l)^4 - 7\cdot809{,}518\,(x/l)^5\}$,

(vi) $y = 35\cdot7143\,\{2\cdot041{,}057 - 4\cdot026{,}295\,(x/l) - 4\cdot172{,}701\,(x/l)^2 + 11\cdot474{,}432\,(x/l)^3 + \cdot249{,}170\,(x/l)^4 - 7\cdot809{,}518\,(x/l)^5 + 2\cdot100{,}062\,(x/l)^6\}$.

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* These are not the proper formulæ if we considered Tbiele's observations as the areas of a frequency curve, but what we are here doing is to fit a series of curves as closely as possible to a polygonal area.

The ordinates for these curves corresponding to the original observations, i.e. $\pm x/l = 0$, 1/7, 2/7, 3/7, 4/7, 5/7, 6/7, 1, were ascertained, and are given in the accompanying Table. The curves are plotted in Fig. 9.

Ordinates of Thiele's Observations.

Actual Numbers		Skew Frequency	Parabolas					
			1st	2nd	3rd	4th	5th	6th
6	*0*	·06	18·3	−40·3	−·3	25·1	−10·3	20·7
7	*8*	1·89	20·8	−14·5	−3·0	−3·4	5·8	−7·8
8	*7*	13·55	23·25	7·7	1·3	−8·8	4·8	14·4
9	*35*	42·26	25·7	26·3	10·7	1·0	3·9	50·5
10	*101*	77·29	28·2	41·4	23·6	19·3	11·2	79·7
11	*89*	97·49	30·7	52·8	38·0	40·5	28·25	92·9
12	*94*	93·54	33·2	60·7	52·4	60·1	51·5	89·2
13	*70*	72·76	35·7	65·0	65·0	74·5	74·5	72·9
14	*46*	47·94	38·2	65·7	74·0	81·6	90·2	50·5
15	*30*	27·60	40·7	62·8	77·6	80·1	92·4	28·8
16	*15*	14·21	43·2	56·3	74·2	69·9	78·0	12·9
17	*4*	6·46	45·7	46·3	61·9	52·2	49·3	5·1
18	*5*	2·88	48·2	32·6	39·05	28·9	15·3	3·9
19	*1*	1·16	50·7	15·4	3·9	3·4	−5·8	3·7
20	*0*	·44	53·2	−5·4	−45·3	−19·9	15·5	−5·1

Taking the 6th parabola as the best let us compare the results found from it with those obtained from the skew curve. Let Δ_1 be the difference from observation in the latter case, Δ_2 in the former. We find

Δ_1	Δ_2	Δ_1	Δ_2
− ·1	− 20·7	− 1·9	− 4·5
+ 1·1	+ 10·8	+ 2·4	+ 1·2
− 6·6	− 7·4	+ ·8	+ 2·1
− 7·3	− 15·5	− 2·5	− 1·1
+ 23·7	+ 21·3	+ 2·1	+ 1·1
− 8·5	− 3·9	− ·2	− 2·7
+ ·5	+ 4·8	− ·4	+ 5·1
− 2·8	− 2·9		

Now whether we measure the goodness of fit by the mean Δ without regard to sign, by the mean square error, or by the value of $S(\Delta^2/y)$, we reach the same result,—there is an overwhelming balance in favour of the exponential curve over the algebraic curve. We can make as Thiele* actually does a curve of factorials or even a parabola of the 15th order to go through all the 15 observations, but although we shall thus of course get a better fit than by using a three-constant curve the fit is purely artificial; a little beyond the range of the actual

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* *Forelaesninger over Almindelig Iagttagelseslaere* (Kjøbenhavn, 1889), p. 12.

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On an Inversion of Ideas as to the Structure of the Universe

(The Rede Lecture, June 10, 1902)

by

Osborne Reynolds
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Professor of Engineering in the
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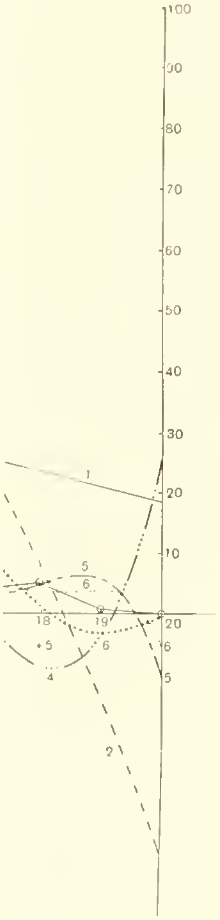

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Actual Numbers	
6	0
7	3
8	7
9	35
10	101
11	89
12	94
13	70
14	46
15	30
16	15
17	4
18	5
19	1
20	0

Taking the 6th
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* *Forelaesninger over Almindelig Iagttagelseslaere* (Kjøbenhavn, 1889), p. 12.

curve, the fit is purely artificial: a little beyond the range of the actual observations, the parabola will diverge immensely from the sort of result that we could possibly reach if we multiplied our observations so as to get, say, the frequency at $x=5$, or $x=23$. The representation is artificial, the parabolic curve cannot give the limited range, or the high contact at its terminals,—well-known characters of such frequency distributions—which can be provided by other curves with, perhaps, a quarter the number of constants. Hence the sort of statement so frequently heard,—"Yes, of course, more constants make better fits,"—is only a half truth, and the manner in which engineers, physicists and actuaries so readily use parabolic curves is open to considerable criticism. There are often considerations, lying outside the actual data, which suffice to indicate that trigonometrical, exponential or other types of curves will give better results than parabolas. A parabola which passes even through all the observations may indeed

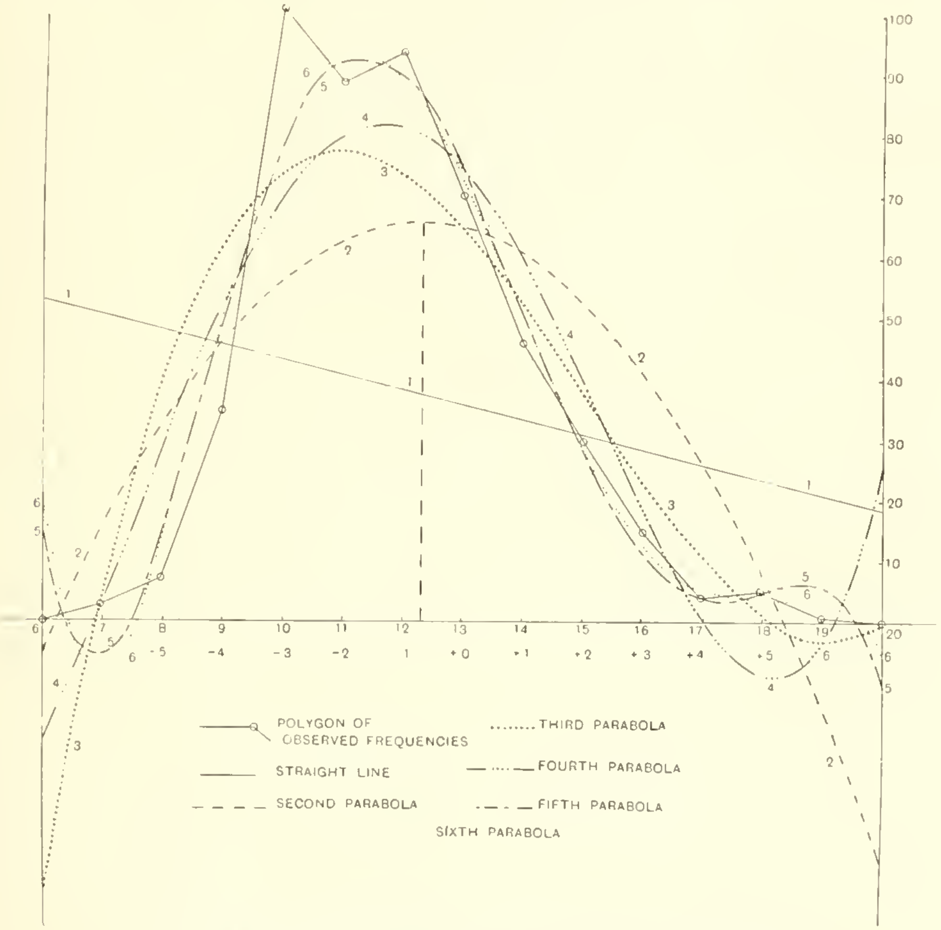

FIG. 9. Example of fitting parabolas (Thiele's frequency observations).

be a most undesirable representation of the facts, for it has twisted and curled to account for error as well as to give the general sweep of the observations.

(14) *Illustration VII.* As a second illustration of parabola-fitting I will take from the Italian marriage statistics of Perozzo the modal ages of bridegrooms of brides of given ages. With some simple interpolations I have determined these from Perozzo's tables* as approximately the following:

Age of Bride	Probable Age of Groom	Age of Bride	Probable Age of Groom
15·5	25	31·5	33
16·5	25·2	32·5	33·5
17·5	25·4	33·5	34
18·5	25·5	34·5	34·5
19·5	25·5	35·5	36
20·5	25·5	36·5	37
21·5	25·75	37·5	38·5
22·5	26	38·5	39·5
23·5	26	39·5	41·5
24·5	26·8	40·5	41·5
25·5	27	41·5	42·5
26·5	27·5	42·5	43·5
27·5	28	43·5	43·5
28·5	29	44·5	43·5
29·5	30	45·5	43·5
30·5	32		

Now let us take 43·5 as the origin of age for the man, and 30·5 as that for the woman; then if y be the man's age and x the woman's, we have the following series of points:

$x =$ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,
$y =$ 11·5, 10·5, 10, 9·5, 9, 7·5, 6·5, 5, 4, 2, 2, 1, 0, 0, 0, 0,
$x =$ − 1, − 2, − 3, − 4, − 5, − 6, − 7, − 8, − 9, − 10, − 11, − 12, − 13, − 14, − 15,
$y =$ 13·5, 14·5, 15·5, 16, 16·5, 16·7, 17·5, 17·5, 17·75, 18, 18, 18, 18·1, 18·3, 18·5.

The range is thus $2l = 30$, and concentrating and then using the corrective formulæ for trapezoidal polygons given in § 10 we easily find

$$y_0 = 10{\cdot}7867,$$

$$\lambda_1 = -{\cdot}348{,}5000, \qquad \lambda_4 = {\cdot}173{,}6922,$$

$$\lambda_2 = {\cdot}299{,}9564, \qquad \lambda_5 = -{\cdot}137{,}7881,$$

$$\lambda_3 = -{\cdot}198{,}1573, \qquad \lambda_6 = {\cdot}123{,}0257.$$

The c's were then determined from the λ's by the formulæ of § 12 and we have for the series of parabolas, if $l = 15$ years:

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* *Reale Accademia dei Lincei.* Anno CCLXXIX, 1882. Nuove Applicazioni del Calcolo delle Probabilità.

1st parabola:

$$y = 10\cdot7867\left\{1 - 1\cdot0455\left(\frac{x}{l}\right)\right\}.$$

2nd parabola:

$$y = 10\cdot7867\left\{1\cdot125{,}1635 - 1\cdot0455\left(\frac{x}{l}\right) - \cdot375{,}4905\left(\frac{x}{l}\right)^2\right\}.$$

3rd parabola:

$$y = 10\cdot7867\left\{1\cdot125{,}1635 - 1\cdot332{,}7459\left(\frac{x}{l}\right) - \cdot375{,}4905\left(\frac{x}{l}\right)^2 + \cdot478{,}7431\left(\frac{x}{l}\right)^3\right\}.$$

4th parabola:

$$y = 10\cdot7867\left\{1\cdot159{,}1392 - 1\cdot332{,}7459\left(\frac{x}{l}\right) - \cdot715{,}2475\left(\frac{x}{l}\right)^2 + \cdot478{,}7431\left(\frac{x}{l}\right)^3 + \cdot396{,}3832\left(\frac{x}{l}\right)^4\right\}.$$

5th parabola:

$$y = 10\cdot7867\left\{1\cdot159{,}1392 - 1\cdot428{,}4973\left(\frac{x}{l}\right) - \cdot715{,}2475\left(\frac{x}{l}\right)^2 + \cdot925{,}5752\left(\frac{x}{l}\right)^3 + \cdot396{,}3832\left(\frac{x}{l}\right)^4 - \cdot402{,}1555\left(\frac{x}{l}\right)^5\right\}.$$

6th parabola:

$$y = 10\cdot7867\left\{1\cdot108{,}0239 - 1\cdot428{,}4973\left(\frac{x}{l}\right) + \cdot358{,}1736\left(\frac{x}{l}\right)^2 + \cdot925{,}5752\left(\frac{x}{l}\right)^3 - 2\cdot823{,}8816\left(\frac{x}{l}\right)^4 - \cdot402{,}1555\left(\frac{x}{l}\right)^5 + 2\cdot361{,}5275\left(\frac{x}{l}\right)^6\right\}.$$

The ordinates of these six curves were then calculated for the 31 values of x, and the curves themselves with the observations plotted in Fig. 10. We have again an instructive graphical representation of the closer and closer approach of a series of parabolas to a fairly smooth system of observations. It will be seen that the parabola of the 4th order gives quite an excellent representation of the observations, better indeed than the 6th parabola which has too many points of inflexion to dispose of.

(15) With this illustration I close my discussion for the present of curve-fitting. I have endeavoured to show:

(1) that the method of moments must theoretically give good fits;

(2) that it provides a systematic method of fitting a great variety of curves;

(3) that it is over and over again available when the method of least squares fails, or can only be applied with excessive labour;

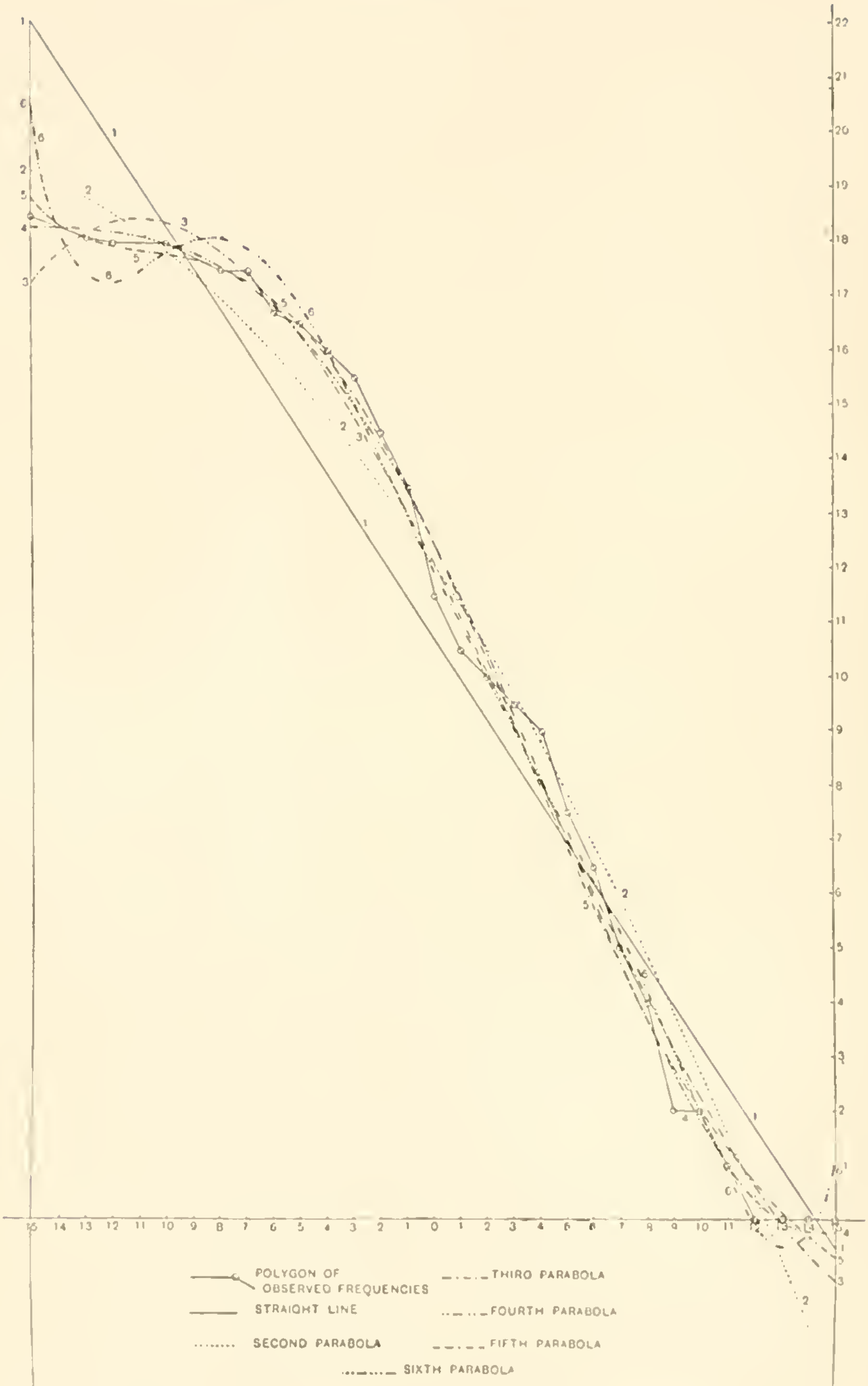

FIG. 10. Example of fitting parabolas (Italian marriage statistics).

(4) that in all cases much depends on the method by which the successive moments are calculated, and that frequency observations and physical measurements require to some extent different processes;

(5) that with a good moment formula the method of moments will give for practical purposes as good fits as the method of least squares when both are applicable;

(6) that for fitting parabolas the method of moments has theoretically the same basis as that of least squares, but in application it is much easier;

and lastly:

(7) that far more really depends for goodness of fit on the equation to the curve selected than on the number of its constants, and that the multiplication of constants to improve fit is not only theoretically undesirable, but does not necessarily lead to the required result.

QUANTITATIVE STUDY OF THE EFFECT OF ENVIRONMENT UPON THE FORMS OF NASSA OBSOLETA AND NASSA TRIVITTATA FROM COLD SPRING HARBOR, LONG ISLAND.

By ABIGAIL CAMP DIMON.

(1) *Introductory.* The aims of this paper are to make a quantitative inquiry into the effect of diverse environmental conditions upon the form of two gastropod species, *Nassa obsoleta* and *Nassa trivittata*, from Cold Spring Harbor, Long Island, and also to record the characteristics of their shells and thus to determine the "place mode" for these shells in that locality. The characters selected for measurement were those described in systematic works as distinguishing the species as far as those characteristics could be easily determined quantitatively. The work was done under the general direction of Dr Charles B. Davenport.

(2) *General Description of Nassa, its range and habits.* *Nassa* is a genus of prosobranch gastropods containing many species distributed over the whole world, chiefly in shallow water The individuals are usually small, with an ovate shell and a large foot, which is notched behind and carries a horny operculum. The two species, *Nassa obsoleta*, Say (*Ilyanassa obsoleta*, Stimpson; *Buccinum obsoletum*, Gould), and *Nassa trivittata*, Say (*Tritia trivittata*, Adams; *Buccinum trivittatum*, Gould), found commonly at Cold Spring Harbor, are American forms, and with the less common *Nassa vibex* constitute the only recognised littoral species of the genus found on the middle Atlantic coast of the United States. Both species range from the Gulf of St Lawrence to Florida. Verrill ('73) reports *N. trivittata* as abundant at Casco Bay, Maine, and in Vineyard Sound and Buzzard's Bay, and as common along Long Island Sound: whereas *N. obsoleta* he reports as very abundant south of Cape Cod and more local further north. *N. trivittata*, therefore, reaches its maximum numbers further north than *N. obsoleta*. Geologically, *N. trivittata* is older than *N. obsoleta*, having been found in the Miocene of Maryland, Virginia and South Carolina, while *N. obsoleta* has not been reported from further back than the Pliocene.

Comparing in detail the habitats of the two species, it is seen that *N. obsoleta* lives in large numbers on flats and shores left bare by the tide during part of the day, and is not found at any considerable depth; while *N. trivittata* is found in some places at a depth of forty fathoms, and is not found above low water line. Verrill ('73) states that about Vineyard Sound *N. obsoleta* occurs in bays and sounds on sandy and muddy shores and bottoms, and on submerged wood-work, such as the piles of wharves, but that it does not occur on rocky shores and bottoms. *N. obsoleta* was found in brackish water on sand, mud, oyster-beds, eel-grass, and submerged wood-work; but it was not found in the open ocean. *N. trivittata* was found by Verrill in all the habitats of *N. obsoleta* except the muddy shores of the bays and sounds and the muddy shores and bottoms, oyster-beds and eel-grass in the brackish water; it was found in the open ocean on shores and bottoms. With these observations my own experience with the distribution of the two mollusca at Cold Spring Harbor and elsewhere is in complete accord.

The breeding season of *N. obsoleta* is given by Mead ('98) for Woods Holl as the latter part of April. In July and August when nearly all my collecting was done there would often be found in pools left above low water mark large numbers of very small individuals which were evidently the brood of that year. The older snails, however, could not be separated into broods of different ages on the basis either of size or the number of whorls, so that either the growth after the first year is extremely slow or else the snails do not live until the third summer.

(3) *Localities from which the shells were collected.* At Cold Spring Harbor we have, within a small district, several localities in which *Nassa* may be found. The Harbor is a branch of Long Island Sound, five miles long by one mile wide. It is divided into an inner and an outer harbor by a sand-spit that extends nearly across it at about half a mile from its head. The inner harbor is fed by a stream which makes its water decidedly brackish, especially at the surface. For near the mouth of the creek, at the surface, the density may be as low as 1·006 while at the bottom it is 1·016. Under other conditions of wind and tide the density will be about 1·016 throughout. The average height of tide is about 7·5 feet in the inner harbor, and at low tide about half the surface is left bare, exposing flats of black mud on which *Ulva* grows in abundance. In the outer harbor the density of the water is from 1·018 to 1·020. The bottom of the outer harbor consists of mud, oyster-beds or sand, with a good deal of eel-grass; the shores are sandy or muddy with a greater slope than those of the inner harbor, so that there are no extended mud-flats exposed at low tide. At the mouth of the harbor, on the east, the shores are sandy and gravelly and have a considerable slope, and the situation is far less sheltered than within the harbor.

The *Nassa* measured were collected from the three localities marked 1, 2, and 3 on the map, Figure 1. Those from 1 (Laboratory Dock) were gathered from the mud-flats at low tide; those from 2 (Laurelton Dock) were taken from the sandy beach at low tide or dredged from a few feet of water at the same locality; and those from 3 (Lloyd Point) were collected from pools left on the sand beach at low tide. It was not possible to find *N. trivittata* in abundance in all three localities, so this species was taken only from 3, where the shells were washed up on the beach in considerable numbers and easily collected. Only one or two specimens were dredged from locality 2, and no specimen has been found in the inner harbor.

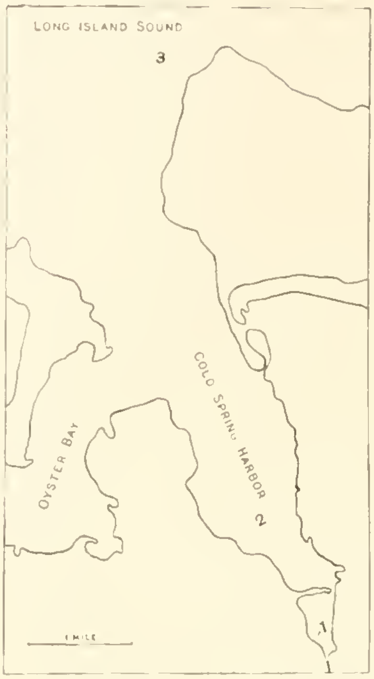

Fig. 1. Sketch Map of Cold Spring Harbor.

(4) *Qualitative comparison of the shells from the different localities.* If handfuls of *N. obsoleta* from each of the three localities marked on the map be compared a decided difference in size will be at once noticed. The shells from 3, the most exposed locality, are much the largest. On the other hand they are much more numerous in the inner harbor. In comparing *N. trivittata* with *N. obsoleta* it is seen that the shells of *N. trivittata* are not covered with algae and are not eroded, while the shells of *N. obsoleta* are covered by algae and much eroded at tip, probably in consequence. Also *N. trivittata* is almost white, whereas *N. obsoleta* is blackish purple, its apical angle is more acute, and its shell is smaller.

(5) *Method of measuring.* The characteristics of the shells of which it was sought to get a quantitative expression were size, shape, number of whorls, color, and roughness of surface. To ensure accuracy the dimensions of the shell and the angle at its apex were all measured two or three times. On account of the length of the process or the great effect of a previous reading on the judgment

the determinations of the number of whorls, color, and roughness were made only once, so these determinations are less accurate. To measure length a micrometer gauge reading to hundredths was used. In Figure 2 A, the line *AD* represents the direction in which length was measured, *B* the direction for diameter, and *C* for greatest length of aperture. The shape of the shell was given by the angle at the apex and the ratios of the diameter to the length and of the aperture to the length. The ratios were calculated from the measurements; the angle was measured directly by means of a bevel protractor. The erosion of the apex disturbed the measurement of the angle in the case of *N. obsoleta*, and it was finally decided to take an angle the direction of each side of which would be a compromise between the directions of lines drawn between the center points of successive whorls. *D*, Figure 2 A, shows such a compromise between lines 1, 2, 3, on one side, and 4, 5, on the other, and represents what I have called the apical angle. Even after the more eroded shells had been discarded, the bluntness of the apex often affected the angle, the general tendency being to read it too large, as shown in Figure 2 B. The mean angles of shells from the different localities are therefore to be compared only with caution.

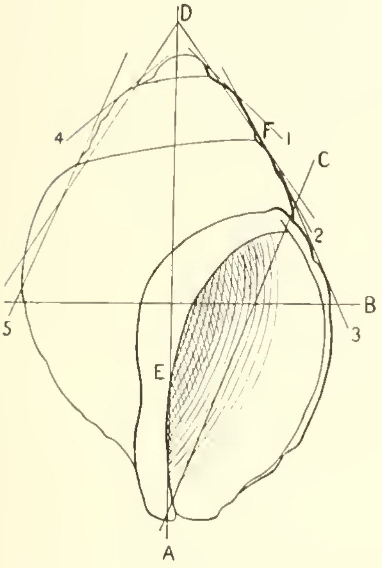

Fig. 2 a.

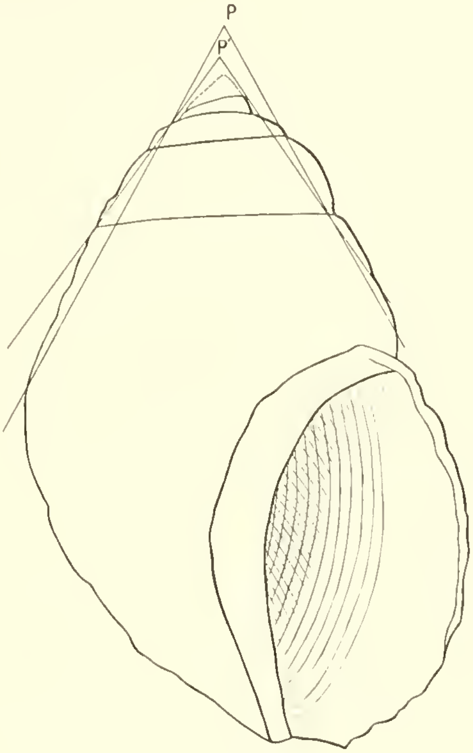

Fig. 2 b.

The number of whorls was counted directly. Color was measured by the color top in the region *E*, Figure 2 A; the results are, however, rather rough. The depth of the suture between the whorls was chosen as an approximate measure of roughness. It was measured between the last two whorls, near *F*, Figure 2 A, by laying the shell so that it rested on its aperture and making a camera drawing magnified eight or nine times of the part of the line it was desired to measure. Then the depth of the depression from a tangent drawn to the adjoining elevations was measured on the drawing, and this distance was divided by the magnification of the figure. The dimension thus obtained depends, however, not solely upon the roughness of the shell but also upon the convexity of the individual whorls. Figure 3 shows some types of outlines obtained in the manner described; *b* represents an extreme case of a smooth shell with a deep suture.

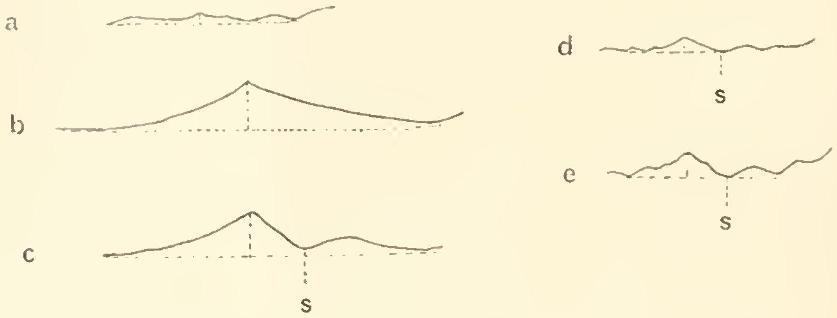

FIG. 3. Camera drawings illustrating the depth of suture. S = shoulder.

(6) *Table of constants of the curves.* In the following table some of the results of the quantitative determinations are given. 1, 2, and 3 represent lots of *N. obsoleta* from the localities referred to; 3*a* is the lot of *N. trivittata* from Lloyd Point.

Discussion of constants. Means. Setting aside for the present *N. trivittata*, a comparison of the means of *N. obsoleta* from the three localities brings out certain points. The shells from 3, the mouth of the harbor, are much longer than the others; those from 1, the inner basin, are the most globose, and have the largest aperture in proportion to their size, and the fewest whorls. In depth of suture the shells from the mouth of the harbor are extreme. The probable error of the color measurement is so great that no stress can be laid on the slight differences recorded. As to the apical angle, the effect of erosion at the apex upon this reading is, as already stated, so considerable that lot 1 (the most eroded) is not strictly comparable with the other lots.

If we correlate these differences with the environmental differences noted above for the three localities, we find the individuals living in less dense water—in the inner harbor—are smaller, more globose, with a larger aperture in proportion to their length, and a shallower suture. Though the series is not in each case a regular one, with the shells from 1 and 3 at the extremities and those from 2 intermediate, yet the general relationship holds. The tendency of the shells from 2 toward an extreme position is not necessarily inconsistent. Cooke ('95)

TABLE I.

Quantitative data. Table of Constants of the Curves.

	Mean (M)	Probable Error of M	Observed Mode	Theoretical Mode	Standard Deviation (σ)	Probable Error of σ	Coefficient of Variability	Type	Skewness
Length: *1*	13·92 mm.	± ·0336 mm.	13·50 mm.	13·79 mm.	1·120 mm.	± ·0238 mm.	7·86 °/o	IV	+ ·1206
2	12·75 ,,	± ·0372 ,,	13·00 ,,	12·76 ,,	1·232 ,,	± ·0263 ,,	9·10 °/o	IV	− ·0051
3	17·14 ,,	± ·0664 ,,	17·00 ,,	16·46 ,,	1·890 ,,	± ·0469 ,,	11·03 °/o	I	+ ·3590
3a	11·96 ,,	± ·0814 ,,	12·00 ,,	12·30 ,,	2·590 ,,	± ·0575 ,,	21·66 °/o	I	− ·1328
$\frac{\text{Diameter}}{\text{Length}}$: *1*	59·74 °/o	± ·109 °/o	57·50 °/o	59·09 °/o	3·614 °/o	± ·0773 °/o	6·05 °/o	I	+ ·1796
2	58·30 °/o	± ·103 °/o	58·50 °/o	58·37 °/o	3·416 °/o	± ·0760 °/o	5·86 °/o	IV	− ·0216
3	58·71 °/o	± ·099 °/o	59·00 °/o	55·56 °/o	2·825 °/o	± ·0714 °/o	4·82 °/o	IV	+ ·0608
3a	52·14 °/o	± ·118 °/o	51·50 °/o	51·30 °/o	3·740 °/o	± ·0835 °/o	7·17 °/o	IV	+ ·2238
$\frac{\text{Aperture}}{\text{Length}}$: *1*	64·12 °/o	± ·107 °/o	64·50 °/o	64·25 °/o	3·522 °/o	± ·0754 °/o	5·49 °/o	IV	− ·0354
2	62·32 °/o	± ·080 °/o	62·50 °/o	62·63 °/o	2·646 °/o	± ·0565 °/o	4·25 °/o	IV	− ·1178
3	62·71 °/o	± ·092 °/o	63·00 °/o	62·84 °/o	2·626 °/o	± ·0652 °/o	4·19 °/o	IV	− ·0481
3a	51·24 °/o	± ·086 °/o	49·00 °/o	50·87 °/o	2·727 °/o	± ·0608 °/o	5·32 °/o	IV	+ ·1350
Apical angle: *1*	53·44°	± ·176°	57·00°	53·93°	5·832°	± ·1241°	10·91 °/o	I	− ·0844
2	55·46°	± ·151°	57·00°	55·79°	5·046	± ·1071°	9·10 °/o	I	− ·0659
3	53·09°	± ·200°	53·00°	53·30°	5·702°	± ·1416°	10·74 °/o	I	− ·0361
3a	47·48°	± ·129	48·00°	47·17°	4·102°	± ·0910°	8·64 °/o	I	+ ·0766
Number of whorls: *1*	4·88	± ·017	5·00	4·63	·560	± ·0120	11·48 °/o	I	+ ·3790
2	5·49	± ·015	5·25	5·43	·496	± ·0106	9·03 °/o	IV	+ ·1161
3	5·34	± ·014	5·38	5·36	·404	± ·0102	7·57 °/o	I	− ·0379
3a	7·01	± ·019	6·87	7·16	·595	± ·0113	8·49 °/o	IV	− ·2480
P. c. of Red in color: *1*	10·44 °/o	± ·063 °/o	10·00 °/o	—	2·063 °/o	± ·0447 °/o	19·76 °/o	—	—
2	10·09 °/o	± ·076 °/o	9·50 °/o	—	2·546 °/o	± ·0542 °/o	25·17 °/o	—	—
3	10·51 °/o	± ·065 °/o	10·00 °/o	9·20 °/o	1·860 °/o	± ·0462 °/o	17·29 °/o	I	+ ·7018
P. c. of Black in color: *1*	88·44 °/o	± ·072 °/o	90·00 °/o	—	2·336 °/o	± ·0506 °/o	2·64 °/o	—	—
2	88·38 °/o	± ·084 °/o	89·50 °/o	—	2·798 °/o	± ·0596 °/o	3·17 °/o	—	—
3	88·11 °/o	± ·083 °/o	89·00 °/o	88·92 °/o	2·352 °/o	± ·0585 °/o	2·12 °/o	I	− ·3465
Depth of suture: *1*	25·98 $\frac{\text{mm.}}{100}$	± ·258 $\frac{\text{mm.}}{100}$	24·00 $\frac{\text{mm.}}{100}$	22·80 $\frac{\text{mm.}}{100}$	8·157 $\frac{\text{mm.}}{100}$	± ·1822 $\frac{\text{mm.}}{100}$	31·40 °/o	I	+ ·3895
2	19·83 ,,	± ·194 ,,	18·50 ,,	17·61 ,,	6·423 ,,	± ·1405 ,,	32·39 °/o	IV	+ ·3458
3	33·35 ,,	± ·418 ,,	28·00 ,,	27·20 ,,	11·890 ,,	± ·2956 ,,	35·65 °/o	I	+ ·5177
3a	42·15 ,,	± ·299 ,,	43·00 ,,	41·75 ,,	9·530 ,,	± ·2117 ,,	22·61 °/o	IV	+ ·0420

has noted that where closely related forms live in brackish water and in sea water, the forms in brackish water will in general have a less solid shell than the forms in sea water, yet as the water becomes still more dilute, the forms inhabiting it have more solid shells. Possibly there are conditions of the same kind holding for the size and shape of the shells from the three localities. The stunting influence, for example, may be checked to some degree in the inner harbor, because there the food supply is most abundant.

The effect of density of water on the size and shape of animals living therein has been often noticed. Bronn ('62), in a section on the effect of salt in the water, states that in the Baltic, containing only 4—6 p.m. salt, *Buccinum undatum* and *Littorina litorea* live, but are always small and stunted. In the Mediterranean, containing 37 p.m. salt, oysters are small: at 30—20 p.m. they are, if not largest, of the best flavor; and at 18—17 p.m. they are stunted: which would seem to show an optimum content of salt, above and below which the animals are dwarfed. Simroth ('91) gives as the peculiarity of brackish water fauna that fewer species but more individuals are present; and quotes Möbius and Heincke to the effect that fewer species *of smaller size* are characteristic. Walther ('93) says that the less dense water of the Baltic dwarfs many of the animals living therein, and he gives illustrations among mollusks and worms. Bumpus ('98) in an article on the variations of *Littorina litorea* gives statistics that show that the more dense the water the less globose are the shells of the animals living therein. The means of the curves given by Bumpus in his paper, as calculated by Duncker ('98), express more nearly the condition of the whole population than do Bumpus's own figures, and are therefore used in comparison. The water at Bristol Narrows and Warren River, in Rhode Island, is less dense than the water at Seaconnet and Newport. The globosities are 90·77 °/₀ and 91·08 °/₀ at Bristol Narrows, 92·69 °/₀ at Warren River, 89·72 °/₀ at Seaconnet, and 89·18 °/₀ at Newport; showing decidedly greater globosity in the less dense water. The data as to relative densities of the water in other localities from which Bumpus collected shells are not given, so no further conclusions can be drawn. The foregoing evidence is in accord with the observations noted above on *N. obsoleta*.

If the means of the different characteristics of *N. trivittata* be compared with the means of *N. obsoleta*, the observations that were made qualitatively will be verified. The shells of *N. trivittata* are rougher, smaller, less globose than the shells of *N. obsoleta*, and with a more acute angle at the apex. The range of each character measured overlaps considerably, and even the means are not very far apart.

(7) *Comparison of Cold Spring Harbor type with the normal type.* If we compare the dimensions of the *Nassa* of Cold Spring Harbor with typical measurements given by Apgar ('91) and Gould ('70) for the two species, we find various interesting differences. After looking, by the courtesy of Professor Pilsbury, over the collection of *Nassa* shells from different localities in the museum of the Philadelphia Academy of Sciences, it seemed fair to accept an

average between the dimensions given by Apgar and Gould as representing the normal type of *Nassa*, found in more exposed situations than Cold Spring Harbor. The second and fourth columns of Table II. give this average for *N. obsoleta* and *N. trivittata* respectively, while the first column gives an average of the means of *N. obsoleta* from localities 1, 2, and 3, and the third column gives the means of *N. trivittata* from 3. Table II. shows that *the Cold Spring Harbor shells of N. obsoleta are plainly depauperate,—smaller, more globose, and with a larger aperture and fewer whorls than the normal. All these signs of depauperization may be associated with the lack of density of the water, and the sheltered situation as compared with the open ocean.*

TABLE II.

Comparison of Cold Spring Harbor type of snails with the normal type.

	Cold Spring Harbor *N. obsoleta*	Normal *N. obsoleta*	Lloyd Point *N. trivittata*	Normal *N. trivittata*
Length	14·6 mm.	25·0 mm.	12·0 mm.	17·7 mm.
Diameter	8·6 „	12·3 „	6·2 „	7·8 „
$\frac{\text{Diameter}}{\text{Length}}$	59 %	49 %	52 %	44 %
$\frac{\text{Aperture}}{\text{Length}}$	63 %	50 %	51 %	40 %
Angle	54°	52·5°	47·5°	45°
Number of Whorls	5·5	6·0	7·0	7·0

This depauperization is evident in the case of *N. trivittata* as well as *N. obsoleta*, and may be explained in the same way. Long Island Sound, as a whole, presents conditions different from those found in the open ocean, its water being less dense on account of the rivers that empty into it, and its situation more sheltered. *N. trivittata*, in fact, is found much more abundantly in the open ocean, and the locality from which the lot measured was collected represents the furthest limit of its intrusion into sheltered, relatively fresh water, in which respect it may be compared with locality 1 as a habitat for *N. obsoleta*. It is what we should expect, therefore, that *the N. trivittata from Cold Spring Harbor show the same relation to normal N. trivittata as the N. obsoleta from locality* 1 *show to normal N. obsoleta.*

(8) *Coefficients of variability.* In considering the coefficients of variability it must be kept in mind that comparisons must be made with caution. Thus, for example, the variability of red is given in Table I. as from 17 % to 25 %, and the variability of black as from 2 % to 3 %. It is obviously incorrect to say that red is from eight to twelve times as variable as black, so that here the coefficient of variability has evidently a different significance in the case of the two colors.

Likewise it cannot be assumed that the coefficient of variability has the same significance for any two characteristics.

If now we run over the coefficients of variability we see that the most variable character is depth of suture, where the variability is about one-third of the depth. It is interesting to observe that this most variable character is to general observation one of the most distinctive of all those measured. That is, it is what Brewster ('99) would call a "conspicuous" character, and to the degree to which it represents roughness of the shell it would have great weight in determining the species. Its great variability is, therefore, in accord with the thesis of Brewster ('97) that "any measurable quality is, in general, variable in individuals in proportion as it is a distinguishing character of the group to which the individuals belong," though the variability is in this case much more extreme than in the case of any evidence Brewster offers.

(9) *Correlations.*

Table III. shows the amount of correlation between some of the characters. The characters of *N. trivittata* show a much greater amount of correlation than those of *N. obsoleta*, and the greatest amount of correlation is shown between the angle and the ratio of diameter to length (which we have called globosity). Since globosity is the most important factor in determining the apical angle, it is natural that the correlation between these two characters should be great. The relatively small correlation between length and number of whorls leads us to look for other factors in the determination of length than number of whorls. These factors we find in the size of the whorls and the inclination of the axis of the whorls to the axis of the shells. The values of the correlations between the characters length, index, and apical angle are largely determined, not only in magnitude but in sign, by the fact that, other things being constant, the index

TABLE III.

The Amount of Correlation between Various Characteristics.

Locality and Species	First Character	Second Character	Coefficient of Correlation
3. N. obsoleta ...	Length	Number of Whorls	·0640
3. N. trivittata ...	,,	,, ,,	·4178
1. N. obsoleta ...	,,	,, ,,	·2860
3. ,,	,,	$\frac{\text{Diameter}}{\text{Length}}$	– ·2641
3. N. trivittata ...	,,	,,	– ·7193
3. ,,	Length	Apical Angle	– ·6577
1. N. obsoleta ...	$\frac{\text{Diameter}}{\text{Length}}$	,,	·4787
3. N. trivittata ...	,,	,,	·7941
3. N. obsoleta ...	Whorls	Angle	·0077

and apical angle vary *inversely* as the length. Hence "spurious correlation" occurs, and we should *a priori* expect index and angle to be negatively correlated with length and positively correlated with each other. The correlation of diameter and length would have to be determined in order to measure the amount of "spurious correlation" between index and length.

(10) *Description of Abnormal Snail.* About one hundred shells from each locality were measured while the animals inhabiting them were still alive, and of this number one animal was seen with the left tentacle double and bearing two eyes, as represented in Figure 4. All the rest of the specimens were killed before measuring, and as the foot was retracted, the tentacles could not be seen. No other sport of any kind was observed. Bateson ('94) has reported cases of doubling of tentacles and eyes as follows: *Patella vulgata*, tentacle and eye repeated on the left side; *Physa acuta*, right tentacle bifid; three cases of a supernumerary eye without doubling of the tentacle; and one case (*subemarginula*) where both eye-stalks were double, and each bore a supernumerary eye. This condition, then, would seem to be not uncommon, and is mentioned and figured merely as adding one more to the cases of similar abnormality already known.

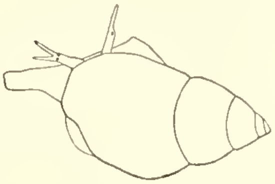

FIG. 4. Freehand outline sketch of snail from locality 2.

Summary.

I. There are in Cold Spring Harbor and vicinity two species of *Nassa*, with no forms intermediate in all characters, though their habitats overlap.

II. When compared with the normal types for the species, both these species are depauperate, being smaller, more globose, with a larger aperture in proportion to their length, more obtuse apical angle; and in the case of *N. obsoleta*, fewer whorls. This depauperization is connected with the slight density of the water in Cold Spring Harbor, as compared with the open ocean.

III. If the three lots of *N. obsoleta* be compared with one another, those from within the harbor are seen to be more depauperate than those nearer its mouth. This depauperization is also connected with the density of the water in the three localities from which the specimens were collected.

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APPENDIX A.

Frequency Data. The class is designated both by a serial number and by its middle absolute value expressed, in the case of length in millimeters, in the ratios of diameter to length and aperture to length in percentages, in the case of angle in degrees, of whorls in units of one whorl, of color in percentages, and of depth of suture in units of one-hundredth of a millimeter. 1, 2, 3, and 3*a*, stand, as in Table I., for the lots of *N. obsoleta* shells from localities 1, 2, and 3, and the *N. trivittata* shells from 3, respectively.

Length: Locality 1. Total frequency, 504.

Class …	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
	10·50	11·0	11·5	12·0	12·5	13·0	13·5	14·0	14·5	15·0	15·5	16·0	16·5	17·0	17·5
Frequency	2	0	1	16	53	65	91	84	82	52	32	9	7	5	2

Length: Locality 2. Total frequency, 499

Class …	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*
	7·5	8·0	8·5	9·0	9·5	10·0	10·5	11·0	11·5	12·0	12·5	13·0	13·5	14·0	14·5	15·0	15·5	16·0	16·5	17·0	17·5
Frequency	1	2	0	1	2	2	8	28	42	83	86	89	64	44	22	13	7	3	0	1	1

Length: Locality 3. Total frequency, 368.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
	13·0	14·0	15·0	16·0	17·0	18·0	19·0	20·0	21·0	22·0
Frequency	5	19	51	68	78	56	46	31	13	1

Length: Locality 3a. Total frequency, 461.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
	5·0	6·0	7·0	8·0	9·0	10·0	11·0	12·0	13·0	14·0	15·0	16·0	17·0	18·0	19·0
Frequency	2	8	15	17	39	50	55	76	71	46	44	28	8	1	1

$\frac{\textit{Diameter}}{\textit{Length}}$: *Locality 1.* Total frequency, 497.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
	51·5	53·5	55·5	57·5	59·5	61·5	63·5	65·5	67·5	69·5	71·5
Frequency	6	19	56	117	104	94	54	28	12	6	1

$\frac{\textit{Diameter}}{\textit{Length}}$: *Locality 2.* Total frequency, 498.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
	46·5	48·5	50·5	52·5	54·5	56·5	58·5	60·5	62·5	64·5	66·5
Frequency	1	0	3	20	50	114	150	100	45	10	5

$\frac{\textit{Diameter}}{\textit{Length}}$: *Locality 3.* Total frequency, 369.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*
	51·0	52·0	53·0	54·0	55·0	56·0	57·0	58·0	59·0	60·0	61·0	62·0	63·0	64·0	65·0	66·0	67·0
Frequency	1	8	2	10	17	41	45	48	64	39	37	28	15	6	1	3	4

$\frac{\textit{Diameter}}{\textit{Length}}$: *Locality 3a.* Total frequency, 456.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
	41·5	43·5	45·5	47·5	49·5	51·5	53·5	55·5	57·5	59·5	61·5	63·5
Frequency	2	1	9	49	103	112	74	48	32	12	12	2

$\frac{\textit{Aperture}}{\textit{Length}}$: *Locality 1.* Total frequency, 497.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
	52·5	54·5	56·5	58·5	60·5	62·5	64·5	66·5	68·5	70·5	72·5	74·5
Frequency	3	3	8	25	54	115	133	83	42	21	9	1

$\frac{\textit{Aperture}}{\textit{Length}}$: *Locality 2.* Total frequency, 499.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
	48·5	50·5	52·5	54·5	56·5	58·5	60·5	62·5	64·5	66·5	68·5
Frequency	1	0	1	2	8	42	124	163	116	34	8

$\frac{\textit{Aperture}}{\textit{Length}}$: *Locality 3.* Total frequency, 369.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*
	54·0	55·0	56·0	57·0	58·0	59·0	60·0	61·0	62·0	63·0	64·0	65·0	66·0	67·0	68·0	69·0	70·0
Frequency	2	1	2	4	8	20	28	51	54	63	54	27	30	12	8	4	1

$\frac{\textit{Aperture}}{\textit{Length}}$: *Locality 3a.* Total frequency, 458.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*
	41·0	42·0	43·0	44·0	45·0	46·0	47·0	48·0	49·0	50·0	51·0	52·0	53·0	54·0	55·0	56·0	57·0	58·0	59·0	60·0	61·0
Frequency	1	0	0	2	1	7	15	27	72	71	63	63	51	35	21	14	7	2	1	4	1

Angle: Locality 1. Total frequency, 502.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*
	35	37	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69	71
Frequency	1	3	3	8	12	24	39	48	56	65	65	73	43	35	14	8	4	0	1

Angle: Locality 2. Total frequency, 505.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*
	41	43	45	47	49	51	53	55	57	59	61	63	65	67
Frequency	3	5	10	21	31	56	61	73	77	73	50	33	9	3

Angle: Locality 3. Total frequency, 369.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
	39	41	43	45	47	49	51	53	55	57	59	61	63	65	67	69
Frequency	2	6	15	21	29	38	35	58	52	40	30	26	9	5	2	1

Angle: Locality 3a. Total frequency, 462.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
	38	40	42	44	46	48	50	52	54	56	58
Frequency	5	19	35	73	77	94	77	36	31	10	5

Whorls: Locality 1. Total frequency, 493.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
	4·00	4·25	4·50	4·75	5·00	5·25	5·50	5·75	6·00	6·25
Frequency	24	75	71	81	110	51	29	23	23	6

Whorls: Locality 2. Total frequency, 494.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
	4·25	4·50	4·75	5·00	5·25	5·50	5·75	6·00	6·25	6·50	6·75
Frequency	1	4	11	62	138	102	94	60	15	4	3

Whorls: Locality 3. Total frequency, 357.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
	4·13	4·38	4·63	4·88	5·13	5·38	5·63	5·88	6·13	6·38
Frequency	1	7	7	65	61	102	55	48	9	2

Whorls: Locality 3a. Total frequency, 439.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
	4·88	5·13	5·38	5·63	5·88	6·13	6·38	6·63	6·88	7·13	7·38	7·63	7·88	8·13	8·38	8·63
Frequency	2	1	2	6	20	13	35	37	86	84	68	42	36	4	2	1

Redness: Locality 1. Total frequency, 484.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
Frequency	11	53	84	150	81	49	23	11	12	2	1	3	2	0	2

Redness: Locality 2. Total frequency, 502.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
	6·5	7·5	8·5	9·5	10·5	11·5	12·5	13·5	14·5	15·5	16·5	17·5	18·5	19·5	20·5	21·5
Frequency	6	87	94	118	73	41	24	19	16	3	8	4	3	4	0	2

Redness: Locality 3. Total frequency, 368.

Class ...	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
	6	7	8	9	10	11	12	13	14	15	16	17
Frequency	2	5	40	47	113	68	44	30	9	8	0	2

Blackness: Locality 1. Total frequency, 484.

Class …	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*
	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93
Frequency	1	1	1	1	4	5	3	8	25	34	49	76	97	106	55	15	3

Blackness: Locality 2. Total frequency, 502.

Class …	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*
	76·5	77·5	78·5	79·5	80·5	81·5	82·5	83·5	84·5	85·5	86·5	87·5	88·5	89·5	90·5	91·5	92·5
Frequency	2	1	2	4	6	7	7	11	16	15	33	65	91	99	73	61	9

Blackness: Locality 3. Total frequency, 368.

Class …	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*
	81	82	83	84	85	86	87	88	89	90	91	92	93	94
Frequency	2	5	6	16	28	27	41	60	73	65	28	14	2	1

Depth of Suture: Locality 1. Total frequency, 456.

Class …	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51
Frequency	6	15	32	48	60	65	64	62	41	28	15	6	8	2	4

Depth of Suture: Locality 2. Total frequency, 498.

Class …	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*
	8	11	14	17	20	23	26	29	32	35	38	41	44	47	50	53	56	59
Frequency	6	42	76	102	102	78	46	15	14	9	4	2	1	0	0	0	0	1

Depth of Suture: Locality 3. Total frequency, 368.

Class …	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
	8	13	18	23	28	33	38	43	48	53	58	63	68	73	78	83
Frequency	1	5	27	67	81	53	47	34	20	11	14	3	3	0	1	1

Depth of Suture: Locality 3a. Total frequency, 461.

Class …	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*
	18	23	28	33	38	43	48	53	58	63	68	73	78
Frequency	4	15	28	53	99	109	66	50	30	3	3	0	1

APPENDIX B.

Correlation Tables. The classes are defined by the class numbers of the variation statistics.

a. Correlation between length and number of whorls of shells of *N. obsoleta* from locality 3.

Length :
Mean = 5·0983.
Standard Deviation = 1·868.

Number of Whorls :
Mean = 5·8286.
Standard Deviation = 1·573.

Correlation = ·0640.

Total frequency = 356.

Number of Whorls

Length	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1	—	—	—	1	1	2	1	—	—	—
2	—	—	—	2	6	6	4	1	—	—
3	—	1	1	11	6	9	13	7	1	—
4	1	2	1	13	12	14	9	12	1	1
5	—	1	2	20	13	23	9	8	2	—
6	—	—	2	9	12	13	4	10	1	—
7	—	3	—	8	6	15	5	4	3	—
8	—	—	1	—	5	14	7	2	1	—
9	—	—	—	1	—	6	3	3	—	—
10	—	—	—	—	—	—	—	—	—	1

b. Correlation between length and number of whorls of shells of *N. trivittata* from locality 3.

Length :
Mean = 7·879.
Standard Deviation = 2·594.

Number of Whorls :
Mean = 9·5548.
Standard Deviation = 2·373.

Correlation = ·4178.

Total frequency = 438.

Number of Whorls

Length	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
1	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—
2	—	—	—	—	5	2	—	1	—	—	—	—	—	—	—	—
3	—	—	1	—	1	2	5	2	3	—	1	—	—	—	—	—
4	—	—	—	1	—	1	5	1	8	1	—	—	—	—	—	—
5	—	—	—	1	—	1	4	10	13	5	2	2	—	—	—	—
6	—	—	—	1	3	1	1	10	14	12	4	2	2	—	—	—
7	1	—	1	—	2	—	2	5	13	14	13	1	1	—	—	—
8	1	—	—	—	1	2	2	4	17	24	12	6	3	1	—	—
9	—	1	—	—	—	—	5	4	10	13	15	9	8	—	—	—
10	—	—	—	2	2	—	3	—	5	7	12	11	1	2	—	—
11	—	—	—	—	2	—	6	—	2	4	4	7	12	—	—	—
12	—	—	—	—	3	3	2	—	—	3	4	2	5	1	1	1
13	—	—	—	—	—	—	—	—	1	1	1	1	3	—	1	—
14	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—
15	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—

c. Correlation between length and number of whorls of shells of *N. obsoleta* from locality 1.

Length :
Mean = 7·856.
Standard Deviation = 2·2151.

Number of Whorls :
Mean = 4·505.
Standard Deviation = 2·1612.

Correlation = ·2860.
Total frequency = 493.

Number of Whorls

Length	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
1	—	—	1	—	—	—	—	1	—	—
2	—	—	—	—	—	—	—	—	—	—
3	—	3	1	—	—	—	—	—	—	—
4	5	3	1	4	1	—	1	—	1	—
5	3	15	10	10	8	—	1	—	3	1
6	8	13	8	10	14	5	1	2	1	—
7	6	14	16	17	19	6	3	2	5	1
8	2	11	11	17	19	11	4	6	1	1
9	—	8	7	11	19	10	11	6	8	—
10	—	5	9	5	16	10	3	1	1	1
11	—	3	2	4	10	4	4	2	1	2
12	—	—	2	1	2	1	—	1	2	—
13	—	—	1	2	—	3	—	1	—	—
14	—	—	2	—	1	1	1	—	—	—
15	—	—	—	—	1	—	—	1	—	—

d. Correlation between length and ratio of diameter to length of shells of *N. obsoleta* from locality 3.

Length :
Mean = 5·136.
Standard Deviation = 1·902.

$\frac{\text{Diameter}}{\text{Length}}$:
Mean = 8·689.
Standard Deviation = 2·797.

Correlation = − ·2641.
Total frequency = 367.

Index

Length	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*
1	—	—	—	—	—	—	—	—	2	1	2	—	—	—	—	—	—
2	—	—	—	—	1	1	1	2	6	1	3	1	3	—	—	—	—
3	1	2	1	—	2	2	5	6	6	5	4	9	5	2	—	1	—
4	—	—	—	2	3	7	6	7	7	9	13	8	2	—	—	2	2
5	—	3	1	5	1	8	6	13	16	6	7	6	3	2	—	—	1
6	—	2	—	—	2	2	10	7	13	10	3	2	2	1	1	—	—
7	—	—	—	1	3	10	8	5	8	6	4	—	—	1	—	—	—
8	—	—	—	1	3	10	6	4	3	1	1	2	—	—	—	—	—
9	—	—	—	1	2	1	2	4	3	—	—	—	—	—	—	—	—
10	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

e. Correlation between length and ratio of diameter to length of shells of *N. trivittata* from locality 3.

Length :
Mean = 7·9629.
Standard Deviation = 2·612.

$\frac{\text{Diameter}}{\text{Length}}$:
Mean = 6·3158.
Standard Deviation = 1·870.

Correlation = −·7193.
Total frequency = 456.

Index

Length	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
1	—	—	—	—	—	—	—	—	1	—	1	—
2	—	—	—	—	—	—	—	—	—	1	5	2
3	—	—	—	—	—	—	—	—	6	5	4	—
4	—	—	—	—	1	—	1	7	3	3	2	—
5	—	—	—	—	2	2	11	15	8	1	—	—
6	—	—	—	—	2	12	10	15	8	2	—	—
7	—	—	—	—	11	18	15	6	4	—	—	—
8	—	—	—	7	21	25	17	3	1	—	—	—
9	—	—	—	10	24	26	10	1	—	—	—	—
10	—	—	3	10	15	12	5	—	1	—	—	—
11	—	—	1	10	17	10	4	1	—	—	—	—
12	2	1	2	7	9	7	—	—	—	—	—	—
13	—	—	1	5	1	—	1	—	—	—	—	—
14	—	—	1	—	—	—	—	—	—	—	—	—
15	—	—	1	—	—	—	—	—	—	—	—	—

f. Correlation between length and apical angle of shells of *N. trivittata* from locality 3.

Length :
Mean = 7·695.
Standard Deviation = 2·590.

Angle :
Mean = 5·722.
Standard Deviation = 2·047.

Correlation = −·6577.
Total frequency = 461.

Angle

Length	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
1	—	—	—	—	—	—	—	—	2	—	—
2	—	—	—	—	—	—	—	2	1	2	3
3	—	—	—	—	1	—	1	2	6	4	1
4	—	—	—	—	—	2	5	3	6	1	—
5	—	—	—	—	1	11	12	12	3	—	—
6	—	—	—	3	2	12	16	5	9	2	1
7	—	1	2	4	11	16	13	5	2	1	—
8	1	—	3	17	19	17	14	4	1	—	—
9	—	4	9	19	15	17	7	—	—	—	—
10	1	4	6	12	14	5	2	2	—	—	—
11	—	4	5	9	10	10	5	1	—	—	—
12	2	5	4	9	3	3	2	—	—	—	—
13	—	1	5	—	1	1	—	—	—	—	—
14	1	—	—	—	—	—	—	—	—	—	—
15	—	—	1	—	—	—	—	—	—	—	—

g. Correlation between the ratio of diameter to length and the apical angle of shells of *N. obsoleta* from locality 1.

$\frac{\text{Diameter}}{\text{Length}}$:
Mean = 5·0973.
Standard Deviation = 1·759.

Angle:
Mean = 10·2292.
Standard Deviation = 2·946.

Correlation = ·4787.

Total frequency = 493.

Angle

Index	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*
1	—	—	1	—	—	—	1	2	1	—	—	1	—	—	—	—	—	—	—
2	1	—	—	—	5	1	2	—	3	2	4	1	—	—	—	—	—	—	—
3	—	2	1	2	—	5	7	6	9	10	7	3	3	—	1	—	—	—	—
4	—	1	1	5	4	9	20	16	12	13	15	17	1	2	—	1	—	—	—
5	—	—	—	—	2	6	6	11	16	11	16	16	12	7	—	—	—	—	—
6	—	—	—	—	1	1	2	10	9	16	13	18	9	9	3	2	1	—	—
7	—	—	—	—	—	1	—	3	3	7	6	9	8	8	5	2	2	—	—
8	—	—	—	—	—	—	—	—	2	1	1	6	5	5	3	3	1	—	—
9	—	—	—	—	—	—	—	—	1	1	1	1	2	2	2	—	—	—	1
10	—	—	—	—	—	—	—	—	—	1	1	1	1	1	—	—	—	—	—
11	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—

h. Correlation between the ratio of diameter to length and the apical angle of shells of *N. trivittata* from locality 3.

$\frac{\text{Diameter}}{\text{Length}}$:
Mean = 6·3158.
Standard Deviation = 1·870.

Angle:
Mean = 5·735.
Standard Deviation = 2·054.

Correlation = ·7941.

Total frequency = 456.

Angle

Index	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
1	—	—	1	1	—	—	—	—	—	—	—
2	1	—	—	—	—	—	—	—	—	—	—
3	2	4	—	2	1	—	—	—	—	—	—
4	2	10	16	16	2	2	1	—	—	—	—
5	—	4	17	33	26	18	5	—	—	—	—
6	—	1	1	17	35	42	13	3	—	—	—
7	—	—	—	3	8	19	33	7	2	2	—
8	—	—	—	—	1	10	16	17	4	—	—
9	—	—	—	—	1	3	7	6	13	1	1
10	—	—	—	—	—	—	—	1	8	3	—
11	—	—	—	—	—	—	1	2	3	3	3
12	—	—	—	—	—	—	—	—	—	1	1

i. Correlation between number of whorls and apical angle of shells of *N. obsoleta* from locality 3.

Number of Whorls:	Angle:
Mean = 5·832.	Mean = 8·0784.
Standard Deviation = 1·615.	Standard Deviation = 2·859.

Correlation = ·0077.

Total frequency = 357.

Angle

Number of Whorls	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
1	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—
2	—	—	1	—	1	—	—	—	2	1	1	—	1	—	—	—
3	—	1	—	—	—	2	1	1	2	—	—	—	—	—	—	—
4	—	—	3	6	1	9	7	12	8	5	3	6	3	—	2	—
5	—	1	1	2	8	7	3	15	4	6	2	5	3	3	—	1
6	1	2	5	10	11	10	11	13	14	11	6	6	1	1	—	—
7	—	1	3	2	3	4	3	8	10	5	7	7	1	1	—	—
8	—	—	2	1	2	3	5	6	11	7	9	2	—	—	—	—
9	—	—	—	—	1	1	3	1	—	2	1	—	—	—	—	—
10	1	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—

ON THE AMBIGUITY OF MENDEL'S CATEGORIES.

BY W. F. R. WELDON, F.R.S.

I. *Introductory.*

IN the early part of this year I published an account of the results obtained by Mendel in his experiments with certain races of cross-bred peas, and of the subsequent attempts to show that results similar to his were obtainable in other cases*. I see no reason to modify the statements I then made.

Since my article appeared the Royal Society has published a Report by Mr Bateson and Miss E. R. Saunders†, in which a large number of experiments in cross breeding are described; Mendel's results are re-stated, and most of the observations recorded are held to be in substantial agreement with Mendel's laws. In this Report, and in a separate essay by Mr Bateson‡, an altogether new interpretation of Mendel's results is suggested, and an issue of considerable importance is raised.

Mendel says that the races of peas, used in his experiments, were observed during several generations, and were found to differ constantly in certain characters, for example in the colour of their cotyledons, which was green in some races, yellow in others. Now "green" and "yellow" are not quantitatively definite terms; each includes a considerable range of recognisably different colours, and every known race of peas produces seeds whose cotyledons vary in colour. All that can be inferred from Mendel's statement, therefore, is, that the range of variation in cotyledon colour, which some of his races exhibited, fell entirely within the range of colours called "green," while that of other races fell entirely within the range of colours called "yellow." Mendel accounts for the behaviour of hybrids between these races, as he describes them, by assuming that each hybrid bears gametes, male and female, of two kinds and in equal numbers, "und dass diese Keim- und Pollenzellen "ihrer inneren Beschaffenheit nach den einzelnen Formen entsprechen§." The issue raised by Mr Bateson and Miss Saunders concerns the meaning which it is necessary, in the light of known facts, to give to this expression.

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* See *Biometrika*, Vol. I. p. 228.

† *Reports to the Evolution Committee: I. Experiments undertaken by W. Bateson, F.R.S., and Miss E. R. Saunders*, Royal Society, 1902.

‡ *Mendel's Principles of Heredity, a Defence*: by W. Bateson. Cambridge University Press, 1902.

§ *Verhandl. d. naturforsch. Vereins Brunn*, Bd. IV. p. 29.

The result, described by Mendel, may be illustrated by reference to the following diagram:

Let *AD* be a scale of colour,—a map of a spectrum, or some similar thing,—of which the range *AB* includes the various shades of green, the range *CD* those of yellow. A race of peas has cotyledons whose colour varies, in individual cases, within the range *AB*; a second race has cotyledons which vary within the range *CD*. If these races be crossed, the hybrid cotyledons are said to fall within the colour-range *CD*; and the plants to which they give rise will, on Mendel's hypothesis, produce gametes of two kinds, in equal numbers; those of the first kind, if paired, give rise to plants whose cotyledon colour (and that of their descendants) lies within the range *AB*; those of the second kind, if paired, give rise to descendants whose cotyledon colour lies within the range *CD*.

That is to say, the descendants of such a cross fall into two groups, each referable, so far as cotyledon colour is concerned, to the same category as one of the ancestors used in the original cross. We are quite unjustified, from the data given by Mendel or by any of those who follow him, in saying more than this. Thus the green-seeded descendants of such a cross are described as "green": they are not described, by Mendel or by any of his followers, as being of the same shade of green as the particular member of a variable race used in the original cross. The information given is therefore compatible with either of several theories of the constitution of those gametes in the hybrid from which "recessive" individuals arise. For anything we are told to the contrary, the character transmitted by such gametes may be a blend of all the kinds of green exhibited by their green-seeded ancestors in various proportions; or different gametes may revert directly to the colour of different individual ancestors; or finally all the recessive gametes may transmit the characters of the green-seeded ancestor which took part in the original cross, and of no others. The first two hypotheses involve the belief that the composition of gametes of either kind, whether "dominant" or "recessive," is affected by that of a whole series of ancestors. Such belief seemed (and still seems) to me a necessary consequence of the facts. Mr Bateson and Miss Saunders have however adopted the view that "the pure dominant and the pure recessive members "of each generation are not merely like, but identical with the pure parents" (*l. c.* p. 12); and Mr Bateson denounces in no measured terms my attempt to regard the inconsistent results obtained by those who have repeated Mendel's experiments as due to differences in the ancestry of the races used.

Mr Bateson fully admits the variability of all races of peas; and he admits that the range of colour-variation in many races is so great that they include colours of both the "green" and the "yellow" category. Nevertheless, he believes that many of these races behave, when crossed, like those described by Mendel. This admitted variability of the races makes it possible for Mr Bateson to test his view experimentally; for if a "recessive" plant, resulting from a cross, is "not merely like but identical with" one of the pure-bred plants used in the cross, then the peas on the recessive plant must exhibit exactly the same series of colour-varieties in their cotyledons as those exhibited by the pure-bred plant which it resembles. Before Mr Bateson can justify the view he has put forward, he must therefore not only find two races which obey Mendel's laws when crossed, which he says he has done, but he must determine

(1) The variability of cotyledon colour in each race;

(2) The mean cotyledon colour and its variability in each plant used in crossing;

(3) The mean cotyledon colour and its variability in each "recessive" and in each "dominant" plant descended from the cross.

Until the result of such determinations is known, it is impossible to distinguish between the resemblance of a series of cross-bred plants to one of the ancestral races, and their resemblance to an individual plant of that race; so that Mr Bateson's contention cannot be supported by evidence.

The confusion between resemblance to a race and resemblance to an individual involved in Mr Bateson's treatment of Mendel's work is one of the many unfortunate results which follow when Mendel's system of dividing a set of variable characters into two categories, and of using these categories as statistical units, is carried too far. Unless the range of characters actually included in each category be constantly borne in mind, the degree of resemblance between two individuals, implied by placing them in the same category, cannot be estimated; and when, as constantly happens, the range of variation in one of the alternative categories differs widely from that included in the other, the Mendelian system becomes absolutely misleading without some explanation (nearly always withheld) of the real limits implied by the terms used. Thus if two plants are said to be glabrous, we know that they are absolutely similar in so far that they possess no hairs; if they are said to be hairy, we know that they both possess hairs, but one may, for anything we are told to the contrary, have ten times as many hairs, per unit of surface-area, as the other. Again the tendency to apply two categories, found suitable for a particular race, to other races of the same or allied species leads to very harmful results; for example the classification of peas into those with green and those with yellow cotyledons leads to a quite erroneous conception of the distribution of cotyledon colour in most existing races of peas, although it may have expressed the facts observed by Mendel in the races which he used.

The Report of Mr Bateson and Miss Saunders contains many statements which would, I think, never have been made if the authors had not been misled by the use of Mendelian categories. It is impossible to realise the meaning of evidence, brought forward to prove that particular hybrids behave in the manner described by Mendel, unless the meaning of the categories employed in each special case is clearly understood. I therefore propose, as opportunity offers, to describe the variability of such races as I can obtain, among those which are said to obey Mendel's laws, and to consider how far the statements made concerning them are affected by the employment of Mendel's very imperfect system of units.

II. *Lychnis diurna* and *Lychnis vespertina*.

In the Report by Mr Bateson and Miss Saunders the results of crossing normal *L. vespertina* and *L. diurna* with white and red-flowered glabrous varieties are described; it is said that the phenomena "follow Mendel's law with considerable accuracy, and no exceptions that do not appear to be merely fortuitous were discovered" (*l.c.* p. 15). Apart from the great uncertainty involved in the use of "hairy" as a definite category alternative to "glabrous" (which will be dealt with later) these experiments are of interest, because a parallel series of crossings had already been carried out by Professor de Vries, who used the same glabrous varieties as those used, at least in some cases, by our authors. They express themselves as "specially indebted to Professor de Vries" for them; and they add in a footnote, "The discovery by de Vries of a wild specimen of *L. vespertina* var. "*glabra*, and the artificial production from it of a smooth red-flowered form are "described in his 'Erfelijke Monstrositeiten,' p. 10...." Now I have not been able to find a work by Professor de Vries, under this title, which contains a page 10; but in the *Botanisch Jaarboek*, Jaargang IX. 1897, pp. 62—93, there is a paper by him entitled "Erfelijke Monstrositeiten in den ruishandel der botanischen tuinen." Under the heading *Lychnis vespertina glabra* I find the following statement (p. 71): "In August 1888 I collected seed of *Lychnis vespertina* in a wood "near Hilversum. Among the plants produced in my experimental garden in "the following year were some completely glabrous examples. I isolated these, "and in the course of some generations the glabrous race has been rendered "stable (geheel standvastig) as a result of continued selection (door voortdurende "selectie). It seems never, or only very rarely, to produce atavistic individuals.

"Traces of hairs are still to be found here and there, especially upon the young "plants. This point is worth closer investigation."

This race of "glabrous" plants was therefore established by a process of selection, and seven or eight years after it was first observed it still produced traces of hairs. There is no evidence in the account given to show whether the seed came originally from a glabrous wild plant, as suggested by Mr Bateson and Miss Saunders, or not; but they may have fuller information from Professor de Vries. In any case the first glabrous plants observed did not behave as "mutations," but as we should expect extreme variations from the hairy type to behave; so that several generations of selection were required in order to fix a stable race with their characters. Although its establishment by continued selection suggests that this glabrous race had not at first the properties of a "recessive" race in Mendel's sense, yet it is said to have behaved as a strictly recessive form when crossed with hairy races later on. In 1892 it was crossed with normal forms of *Lychnis diurna*, and the resulting generations were held to obey Mendel's laws. The hybrids of the first generation were all hairy; and the offspring obtained by pairing these were in part hairy and in part glabrous. From the glabrous hybrids a stable glabrous variety is said to have been raised, and there is no record that these glabrous "recessive" forms ever give rise to "hairy" plants. The gametes of the glabrous *Lychnis vespertina*, as fixed by selection, are therefore said to behave in this cross as if they were what writers on Mendel's work call "pure." Mr Bateson rejects the view that the characters of cross-bred individuals, derived in part from such "pure" parents, can be regarded as depending upon the characters of the ancestors from which the "pure" parents are descended; he declares that they depend entirely upon the characters of the "pure" individuals used in making the cross. It is therefore to be regretted that he has abstained from discussing an experiment in which Professor de Vries crossed this "pure" glabrous *Lychnis vespertina* with *Silene noctiflora*, and obtained hybrid offspring which were indeed hairy, but their hairs were of the type proper to *Lychnis vespertina*, and not of the type of *Silene noctiflora*. Surely we have here a clear proof that the "dominant" character, hairiness, may on the application of a suitable stimulus be manifested by the fertilised germ-cells of what is said to be a purely recessive plant; so that the theory of pure parental gametes, on which Mr Bateson lays such stress, is shown to be inadequate for this case, and a theory of inheritance, with reversion to particular ancestors, is indicated as likely to express the facts of "Mendelian" cases also.

As has already been said, Professor de Vries crossed the glabrous *L. vespertina* here described and normal hairy *L. diurna*, always apparently using *L. diurna* ♀ × *L. vespertina* ♂; the first generation of hybrids contained only hairy individuals. The second generation is not very fully described; in the *Erfelijke Monstrositeiten* (p. 72) we are told that about $\frac{2}{3}$ of the individuals were hairy, and $\frac{1}{3}$ glabrous; in the *Comptes Rendus*, March 1900, we are told that 28 per cent. were glabrous; and this statement is repeated in subsequent accounts.

Although the actual numbers of individuals are not given, so that the probable errors of these results cannot be calculated, it is clear from the adoption of the round numbers $\frac{1}{3}$ and $\frac{2}{3}$ that the impression produced was not that of $\frac{1}{4}$ recessive and $\frac{3}{4}$ dominant or dominant-hybrid individuals, as it should have been on Mendel's hypothesis. Several hundred of individuals are said to have been observed. The odds against a deviation so largely exceeding Mendel's result with 500 individuals would be about 17 to 1. The glabrous variety produced in these experiments appears to have furnished some, at least, of the glabrous, red-flowered forms used by Mr Bateson and Miss Saunders, and in their hands it is said to have given results in good accord with Mendel's law; they pass over the deviations from Mendel's law, observed during its earlier history, without notice.

So much for what is known about the previous history of the races used by Mr Bateson and Miss Saunders. It is sufficient to show that even the use of Mendel's categories is not enough to bring the phenomena described into anything like *exact* accord with Mendel's laws. Before going further, we will examine a little more closely the conditions included under the two categories "glabrous" and "hairy."

By *glabrous* Professor de Vries understands (at least when speaking of *L. vespertina*) the absence of perfect hairs from all parts of the plant, although he calls a variety "glabrous" when especially in the young state it has "traces of hairs" here and there. Mr Bateson and Miss Saunders leave their meaning a little doubtful. On p. 16 they say "in the glabrous varieties no hairs were observed on any part of the plants at any time"; but since when speaking of cross-bred forms they always refer to hairiness as a character of *leaf-surface*, it is not quite clear to me whether a cross-bred plant with glabrous leaves and a hairy stem was ever observed by them, and if so, whether they would classify it as glabrous or as hairy. The only glabrous plant I have myself found wild (*L. vespertina*) had no hairs on any part of its surface; and I am not aware that plants with hairy stems or calices and glabrous leaves have been described. For this reason the repeated statement that hairiness is a character of leaf-surface is difficult to understand.

The category "*glabrous*," whether merely applied to leaf-surface or to the whole plant, certainly includes only a small range of conditions. It is far otherwise with the category "*hairy*." In accordance with the differences in habit of the two species, the hairs of *L. diurna* and of *L. vespertina* differ considerably in their distribution, even when leaf-surface only is considered; and while the average condition of the hairs in the two "species" is different, each "species" varies from race to race, and from individual to individual.

The hairs are multicellular, and may vary in length, on the same leaf, from about 0·2 mm. to about 1·9 mm.; the cell sap may be red, or blue, or nearly colourless; some of the hairs on a leaf may be glandular, the percentage of such hairs varying greatly. Further, when a race of *L. diurna* and one of *L. vespertina*, both being hairy, are crossed, the hybrids are said by Gagnepain to be intermediate between the parent races in the condition of their hairs (*Bull. Soc. Bot. de France*, T. XLIV. p. 445).

In order to avoid the effects of differentiation among the leaves, it is necessary to compare corresponding leaves of the different plants examined. To illustrate the variation in the frequency with which hairs occur in unit area of leaf-surface Table I. has been compiled. It shows the number of hairs per square centimetre of epidermis on the lower side of a leaf from the node below that which bears the terminal flower. The observations were made (*a*) upon a race found in a little copse on the Berkshire (right) bank of the Thames, about four miles by road below Oxford, and close to Sandford Lock; (*b*) upon a race found on the slopes of Cooper's Hill, Surrey, also on the right bank of the Thames, but more than 35 miles distant from Sandford in a straight line. The second race was collected, and in large part examined by Miss C. B. Sanders, whom I gladly take this opportunity of thanking for the time and labour she was kind enough to give. The epidermis was removed from the leaf, and mounted in water, with its hairy side uppermost, the hairs being then counted under a low power of the microscope; in this way error due to the accidental breaking of hairs was avoided, because the stump of a broken hair was easily recognised.

TABLE I.

Number of Hairs per square centimetre of Lower Leaf in Lychnis diurna ♀. The area chosen in all cases so as to exclude the great veins.

Number of Hairs	Frequency		Number of Hairs	Frequency	
	Sandford	Cooper's Hill		Sandford	Cooper's Hill
0— 24	0	1	350—374	6	—
25— 49	1	10	375—399	1	—
50— 74	2	14	400—424	6	—
75— 99	5	19	425—449	1	—
100—124	5	17	450—474	1	—
125—149	7	11	475—499	0	—
150—174	17	5	500—524	1	—
175—199	9	8	525—549	0	—
200—224	7	5	550—574	1	—
225—249	9	3	575—599	0	—
250—274	7	2	600—624	0	—
275—299	3	2	625—649	0	—
300—324	7	3	650—674	1	—
325—349	4	—	675—699	1	—
			Totals	102	100

The figures given do not convey an adequate impression of the range of variability throughout the species. Even in localities close to those inhabited by the races described a greater range of variation has been observed; such as they are, however, the two races examined are enough to show how wide is the range, covered by the category "hairy," in *L. diurna* alone. In a single race of *L. vespertina* from Shotover Hill, near Oxford, it is even greater; 112 plants of this race included one glabrous individual, and one with 1106 hairs per square centimetre on the standard leaf. The distribution of hairs in this race is given in Table II, which is again an inadequate picture of the range throughout the species, even in the neighbourhood of Oxford. Odd plants gathered by the wayside have given over 1300 hairs per square centimetre.

TABLE II.

Number of Hairs per square centimetre on Lower Leaf Surface in Lychnis vespertina ♀ from Shotover.

Number of Hairs	Frequency	Number of Hairs	Frequency
0— 24	1	500—524	5
25— 49	0	525—549	4
50— 74	0	550—574	0
75— 99	1	575—599	3
100—124	2	600—624	3
125—149	4	625—649	2
150—174	2	650—674	1
175—199	6	675—699	2
200—224	5	700—724	2
225—249	4	725—749	0
250—274	7	750—774	0
275—299	5	775—799	1
300—324	5	800—824	0
325—349	3	825—849	0
350—374	9	850—874	3
375—399	7	875—899	0
400—424	4	900—924	1
425—449	9	925—949	0
450—474	5	950—974	1
475—499	4	975—999	0
		* *	*
		1100—1124	1
		Total	112

Taking a group of 25 hairs as the unit of measurement, and calling the group containing from 0 to 24 hairs the first, we have, for the constants of the three distributions the following values*:

TABLE III.

Constants of the Frequency. Distribution of Hairiness.

	L. diurna ♀		*L. vespertina* ♀
	Sandford	Cooper's Hill	Shotover Hill
Mean	10·2255	5·4900	16·7143
Mode	7·0284	—	14·4399
σ	4·8863	2·7725	7·9583
μ_3	132·0653	—	441·4132
μ_4	2658·4536	—	15675·9832
β_1	1·281485	—	0·766935
β_2	4·663582	—	3·907930
a_1	4·4337	—	13·6382
a_2	92·7746	—	158·3939
m_1	1·1689	—	2·5372
m_2	24·4607	—	33·6951

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* Notation is that used by Pearson: *Phil. Trans.*, Vol. 186, A, p. 367 et seq.

The probable error of all these values is of course considerable, but they fit the observations as well as could perhaps be expected. Thus the lower limit of hairiness in the race of *L. diurna* from Sandford is 7·0284 − 4·4337 = 2·5947 units, or allowing for the probable error of the start of the range, which is about ·63 units, 52 ± ·16 hairs, the lowest observed number of hairs being 47. I feel bound to call attention to this feature of the distribution, which is some sign that in *L. diurna* the range of variation, in a normal hairy race, does not involve an occasional production of glabrous individuals; it is interesting in this connection to notice (1) that Mr Bateson and Miss Saunders carefully refrain from speaking of a glabrous variety of *L. diurna*, and (2) that the red-flowered hybrid, which Professor de Vries calls *Lychnis diurna glabra*, results from the cross between glabrous *L. vespertina* and hairy *L. diurna* already alluded to. The lower limit to the distribution in *L. vespertina* is 14·4399 − 13·6382 = 0·8017 units, or 8 hairs per centimetre, which is as good an approximation to a lower limit at no hairs as we need expect from the limited number of observations. It must be remembered that the first unit of the grouping adopted has a centre at 13 hairs, while the only individual in the group was glabrous.

The Cooper's Hill material seems heterogeneous, and I have not thought it worth while to resolve this small number of individuals into components.

TABLE IV.

Number of Hairs with Glandular Extremities per Hundred Hairs observed.

Gland Hairs per cent.		*L. diurna* ♀		*L. vespertina* ♀
		Sandford	Cooper's Hill	Shotover Hill
0—4·9	0	89	100*	43
	Less than 1	3	—	12
	1—1·9	3	—	2
	2—2·9	1	—	2
	3—3·9	1	—	6
	4—4·9	2	—	3
	5·0— 9·9	1	—	16
	10·0—14·9	1	—	6
	15·0—19·9	1	—	5
	20·0—24·9	0	—	6
	25·0—29·9	0	—	1
	30·0—34·9	0	—	0
	35·0—39·9	0	—	0
	40·0—44·9	0	—	1
	Totals	102	100	103

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* So my notes say. If we make the utmost allowance for possible breakage of the tips of gland-hairs, so that they cannot be recognised, it is I think certain that no female plant from Cooper's Hill had one per cent. of such hairs even near the base of the leaf.

The foregoing facts show the range of variation in the number of hairs per unit area on the leaves of a few individuals. It is difficult to estimate the percentage frequency of glandular hairs, because they are more frequent near the base of the leaf than near its apex; I have attempted an estimate of their percentage frequency in the basal part of the leaf, and the result is given in Table IV.

The few data here brought together are sufficient to show the way in which the adoption of such a category as "hairy" conceals the facts of variation within the races discussed. In the light of such facts the statements made by Mr Bateson and Miss Saunders are seen to be utterly inadequate, either as a description of their own experiments, or as a demonstration of Mendel's or of any other laws.

When hairy and glabrous plants were crossed we are told that "among the whole number of plants raised, not a single intermediate was observed" (*l.c.* p. 15), but we are not told what "an intermediate" is. The authors must know, or they could not certify its absence; it would have been well if they had thought fit to define an "intermediate" in their Report, for the definition is a vital part of their argument.

The preceding tables show that "hairiness" is not an absolute, invariable quality, but that it is manifested in various degrees. In the few individuals examined it is possible to pass by a series of small steps from the glabrous condition through individuals with various numbers of hairs per square centimetre of leaf-surface, up to a condition of very great hairiness. It is perfectly legitimate to regard those individuals with a small number of hairs per unit area as intermediate between those with a larger number and those with a smaller number or with none. Thus a plant with only 80 hairs per unit area of leaf-surface may be called intermediate between a plant with 200 hairs per unit area and a plant with none.

Other conditions are conceivable, intermediate between that of a plant with a given number of hairs per unit area, the hairs being of known length, and that of a glabrous plant. Thus the transition might take place by a reduction in the length of the hairs through various steps to zero, without reduction in their number; in which case the plant with the shorter hairs would be intermediate between the plant with longer hairs and the glabrous individual; or again the transition might conceivably be effected by the appearance of glabrous patches, so that the intermediate individuals assumed a mosaic character. On the whole, however, the density of the hairs per unit area seems the best measure of "hairiness" for our present purpose.

Whatever we may choose as a measure of hairiness, Mr Bateson and Miss Saunders give no evidence by which we can judge the result of their work. Thus we are told that "hairiness" is dominant, because "of the thousand cross-breds "raised from various unions between hairy and glabrous strains, all, without exception, were hairy" (*l.c.* p. 19); but unless we know how hairy they were, we cannot judge what is the value of the statement that hairiness is dominant. Thus a plant with 40 or 50 hairs per square centimetre is obviously and unquestionably hairy; it is just as clearly intermediate between a plant with 1100 hairs per centimetre and a glabrous plant. Without some proof that the mean number of hairs per unit area was not less in the offspring of a cross between a glabrous and a hairy form than it was in the hairy parent, the result described cannot be distinguished from that most probable on any theory of "blended" inheritance.

It is most unfortunate that the degree of hairiness of each individual was not recorded, because the great variability of the parental races would have given an excellent opportunity of demonstrating the truth of the statement that pure dominants "are not merely like, but identical with the pure parents." We might at least have hoped to learn whether a *Lychnis diurna* with 40 hairs per centimetre, and a *L. vespertina* with 1100 or 1200, produce plants with different degrees of hairiness when crossed with the same glabrous variety, or whether the presence or absence of gland hairs on the leaves of the hairy parent has any effect upon this "character of leaf surface" in the offspring!

Unless all the hairy parents used, of either race, were equally hairy, the first cross-bred generation must either have contained individuals of different degrees of hairiness, or the identity between pure dominants and their parents disappears. On the other hand, if the cross-breds and their parents differed in this character we have no way of distinguishing "dominance" from the result of "blended" inheritance, until we abandon the Mendelian categories, and adopt a rational way of measuring hairiness. The absence of glabrous cross-breds will not help, until we know the variability of the hairy race, and the character of the hairy parent; for the chance of obtaining a glabrous individual among 1000 plants of any ordinary race of either species is admittedly small, and the variability of a few families, in each of which one parent is of fixed character, will on any hypothesis be less than the variability of the race, and the chance that such a series of families would contain a glabrous individual, on any theory of inheritance, cannot be estimated without information which is at present not available.

The difficulties which arise from imperfect description when we consider the question of dominance in the first cross-bred generation are equally formidable in the case of later generations. The category hairy is so wide that it is impossible to judge how the individuals included in it resemble or differ from their parents. The total results of the various crosses recorded are in better agreement with Mendel's statements than is usual, but the published data do not afford material for discussing the question how far any particular theory of inheritance can be successfully applied to them.

It is deeply to be regretted that so many interesting experiments, involving so much time and labour, should be recorded in a form which makes it impossible to understand the results actually obtained, and so gives rise to misconceptions both in the minds of the recorders and in others. Such justification as there may prove to be for classifying the form of inheritance exhibited by the hairs of *Lychnis* with that described by Mendel may be just as easily seen when the variations observed both in parents and in offspring are clearly described; when this has been done, and not before, we shall be able to consider seriously what the issues raised by Mendel's statements really are. In the mean time the accumulation of records, in which results are massed together in ill-defined categories of variable and uncertain extent, can only result in harm.

The confusion introduced by the use of such categories into the record of experiments on the spiny-fruited and smooth-fruited races of *Datura*, performed by Mr Bateson and Miss Saunders, is even greater than that we have examined; but I refrain from discussing it or other cases in detail, until I can illustrate the categories by reference to a full series of specimens.

COOPERATIVE INVESTIGATIONS ON PLANTS.

I. ON INHERITANCE IN THE SHIRLEY POPPY*.

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(1) *Material.* In 1899 at Hampden Farm House on the Chilterns we had at our disposal for the study of homotyposis a considerable strip of garden covered with Shirley poppies. These were extremely fine plants, individuals at the end of the season having often 20 to 30 and occasionally 60 to 80 fruits. An investigation of the stigmatic bands of these capsules showed marked individuality in plants grown under apparently very like conditions of soil, air, rain and individual crowding. Of course plants on the borders of the plot had rather more opportunity for side development and put forth—on the whole late in the season—poor flowers on low shoots. The capsules of these low flowers had rarely any quantity of fertile seed and often none at all. The differentiation of high and low flowers in size of capsule was not accompanied by an equally marked differentiation in the number of stigmatic bands. Owing to this and the relative fewness of outlying individuals the whole plot was dealt with as producing a fairly homogeneous mass of plants growing under like conditions. For a first approximation capsule differentiation was neglected. This rule has been adopted in later crops;

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* Drafted by K. Pearson.

the refinement of separating a small percentage of outlying individuals being more apparently than really advantageous. For, given the same soil and climate, there must, if any considerable number of plants be dealt with, be subtle individual advantages or disadvantages to individual plants in the crop due to slight differences in soil and situation, to thinning, tending or insect attack. Similar advantages or disadvantages, however, occur in the case of individuals, who are offspring of the same parents in men, horses or dogs, and do not therefore destroy all possibility of comparative study.

In 1900 F. W. Oliver also grew a crop of Shirley Poppies at Chelsea. These showed a very marked difference in the mean number of stigmatic bands per capsule as well as in the number of capsules per plant. We have then to recognise the factors: (i) considerable individuality as expressed by the homotypic correlation in plants grown under like environment, (ii) considerable difference in mean result for two crops grown in different environments, i.e. with differences of soil, tending, climate, atmosphere, etc.

The Shirley Poppy seemed excellent material for testing the laws of inheritance in plants, partly because of the healthy strong crops which can be raised under all sorts of conditions with small amount of attention; partly, because of the ease with which the stigmatic bands can be counted. In some of the series to be considered below colour has also been recorded, but the difficulties attaching to a satisfactory study of individual flowers have permitted at present only of limited consideration of this point. Further experiments in this matter are in progress.

In 1900 K. Pearson collected (i) all the seed from every capsule of 24* individuals out of his 176 Hampden plants; (ii) the seed from the capsules of a great variety of poppies sorted into groups of capsules having 8 to 18 bands; (iii) a mass of seed from all capsules and plants without discrimination. We had thus three lots of seed to be known as: (α) Individual Plant Seed: (β) Individual Band Seed and (γ) General Seed. In later years an additional type: (δ) Individual Capsule Seed, was dealt with.

From these lots of seed the following crops were grown in 1900:

(A) *Highgate Crop.* By the courtesy of the Misses Sharpe this was sown in a piece of meadow broken up for the first time for garden purposes. The soil was very poor and purposely not enriched in any way, the crop had an extremely hard struggle for existence at all, only a small percentage of the seed reaching maturity; the crop needed no thinning and was left to itself. There were only two or three capsules per plant, and the sizes of these as well as of the plants themselves were inferior even to F. W. Oliver's Chelsea crop of the preceding year. Only seed of type (α) was dealt with, (γ) failing entirely. In well tended and watered beds of the same garden non-experimental (γ) seed gave strong healthy plants of plenty of blossom. Counting and reduction by K. Pearson.

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* It would doubtless have been better to start with more than 24 individual plants, but besides selecting plants with a sufficient number of capsules, we had by the conditions of the homotypic experiments to wait till the end of the flowering season, and then many of the plants had scattered their seed.

(B) *Oxford Crop.* W. F. R. Weldon had seed types (α), (β) and (γ). The plants were raised in pots, and besides the stigmata a considerable number of characters on the individual plants was observed. The plants were raised in pots for ease in measurement and handling of individuals, but the need for constant watering introduced other difficulties to be referred to later. Counting and reduction by W. F. R. Weldon and in the case of the reduction of "first flower" characters by Alice Lee.

(C) *Chelsea Crop II.* F. W. Oliver had seed types (α) and (β). This crop was hopelessly ruined by a storm. The plants flowered well but the young fruits got beaten down and rotted off. Only 355 capsules were available for counting, one or two to a plant, and the frequency distribution of these seems the only data worth dealing with.

(D) *Crockham Crop.* A. G. Tansley had seed types (β) and (γ), and grew a very healthy crop. The counting and reducing are due to Marie A. Lewenz.

(E) *Bookham Crop.* G. U. Yule grew a crop of (α) seed. The plants were fairly healthy, but the crop was not fully harvested and counted. Counting is due to G. U. Yule and reduction to Alice Lee.

(F) *Enfield Crop I.* W. R. Macdonell grew a good crop from both (α) and (β) and also a control series of (γ). From his seed were taken supplies for the following crops in 1901:

(G) *Enfield Crop II.* This was grown from seed of type (δ). From the capsules of the Enfield crop of 1900, 57 were taken from different plants with bands varying from 9 to 20. 13 of the series failed, but the remaining 44 gave 239 plants with 566 capsules—a poor crop compared with that of 1900. Mrs W. R. Macdonell counted both Enfield crops, and the reductions are by W. R. Macdonell himself.

(H) *Kidderminster Crop.* This was grown by John Notcutt from seed of type (α) for 100 Enfield plants. The crop was a magnificent one, giving 1618 individual plants bearing more than 19,000 capsules. The work of counting the bands on these capsules is due to Margaret Notcutt, Marie A. Lewenz, Edna Lea-Smith and Norman Blanchard. The whole of the labour of reduction is due to W. Palin Elderton, one of whose tables involves upwards of 3,800,000 entries.

It will be seen that the material used for the purpose of the present paper is drawn from fairly diverse districts: London (2 crops), Oxford, Hertfordshire (2 crops), Surrey (2 crops) and Worcestershire. Further, it enables us to appreciate the magnitude of parental, grandparental and fraternal relations in plants, and the influence of environment, in the case of a single character in one species. There are many difficulties and obscurities, which require special investigation, and it is hoped that other experiments now in progress will throw light on some of these matters.

(2) *Complexity of Heredity in Plants.* With plants which are not either artificially crossed, or artificially restricted from crossing, the conception of the fraternal relation becomes very complex. If any artificial crossing or restriction in crossing be adopted, we can confine our attention to one special type of brother-plants, but not only is it very difficult to carry out such crossing or restriction in the large populations which alone seem to give reliable results in heredity, but when carried out we have reached a condition of things very different from what happens in wild life, where except for absolutely self-fertilising plants the diversity in brother-plants is itself a factor of the variation which evolution finds to draw upon. If we examine the case of man we find only three types of fraternal relationship, i.e. whole siblings or half-siblings on mother's or father's side. But in plants great complexity is introduced by the multiplicity of ovaries and pollen sacks, and by the possibility of self- or cross-fertilisation. If we put on one side (i) the division of the ovary into cells, and (ii) the division of the anther into lobes, we have still (iii) the multiplicity of anthers on the same flower and (iv) the multiplicity of flowers on the same plant to deal with. Are we to consider the plant as an individual because it proceeds from a single seed, and to suppose the rudiments of future seed in all cells of the same ovary and in all ovaries of the same plant of equal valency? And again is all pollen whether from anthers of the same flower, or from anthers of different flowers of the same plant, equivalent from the standpoint of parentage? Or, is the plant to be looked upon as a colony of individuals, and the flower to be the unit of parentage? We may even go beyond this and consider the individual cell of the ovary or the individual anther as our unit. It is clear that only very wide-reaching direct experiments on artificial fertilisation would enable us to distinguish whether one or other of the many types of plant parentage give offspring more or less alike. Still some classification of sibship in plant life seems desirable if we are to compare our poppy results with those for insects and animals.

We confine our attention for the time to the flower as the unit of parentage, and denote by a single letter an individual plant; small letters $a, a', a''\ldots$ denote ovaries of different flowers of this plant, large letters $A, A', A''\ldots$ the groups of anthers attached to the corresponding flowers or ovaries $a, a', a''\ldots$. A pair of plants growing from seeds taken from the cells of the same ovary will be termed *co-ovarial*; if they come from two ovaries of the same plant *bi-ovarial*, if from ovaries on different plants *dis-ovarial*. If the pair of plants come from the pollen of anthers on the same flowers, they are *syn-anthic*, if from anthers of different flowers of the same plant *di-anthic*, and if from pollen of different plants *dys-anthic*. When fertilisation is from the anthers attached to a given ovary, we shall term the resulting plant *homotropic*; when fertilisation is from the anthers of another flower of the same plant, the result is *endotropic*, and when the pollen is from a different plant *exotropic*; the crossings for the same are *homogamic**, *endogamic* and *exogamic*. Given a pair of plants sprung from the ovaries of the same plant or from pollen of the same plant, these may be termed siblings or half-siblings, and may be classified by compounding the above terms, thus instead of speaking of a homotropic-homotropic pair we can without confusion speak simply of a homotropic pair, and instead of an exotropic-homotropic or exotropic-endotropic pair, simply of an *exhomotropic* or an *exendotropic* pair. Further an interesting series of subcases arises when there is a mutual interchange of pollen between two flowers or plants. If the interchange be such that the ovary of one flower (a) receives pollen from the anther of a flower on the same or on a second plant (A' or B), while the ovary of this latter flower (a' or b) receives pollen from the anthers (A) associated with the first ovary, we shall speak of this relation as *hypermetatropic*. If the first ovary (a) receives from an anther (A' or B) associated with the second ovary (a' or b), but the second ovary from the anthers of the first plant not associated with the first ovary (A''), then the relationship is *mesometatropic*. Finally if the interchange be between two plants the pollen of one going to the other, but not the pollen from anthers associated with the ovaries fertilised, then the relationship is *prometatropic*. If the interchange is only half completed, then we have the three corresponding types of *hemimetatropy*.

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* Not to be confused with the original use of this word for plants which mature the organs of both sexes at the same time.

We are now able to classify the various forms in plants corresponding to the fraternal relationship.

Full and Half-Sibship in Plants.

	Terminology	Symbol
I.	Co-ovarial synanthic homotropy	$aA . aA$
	„ dianthic endhomotropy	$aA . aA'$
	„ synanthic endotropy	$aA' . aA'$
	„ dianthic endotropy	$aA' . aA''$
	„ dysanthic exhomotropy	$aA . aB$
	„ dysanthic exendotropy	$aA' . aB$
	„ synanthic exotropy	$aB . aB$
	„ dianthic exotropy	$aB . aB'$
	„ dysanthic exotropy	$aB . aC$
II.	Bi-ovarial dianthic homotropy	$aA . a'A'$
	„ synanthic endhomotropy	$aA . a'A$
	„ dianthic endhomotropy	$aA . a'A''$
	„ dysanthic exhomotropy	$aA . a'B$
	„ dianthic hypermetatropy	$aA' . a'A$
	„ dianthic hemi-hypermetatropy	$aA' . a'A''$
	„ dysanthic hemi-hypermetatropy	$aA' . a'B$
	„ synanthic endotropy	$aA'' . a'A''$
	„ dianthic endotropy	$aA'' . a'A'''$
	„ dysanthic exendotropy	$aA'' . a'B$
	„ synanthic exotropy	$aB . a'B$
	„ dianthic exotropy	$aB . a'B'$
	„ dysanthic exotropy	$aB . a'C$

III.	Dis-ovarial synanthic exhomotropy [*]	$aA \,.\, bA$
	„ dianthic exhomotropy	$aA \,.\, bA'$
	„ synanthic exendotropy	$aA' \,.\, bA'$
	„ dianthic exendotropy	$aA' \,.\, bA$
	„ dysanthic hypermetatropy	$aB \,.\, bA$
	„ dysanthic mesometatropy	$aB \,.\, bA'$
	„ dysanthic hemi-hypermetatropy	$aB \,.\, bC$
	„ dysanthic prometatropy	$aB' \,.\, bA'$
	„ dysanthic hemi-prometatropy	$aB' \,.\, bC$
	„ synanthic exotropy	$aC \,.\, bC$
	„ dianthic exotropy	$aC \,.\, bC'$

It will thus be seen that there are 33 different forms of sibling relationship in plants corresponding to whole or half-brotherhood in horses, dogs or men; and further for a particular plant grown as a crop, although the nature of the plant may enable us to cancel certain of these relationships as impossible, there will generally be a considerable number left, and the proportions of each class may be quite unknown to us. For example, if we take seed from the same capsule on a plant whose flowers are capable of either cross- or self-fertilisation we may really have a mixture of nine different types of relationship owing to pollen from the anthers of the same flower, from anthers of other flowers of the same plant, and from anthers of flowers of different plants being scattered on or carried by insects to the same stigma or system of stigmata. Hence it is very difficult to compare the relationship of the bisexual offspring of some animals[†], or even of the parthenogenetic offspring of certain insects with the relationship among offspring, which may range all the way from $aB \,.\, aC$ to $aA \,.\, aA$ in unknown proportions[‡].

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[*] While in every case the names lead to the same relationship symbolically expressed, the symbol may lead us to different names for the same relationship. Thus dis-ovarial synanthic exhomotropy = dis-ovarial synanthic hemi-hypermetatropy, etc., etc. We have reserved, however, the metatropic terminology for those special endotropic and exotropic relationships which cannot be expressed without it.

[†] Similar relations to those of plants may occur in animals having a repetition of gonads; but such animals have not yet been investigated from the standpoint of heredity, so that no comparison with our plant results is possible.

[‡] If the individual flower, as is occasionally the case, possesses a system of ovaries or sub-ovaries ($a_1, a_2, a_3, \ldots$) and these are associated with individual systems of anthers ($A_1, A_2, A_3, \ldots$) then the system of relationships becomes still more complex and breaks up in the case of seed from the same flower head alone into the 37 types:

$$aA \,.\, aA = \begin{cases} a_1A_1 \,.\, a_1A_1;\ a_1A_1 \,.\, a_1A_2;\ a_1A_1 \,.\, a_2A_1;\ a_1A_1 \,.\, a_2A_2;\ a_1A_1 \,.\, a_2A_3;\ a_1A_2 \,.\, a_1A_2; \\ a_1A_2 \,.\, a_2A_1;\ a_1A_2 \,.\, a_2A_3;\ a_1A_3 \,.\, a_2A_3;\ a_1A_2 \,.\, a_1A_3;\ a_1A_3 \,.\, a_2A_1; \end{cases}$$

$$aA \,.\, aA' = a_1A_1 \,.\, a_1A_1';\ a_1A_2 \,.\, a_1A_1';\ a_1A_1 \,.\, a_2A_1';\ a_1A_2 \,.\, a_2A_1';\ a_1A_3 \,.\, a_2A_1';$$

$$aA' \,.\, aA' = a_1A_1' \,.\, a_1A_1';\ a_1A_1' \,.\, a_2A_1';\ a_1A_1' \,.\, a_1A_2';\ a_1A_1' \,.\, a_2A_2';$$

$$aA' \,.\, aA'' = a_1A_1' \,.\, a_1A_1'';\ a_1A_1' \,.\, a_2A_1'';$$

$$aA \,.\, aB = a_1A_1 \,.\, a_1B_1;\ a_1A_2 \,.\, a_1B_1;\ a_1A_1 \,.\, a_2B_1;\ a_1A_2 \,.\, a_2B_1;\ a_1A_3 \,.\, a_2B_1;$$

$$aA' \,.\, aB = a_1A_1' \,.\, a_1B_1;\ a_1A_1' \,.\, a_2B_1;$$

$$aB \,.\, aB = a_1B_1 \,.\, a_1B_1;\ a_1B_1 \,.\, a_1B_2;\ a_1B_1 \,.\, a_2B_1;\ a_1B_1 \,.\, a_2B_2;$$

$$aB \,.\, aB' = a_1B_1 \,.\, a_1B_1';\ a_1B_1 \,.\, a_2B_1';$$

$$aB \,.\, aC = a_1B_1 \,.\, a_1C_1;\ a_1B_1 \,.\, a_2C_1.$$

If instead of collecting seed from a single capsule we collect all the seed from all the capsules of a plant, we may still further widen our range to 20 grades of sibship, and it is not by any means certain that these 20 grades will give as close a general relationship, as the 9 grades from the same capsule even if the plant and not the flower be the individual. For, the more capsules taken the more chance there is for variety in the crossing with other plants, i.e. there is increased admixture of the half-sibling relationships.

It is further obvious how wide must be the range of experiment if we are to determine the relative influence of these various modes of crossing; for, every heredity series to have validity ought to contain at the very least 100 plants, and the majority of the cases discussed require artificial fertilisation, and in some of the metatropic cases this is of a rather complex kind.

The series of experiments considered in this paper do not deal with the question of fertilisation at all, although tentative experiments in this direction have been commenced this year. Shirley poppies grown in masses are both self- and cross-fertilised, the latter principally by bees although the flowers are not honeyed. The relative extent of each form of fertilisation could probably not be determined without special investigation or experiment*. To what extent the cross-fertilisation is really endotropic and not exotropic, in the case of plants having sometimes a dozen flowers in blossom at the same time must also be the subject of a later enquiry. It is clear, however, that the relative proportions of homotropy, endotropy and exotropy may vary very largely from one crop to a second and thus largely influence the 'parental' and 'fraternal' relationships.

Other matters directly influencing the constants are, besides soil and climate, differentiation between early and late flowers and between high and low flowers (which is partly the same thing), and the effect of special treatment of individual plants. We shall refer to these topics again later.

(3) *Influence of Environment on Crop-constants.* It must be remembered that we had three different kinds of seed, and that two of these: Individual Plant Seed (α), and Individual Band Seed (β) were selected, and further the amounts of each subgroup sown in the different crops would naturally differ rather widely. Hence the mean and variability of any crop cannot be effectively compared with that of the first Hampden crop on the basis of either (α) or (β). For comparison the General Seed (γ) was provided, but unluckily this failed in two instances; at Bookham no return for (γ) was made, and at Highgate the experimental patch of (γ) produced only one or two starveling plants. We have the following results:

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* Messrs Sutton & Co. to whom I am indebted for information on this point hold that the relative extent depends on the state of weather at the time of flowering and on other conditions.

TABLE I.

Control Seed (γ), 1900.

Crop	Capsules	Mean	S. D.
Original Hampden ...	4443	12·51	1·898
Highgate	wanting	wanting	wanting
Oxford[*]	531	9·29	2·134
Chelsea[†] II	40	10·45	1·688
Crockham	322	13·01	1·977
Bookham	wanting	wanting	wanting
Enfield I	403	14·26	1·753

Thus the town conditions of Chelsea and the potting at Oxford were unfavourable to the development of many stigmatic bands, while at Crockham and Enfield the soil or climate were more favourable than on the top of the Chilterns. The alterations in variability are not so marked, but the potting in a specially mixed soil while it lowered the mean seems to have increased the variability.

We can deal with the influence of environment in a somewhat different manner on the basis of the seed (α). We can compare what the weighted parents

TABLE II.

Weighted Parents and Local Offspring. Seed (α).

		HAMPDEN PARENTS		LOCAL OFFSPRING	
Crop	No.	Mean	S. D.	Mean	S. D.
Original Hampden ...	706	12·68	1·219[‡]	—	—
Highgate	606	13·21	1·120	12·03	1·890
Oxford...	2728	12·72	1·106	10·76	2·293
Chelsea II	219	12·04	1·694	11·84	1·808
Bookham	1905	12·54	1·368	12·76	2·217
Enfield I	4204	12·76	1·165	13·95	1·622
		ENFIELD I, PARENTS		LOCAL OFFSPRING	
Original Enfield I ...	—	14·00	1·311[§]	—	—
Kidderminster	19,027	14·26	1·321	13·13	1·721

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[*] The Oxford poppies were grown in pots placed on *gravel*. If we add to the above another 45 pots of seed (γ) grown in pots placed upon a *garden bed* we find for the whole series of 967 capsules: mean 10·87, S.D. = 3·001. This shows the great influence of an even slight change of environment.

[†] Chelsea I, the crop of 1899 from different seed gave on the basis of 1020 capsules a mean of 12·37 and a standard deviation of 1·680, the equality of the variabilities is possibly only accidental.

[‡] These are the mean and standard deviation of the selected parent Hampden plants, of which I took all the seed from all capsules. A distinction must always be drawn between plant mean and capsule mean.

[§] These are the mean and standard deviation of the selected 100 Enfield parent plants.

would have done under the Hampden conditions with what the offspring plants actually did in the given locality.

The mean of the 706 capsules on the Hampden parent plants was 12·75 with a standard deviation of 1·783, while the total Hampden crop had a capsule mean of 12·51 and standard deviation of 1·898. Thus my selected parentage had a higher mean and a much reduced variability, however measured. It will be observed, however, that the weighted parental mean, i.e. parents weighted with the number of offspring capsules, always differs from the mean of the parent plants. We cannot, however, determine how much of this was due to environment; the seed of the original 24 parent plants could not be distributed in normal population proportions, the offspring of each plant which survived differed in number according to the amount of seed sown, the thinning, etc. At Chelsea the storm chanced to destroy the offspring groups having parents near the mean, and thus artificially exaggerated the variability of the Chelsea parental selection. At Oxford where W. F. R. Weldon attempted to deal with individual plants by sowing in pots (and so by subjecting individuals to a like treatment a record of the relative fertility of the parent seed was possible) it was found that even the nature of the ground immediately under individual pots influenced individual development. Thus at Chelsea and Bookham, only 12 and 16 of the parents respectively contributed to the offspring, and this fully accounts for the variability of the actual weighted parents being in these cases considerably greater than that of the original Hampden parentage. The conditions at both Highgate and Oxford were adverse to the plant: at Highgate one capsule to a plant was about the average, and at Oxford the average was only 3·2. The low variabilities of the parentages at these places, are therefore probably due to the selection of parent seed by the environmental conditions, but a more definite statement than this it would be unwise to make. Comparing, however, the actual parentages with the local offspring we see at once that the Bookham and Enfield environments were more favourable to the development of stigmatic bands, and Oxford, Highgate and Chelsea less favourable than Hampden. The Kidderminster crop while far heavier than the Enfield in the number of fruit per plant shows a diminution in the mean number of stigmata. Thus the environment largely affects the number of stigmata, but there is no evidence to show that this is appreciably influenced by parentage in bulk. Thus while the selected parentages range only from 12·04 to 13·21, a range of 1·17, the resulting offspring means range from 10·76 to 13·95, or a range of 3·19. Nor is there any system in the arrangement, e.g.

Order of Parentage	Order of Offspring
Chelsea	Oxford
Bookham	Chelsea
Oxford	Highgate
Enfield I	Bookham
Highgate	Kidderminster
Kidderminster	Enfield I

which could be interpreted on the basis of a law of regression applied to the crop means.

Thus, whatever theory of heredity we apply, it must allow for (i) absolute variation of the mean due to treatment and environment of the offspring crop as a whole, and (ii) selection of parentages differing from crop to crop. Now these are the very points so often misinterpreted by those who fail to grasp the theory of regression. The first difficulty is got over by remembering that regression is not towards the parental but towards the filial mean, and the second by noticing that while both parental and filial variabilities and the correlation are changed by selection of parents, the slope of the regression line is not changed, and that the slope of this line would be the coefficient of parental heredity supposing the population to reproduce itself stably, i.e. without change of variation from generation to generation. In other words regression is summed up in:

Probable deviation of individual offspring from filial mean = coefficient of parental heredity × actual deviation of individual parent from parental mean.

It is this "coefficient of parental heredity," equal to correlation between parent and offspring multiplied by the ratio of filial to parental variability, that we must find to test how far heredity in our poppies is in accordance with what we have found it to be for men, horses, etc.

Lastly we may consider seed (β) drawn from a variety of capsules with the same number of stigmata, although we have here of course no measure of the original Hampden parentage. The following table gives the results.

TABLE III.

Weighted Capsules and Local Offspring. Seed (β).

		Parent Capsules		Offspring Capsules	
Crop	No.	Mean	S. D.	Mean	S. D.
Oxford...	1140	12·89	3·143	9·83	2·248
Chelsea II*	96	11·24	3·305	12·19	1·660
Crockham	2991	12·34	3·537	13·46	2·119
Enfield I	2066	12·27	3·319	14·26	2·018
Enfield II	566	14·41	2·203	12·16	1·751

It will be seen at once that the general results are the same, Oxford and Chelsea II are the poorest of the 1900 crops; Oxford is the most variable. But Enfield II was also a poor crop, the mean having fallen back in 1901 below even the Hampden parent value from which it rose in 1900. While in Table II. the variability of the offspring was in each case greater than that of the parents,—because we were comparing a capsule variability in the former, with a mean plant variability in the latter,—in Table III. the parent capsules are markedly more variable than the offspring capsules. This is precisely what we should expect, if the individuality on which heredity depends lies in the plant as a whole and not in the special capsule. We shall investigate this at greater length below.

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* Chelsea II is hardly worth recording with only 96 capsules. 40 out of the 96 capsules obtained were on plants the seed of which was taken from capsules with 7 and 8 bands, whereas at Enfield only 360 out of 2066 capsules came from these low band groups. It is possible that low band seeds were better suited to the unfavourable Chelsea environment.

(4) *Relative Fertility of Different Capsules.* Direct experiments were not made on relative fertility, but one or two points may be noted. In examining the Crockham capsules, we were again* struck by the sterility of the low banded, and to a lesser extent of the high banded poppies. W. R. Macdonell for the second Enfield crop selected 57 capsules and sowed from the seed of these capsules 57 rows. Of these rows 22 were from capsules with 9 to 12 bands, 28 from capsules of 13 to 15 bands and 7 from capsules of 16 to 20 bands. 10 rows of the first group, none of the second and 3 of the third were unproductive, or about 45 per cent. of the low banded and 43 per cent. of the high banded capsules failed, while none of the capsules near the modal value (about 14) were unproductive.

In the Oxford Band Series (β), W. F. R. Weldon sowed 110 pots from capsules 7 to 10, 150 pots from capsules 11 to 15, and 90 pots from 16 to 18, each pot being sown with a few seeds. 43 per cent. of the pots in the first group failed to show germinations, and 21 per cent. in the third group, while 8 per cent. only failed in the group from 11 to 15, the modal value of the original Hampden crop being 12·75 bands per capsule. The poppies were sown on March 25—26, and the productive pots counted from April 12 to 20. Further, on April the 21st, the first group from capsules of 7 to 10 had about 5 seedlings per pot, the second group from capsules of 11 to 15 about 22 seedlings per pot, and the third group from capsules of 16 to 18 had 10 seedlings per pot.

Now of course this seedling result admits of more than one interpretation, for it may be said that there was more seed of the modal capsules and that more of it may have been sown. While it required comparatively few modal Hampden capsules to provide a good amount of seed, almost every available low-banded and many high-banded had to be dealt with to collect enough seed at all from these classes of capsules, thus the seed was certainly not in proportion to the frequency of the capsules in the original crop, nor was it sown in that proportion. Hence Weldon's percentage of productive pots and proportion of seedlings seem on the whole to confirm Macdonell's results, which started not with equal quantities of seed, but with the seed of *single* capsules.

From the Seed (α) it is difficult to draw any conclusions, for we do not know what proportion of the seed came from modal capsules, and it is therefore not possible to test the general principles (i) that the modal capsule has most seed and (ii) that this seed more readily germinates. On the whole there is evidence enough in favour of these principles from the band series to make it desirable that experiments directly bearing on these points should be instituted next year.

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* *Grammar of Science*, p. 444.

(5) *Homotyposis.* We have now eight cases of homotyposis worked out for Shirley Poppies. The results obtained show a very considerable variation in the homotypic constant, and it is undoubtedly affected by the environment and treatment of the crop. Chelsea II was such a failure that no further constants were determined for it. The Highgate crop had very rarely two fruits to the plant and so homotyposis could not be dealt with. Both the Bookham and the Crockham crops were gathered as plants which were tied into bundles for each series, the other crops having the fruits collected off each individual plant into separate receptacles. Thus in the former crops all the capsules from the same series, but possibly not all the capsules from the same plant, have been grouped together. This would to some extent affect the homotyposis of these series[*]. The Chelsea I crop suffered, as has been elsewhere recorded, much from selection after gathering and the Enfield II crop was a remarkably poor one, having on the average only 2 to 3 capsules per plant. The general results are given in Table IV.

TABLE IV.

Homotyposis in Shirley Poppy.

Crop	Year	Plants	Capsules	Capsules per plant	Pairs	Mean	S. D.	Homotyposis
Hampden ...	1899	176	4443	25·2	197,478	12·61	1·885	·5238
Chelsea I ...	1899	325	1020	3·1	2,756	12·37	1·680	·6149
Oxford ...	1900	861	2728	3·2	11,588	11·39	2·173	·5573
Crockham...	1900	355	2991	8·4	61,334	13·39	2·055	·3446
Bookham ...	1900	1029	1905	1·9	4,338	13·14	2·330	·3475[†]
Enfield I ...	1900	907	2066	2·3	19,370	13·78	1·518	·5093
Enfield II ..	1901	239	566	2·4	1,244	12·72	1·717	·6831
Kidderminster	1901	1618	19,110	11·8	329,840	13·18	1·702	·4025
Mean	—	—	—	—	—	—	—	·4978

Now it will be seen at once from this table that the magnitude of the homotyposis is not related at all or not in any simple manner to number of plants, number of capsules, number of pairs, or to average number of capsules per plant used in its determination. Nor does a high or low variability seem significant for changes in the homotypic correlation. On the other hand the four crops with low means have the four highest correlations and the four crops with high means have the four lowest correlations. But even thus the order of correlations is by no means the order of relative means. We hold that, at present, all we can safely conclude is that any treatment or change of environment of a crop which tends to raise its mean, will in this case lower its homotyposis. Perhaps we might even say that 'starveling' conditions, which certainly existed in Chelsea I, Oxford and Enfield II, tend to intensify the homotypic relations, while favourable conditions such as those of the magnificent Kidderminster crop, or the very good Crockham crop tend to reduce the homotyposis (see p. 58).

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[*] G. U. Yule writes: "The plants were sometimes broken, thus capsules belonging to the same poppy may be separated, but capsules entered between bars certainly belong to the same plant." I.e. one broken plant might occasionally be counted as two plants.

[†] The Bookham homotyposis is unsatisfactory; the record of capsules from the same parentage was satisfactory, but that of capsules from the same plant was very defective, as a rule only 1 to 3 capsules were recorded as certainly from same plant, but two plants had between 30 and 40 capsules, and these had to be omitted, or they would have given rise to one-third the total number of pairs.

The general average ·498 of this series of eight agrees excellently with that ·499 found for a former series of thirty-seven cases in the animal and vegetable kingdoms*—only the first two crops being common to the two series. We see that treatment may considerably influence individual results, but, as it appears as likely to tend in one as the other direction, the mean remains steady.

(6) *Methods of measuring Plant-character for Inheritance.* When we are considering quantitative inheritance in plants we are at once confronted by the fact that although the flower, fruit, or leaf may be convenient subjects for observation, they are not as a rule unique in the plant, and consequently we are confronted with a multiplicity of elements for the determination of the character—a very little observation showing that variability within the individual is nearly as marked as the variability of individuals among themselves. Thus a single fruit or flower cannot be taken as a sample character of an individual,—for doing so is at once assuming that homotyposis is perfect or unity, which we know is very far from the case. The consideration therefore of plant inheritance leads us at once owing to the multiplicity of organs to homotypic relations. A like result rarely occurs when we deal with heredity in mammals, for their homotypes are not organs, which directly present themselves as suitable for the problem of inheritance. In dealing with plants accordingly we cannot take a single organ, but must deal either with all such organs, or at least with a sufficiently large random sample of them.

If we express the character of the individual plant by the mean of any quantity observed or measured on such organs, we are at once impressed with the varying degree of weight to be given to this character according to the number of fruits or flowers, etc. upon which the estimate is based. A Shirley poppy may have any number of capsules from one to eighty and the average number of stigmatic bands per capsule, which may be taken to express the individual plant character, will have a great variety of weights according to the number of capsules dealt with. In some plants it might be possible to take a random sample of 25 or 50 flowers or fruits, etc. and strike the average of these. But this would be laborious in the case of a crop of 1000 individuals 5 to 10 as a sample would be easier, but not nearly so reliable, and even so few could not have been obtained in the case of many Shirley poppies. We therefore determined to take all the capsules and include them all in our calculations, but, to avoid the great labour of taking means and weighting, to adopt other processes for linking parents to offspring, using the mean of the capsules of the former and the individual capsule of the latter.

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* *Biometrika*, Vol. I. p. 343.

Let us consider the N offspring plants $o_1, o_2, o_3 \dots$ of one individual parent plant with capsule-mean P, and let these offspring plants have $n_1, n_2, n_3 \dots$ capsules; suppose these capsules to have $c_1, c_1', c_1'' \dots c_2, c_2', c_2'' \dots c_3, c_3', c_3'' \dots$ stigmata respectively.

Then the mean character of an offspring would be $o_1 = S(c_1)/n_1$, and the offspring mean, if all individuals were of equal weight, would be

$$O = S(o_1)/N \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\text{(i)}.$$

If O were plotted to each P, we should have a series of points, from which we could at once deduce the regression line of offspring on parents, and so the intensity of heredity.

If the individual offspring have not equal numbers of capsules, the offspring means ought to be weighted, and we should have instead of (i)

$$O = S(o_1\sqrt{n_1})/S(\sqrt{n_1}) \quad \dots\dots\dots\dots\dots\dots\dots\dots\text{(ii)}.$$

This formula is extremely troublesome for practical calculations, and when we are dealing with perhaps 50 to 100 offspring of each parent practically unworkable.

If we weight, however, with n and not $\sqrt{n}$, we have

$$\begin{aligned} O &= S(o_1 n_1)/S(n_1) \\ &= \frac{\{S(c_1)/n_1\}\, n_1 + \{S(c_2)/n_2\}\, n_2 + \{S(c_3)/n_3\}\, n_3 + \dots}{n_1 + n_2 + n_3 + \dots} \\ &= \text{mean of all capsules in the array of plants due to one parent} \dots \text{(iii)}. \end{aligned}$$

In this case all we have to do is to correlate the individual capsule with the parental mean capsule; this correlation will not be significant for heredity, but if we calculate the slope of the corresponding regression line, it will be the slope of the regression line of parental inheritance, as far as it is legitimate to replace (ii) by (iii). Direct test in a few cases showed that with a considerable number of offspring, not only (ii) and (iii), but even (i) led to very close results.

Accordingly our first method will consist in correlating all offspring capsules with the parental mean capsule, and then determining the slope of the regression line—the measure of heredity, free, as we have seen on p. 65, from the effects of selecting individual parents, and also of environmental change of mean.

Secondly we may obtain a measure of the intensity of inheritance in the following manner. We may compare the average variability of an array of offspring due to a single type of parent plant with the variability of all the offspring population. This method has certain advantages, if we suspect that the environment of a crop has not necessarily been continuous throughout, and that sub-environments may have altered in an arbitrary manner the means of the different parent plants. For example, with only a few parents some may have been more highly favoured by light, soil or water than others. At any rate this method is valuable for purposes of control, although as it involves the labour of finding individual offspring means, it can only be occasionally applied.

Thirdly we may proceed by the homotypic relationship. This requires a brief theoretical treatment.

Let p be the mean character in any individual parent, P be the mean parent; o the mean character in any offspring, O the mean offspring and R parental correlation; then if $\sigma_p\ \sigma_o$ be the standard-deviations of parent and offspring respectively, and N the total number of cases, we have

$$R = \frac{S(p-P)(o-O)}{N\sigma_p\sigma_o} \quad \text{.............................(iv).}$$

But if we weight the offspring as on p. 69 with the capsule numbers of each individual, we shall have

$$R = \frac{S(p-P)(no-nO)}{N'\sigma_p'\sigma_o'},$$

where n = number of capsules in an offspring plant, $N' = S(n)$, σ_p' = standard deviation of parents and σ_o' of offspring weighted with capsule numbers.

If now we use c to denote the capsule, we have $no = \Sigma(c)$, where Σ is the summation for every capsule of an individual. Further

$$O = S(no)/S(n) = S\Sigma(c)/N' = C,$$

where C is the mean capsule of the whole series of offspring. Hence we find

$$R = \frac{S(p-P)\Sigma(c-C)}{N'\sigma_p'\sigma_o'}$$

$$= \frac{S'(p-P)(c-C)}{N'\sigma_p'\sigma_o'} \quad \text{..............................(v),}$$

where S' is a summation of every parent plant and offspring capsule. But if r be the correlation between parent plant and offspring capsule

$$r = \frac{S'(p-P)(c-C)}{N'\sigma_p'\sigma_c}.$$

Hence it follows that

$$R = \frac{\sigma_c}{\sigma_o'}\, r^{*} \quad \text{....................................(vi).}$$

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[*] Multiplying by σ_o'/σ_p' we have the regression coefficient equal to $r\sigma_c/\sigma_p'$ the result of our first method.

But since the present system of weighting makes O and C the same, we have

$$\sigma_c^2 = s_c^2 + \sigma_o'^2,$$

where s_c is the mean S.D. of all the offspring arrays of capsules $= \sigma_c\sqrt{(1-\rho^2)}$, if ρ be the coefficient of homotypic correlation. Thus we have

$$R = r/\rho \quad \text{..................................(vii).}$$

(vii) would probably be a good result to work from if we wanted to find the parental heredity R from r and ρ in material where neither parents nor offspring were selected, and where there was no marked change of environment between the two generations. Possibly the influence of selection of offspring and of their environment, since they affect both r and ρ, may be less marked in (vii) than in $r\sigma_c/\sigma_p'$, the regression given by the first method, for r and ρ (see Table VI.) may tend to rise and fall together, and thus the third method may in some cases give us better results than the first.

TABLE V.

Parental Inheritance. First Method.

Crop	Correlation of Parental Mean and Offspring Capsule	Slope of Regression Line
Highgate ...	·3230	·5451
Oxford ...	·1960	·4064
Bookham ...	·2199	·3412
Enfield I ...	·1864	·2595
Kidderminster	·1220	·1589
Mean	·2095	·3422

(7) *Results for Parental Inheritance.* We have seen that the first method ought to eliminate the effect of a selected parentage, but that it would not be uninfluenced by a selection of offspring. There can hardly be a doubt that the order of Table V. is practically that of the stringency of environment for the five crops,—the most starveling crop being the Highgate one and the most flourishing the Kidderminster. Or, it would seem that the more luxuriant the crop, the less intense is the strength of heredity. The mean value of the slope of the regression line is not far from the $\frac{1}{3}$ originally given for parental heredity in human stature by Galton, but it is considerably less than the value (about ·45) recently obtained for about sixteen cases in man*. We have already pointed out that the restriction of the variability of the general population to be found in an array of offspring due to a single parentage might still be maintained, if different parents had received different treatments. With the assistance of Marie A. Lewenz this point was investigated. She struck the means of all the capsules on each of the 907 Enfield I plants. These plants were then grouped into families and their

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* Unpublished Family Measurements on upwards of 1000 families in the possession of K. Pearson.

individual standard deviations as families taken, as well as the mean and standard deviation of the population of 907 plants. The mean of the plant means was 13·75 stigmata, and their standard deviation 1·370. The average variability (or S.D.) of plants from the same parentage was 1·272, and the weighted mean square deviation—i.e. $\sqrt{S(n\sigma^2)/N}$, where σ is the standard deviation of a fraternity of n, and N the total number of plants—was 1·287. If r be the parental correlation $\Sigma\sqrt{1-r^2}$, Σ being the total plant variation, should be that of an array of offspring. Hence equating this to 1·287 we find $r = ·3427$. Had we equated it to 1·272 we should have found $r = ·3716$. In either case this, our second method, gives a value much closer to that ·3660 of the third method, than the value ·2595 found for the slope of the Enfield crop regression line.

Finally to illustrate the second method more completely a correlation table has actually been formed for the means of parent and offspring plants in the case of the Enfield I crop. The value of the parental mean plant is 12·69 of the offspring plant, 13·75, of their respective standard deviations 1·199 and 1·370; the correlation is ·1561 and the regression coefficient of offspring on parent plant ·1784. Nor is this result really to be much wondered at, for not only are the means of the stigmatic bands much influenced by the number of capsules on the plant, but the flowers that come out early in the season have fewer bands than those which come later. Thus taking the Oxford record for *first* flowers on 322 plants, we divided them into 161 earlier flowers of each family and the 161 later flowers, and found a correlation between the number of stigmatic bands of the first flower and its lateness in coming out of ·2078. The number of capsules to the individual plant, and the dates at which it produces them, tend to obscure the influence of pure heredity, and make the stigmata however easy to count and deal with a by no means ideal character to study heredity upon.

To avoid the difficulty due to differentiation in capsules, a correlation table was formed for 327 *first* flowers and the means of the parent plants in the Oxford crop. We found parental mean 12·74, standard deviation 1·240; stigmata of first flowers of offspring 11·02, standard deviation 1·828; correlation ·2438, and regression of offspring first capsule on parental mean = ·3594. There can be small doubt that, if we had compared the stigmata on the first flower of the parent with those on the first flower of the offspring, we should find heredity in poppies as intense as in forearm or cephalic index for the case of man. It is true that the Enfield I and Kidderminster crops show a remarkably low parental correlation, but both were large crops and it is highly probable that the environment of the strips devoted to special families was to some extent differentiated, and this may well have weakened the apparent strength of heredity*.

Finally if we attempt to obtain the parental correlation by our third method, we have the following table:

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* Mr John Notcutt assures me that for the Kidderminster crop there was no differentiation in environment apparent for either light, shelter or soil.

TABLE VI.

Parental Inheritance. Third Method.

Crop	Correlation of Parental Mean and Offspring Capsule	Homotyposis	Parental Correlation
Highgate ...	·3230	[·6490 ?]	[·4976 ?]
Oxford	·1960	·5573	·3517
Bookham ...	·2199	[·3475 ?]	[·6328 ?]
Enfield I ...	·1864	·5093	·3660
Kidderminster	·1220	·4025	·3031
Mean... ...	·2095	·4897 [·4931 ?]	·3403 [·4302 ?]

The Highgate poppies were a starveling crop with rarely more than one capsule to the plant. Hence no determination of homotyposis was possible. The most like crops were Chelsea I and Enfield II and the mean homotyposis of these two is put in brackets with a query. We have already seen that the Bookham homotyposis result (p. 67, ftn.) is doubtful, and probably this result ought also to be excluded. Omitting these two results we find the mean in good agreement with that obtained from the first method, i.e. ·3422.

We may safely conclude that parental relationship for the stigmatic bands in Shirley poppies is expressed by a correlation lying between ·35 and ·4, but that its value is considerably influenced by the conditions of the individual crop, or even by differential treatment of parts of it.

(8) *Hereditary Influence of the Capsule and of the Plant.* One of the questions proposed to those assisting in the present investigation was the problem of the relative individuality of the plant and its fruit. Was the resemblance of the offspring to the special capsule from which its seed had been extracted greater than to the average capsule of the parent plant? The data for answering this question were to be found in the crops grown from Seed (β). Only three such crops were sown in 1900 and the following table contains the results.

TABLE VII.

Capsule Parentage.

Crop	Correlation of Offspring and Parent Capsules	Regression	
		Offspring on Parent Capsule	Offspring on Parent Plant
Oxford	·1220	·0873	·4064
Crockham ...	·1949	·1168	— *
Enfield I ...	·1492	·0907	·2595
Mean... ...	·1554	·0983	·3320

* No crop was grown at Crockham from Seed (α).

It would seem accordingly that individuality lies in the plant rather than in the fruit, i.e. that an offspring capsule only takes after the parent capsule, because that capsule is grown on the parent plant, and that the individuality of the capsule in the plant does not influence the seed it bears*.

The seed of the above crops was all of the same kind, namely "capsule parentage 14" consisted of all the seed contents of many 14-capsules on *a variety* of Hampden plants. W. R. Macdonell in a crop of 1901 modified this experiment to some extent by selecting the seed of 57 capsules, of which 44 gave rise to 239 plants with 566 capsules—a 'starveling' crop. Thus while the plants at Hampden had an average of 25·2 capsules per plant and this gave a wide variety of capsules on the plant, the Enfield I had only 2·3 capsules on the plant and the Enfield II, the above 239, only 2·4 capsules per plant. The individuality of the plant in Enfield II was thus possibly more likely to be represented by a selection of its few capsules than by a selection from the many capsules on a Hampden plant. The results obtained by Macdonell were:

Crop	Correlation of Offspring and Parent Capsules	Regression of Offspring on Parent Capsule
Enfield II ...	·2819	·2241

The regression here is more than double that found in the earlier series of experiments, but it is still considerably less than the average value of the regression of offspring plant on parent plant, ·3422, determined on p. 73. Macdonell further correlated the capsules of the Enfield II crop with the grandparental Hampden plants,—grandparental means and offspring capsules. He found for the weighted grandparental mean of the 566 offspring 13·11 and for the grandparental S. D. ·767. This shows that much selection of grandparents had taken place for the mean and S. D. of the Hampden parents were 12·68 and 1·219. The correlation of grandparent plant and offspring capsule was ·0066 and the regression of the former on the latter ·0190. Both of these are really insignificant. But it is difficult to appreciate in this case the allowance which has to be made (i) for selecting in the intermediate stage not a parent plant, but an arbitrary capsule off the parent plant, and (ii) for comparing offspring capsule with grandparental plant. The result as far as it goes confirms of course the previous statement, that the closer degree of hereditary relationship is between plant and plant and not between capsule and plant ancestry.

That a slight variation in the material dealt with makes a considerable difference in plants, which are peculiarly susceptible to environmental and seasonal influences in their flowers, is well evidenced by the following results provided by W. F. R. Weldon from his Oxford crop.

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* This conclusion has been further confirmed by the case of *Nigella Hispanica*. There was sensibly no relationship in this plant between the segmentation of the individual capsule on a plant and that of the capsule from which the seed which gave rise to that plant was taken. K. P.

TABLE VIII.

Nature of Group dealt with	Correlation of Parent Plant and Offspring Capsule	Regression of Offspring on Parent Plant
(*a*) Early capsules (apical flowers) of principal plants*	·2323	·4003
(*b*) Capsules on plants—not starvelings, i.e. with at least three capsules	·2430	·4050
(*c*) All capsules on principal plants...	·2295	·4295
(*d*) All capsules on all plants	·1960	·4064

The regression therefore is fairly constant, although the correlation varies. The relationship as expressed by the correlation is greatest, when we avoid differentiation in either the plants themselves or in their fruits, but such differentiation does not widely influence the regression, which for this Oxford crop takes a mean value not very divergent from what has been found for the forearm in man (·42 as mean of four results).

(9) *Grandparental Inheritance.* Our data for grandparental inheritance are very limited and suffer even more than the parental material from changes in crop treatment. Thus the original grandparents were grown at Hampden on the top of the Chilterns, the parents at Enfield and their offspring at Kidderminster. There is thus considerable change not only in soil but treatment; unfortunately we had not the means of carrying on the experiments under uniform supervision in the same environment. From the Kidderminster crop, we had 19,204 capsules due to 100 parent plants and 24 grandparental plants—in all cases the pollen plants being unknown. The grandparental plant mean was 12·64 and the standard deviation 1·1609. The grandchild capsule mean was 13·19 with a standard deviation of 1·7526. The correlation between grandparental plant mean and grandchild capsule was ·0490. The regression of offspring capsule on grandparental plant mean was thus ·0739. If we use the third method and the value of the Kidderminster homotyposis it would be ·1216. The only result with which these can be compared is the Enfield II grandparental capsule result (see p. 74). Dividing this by the product of the two Enfield homotypic values, we should have ·1897 for the grand-

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* On the thinning out one plant, the principal plant was left in a pot, but later other seed germinated and small secondary poppies were found in a number of pots.

parental relationship of plant means. Our meagre results may then be summed up in the following table:

TABLE IX.

Grandparental Inheritance.

Crop	Method	Grandparental Inheritance	Parental Inheritance
Kidderminster	Regression	·0739	·1589
,,	Homotyposis	·1216	·3031
Enfield ...	Homotyposis	·1897	·3660

These results at least indicate that, however determined, the grandparental relationship is fairly *close to half the parental.* It is much to be regretted that we have not wider data, especially on a less luxuriant crop than the Kidderminster one, for the above results are very variable.

(10) *Colour Inheritance.* W. R. Macdonell made an elaborate classification of the colour of 1604 flowers divided according to their parentage into 24 groups. He formed 13 colour classes, which after consideration were classified into three groups according to the intensity of red colouring matter in the flower. The first group embraced all the range from dark red to red-whites; the second the pure pinks and the third all the pink-whites to pure whites. In this classification attention was not paid to the base of the petal, but to the middle portion and margin. The following are the frequencies of the original scale:

(1)	Red, bordered lilac ...	2	(8)	Pink-white, bordered pink	0
(2)	Red	275	(9)	Pink-white	174
(3)	Red, bordered white ...	70	(10)	Pink-white, bordered white	155
(4)	Red-white	31	(11)	White, bordered pink ...	0
(5)	Red-white bordered white	43	(12)	White faint pink... ...	76
(6)	Pink	622	(13)	White	50
(7)	Pink, bordered white ...	106		Total	1604

Classes (8) and (11) had representatives in crops from Seed (β) and (γ). Classes (1) to (5) formed our first general class, (6) our second, and (7) to (13) our third. This grouping enabled us to use the method for dealing with characters not quantitatively measurable, assuming that the distribution of red colouring matter is an approximately normal one. If σ be the standard deviation of the whole population of 1604 offspring flowers, and Σ the mean standard deviation due to the arrays having a common parent $\Sigma = \sigma\sqrt{1 - r^2}$, where r is parental correlation. Now σ was found in terms of p, the pink range, by means of tables of the probability integral, and we have $\sigma = p\,(1{\cdot}0218)$. The standard deviation of each array of offspring was found in like manner, and the pink range eliminate between this and the racial standard deviation. There resulted

$$\Sigma/\sigma = {\cdot}9111,$$

or Parental Correlation in Colour = ·4122.

This result,—in excellent accordance with those for forearm and span in 8 cases of 1000 each in man—suggests that colour, even with the rough classification indicated, may be effectively dealt with. The continuity of the intensity of colouring in Shirley Poppies is fairly obvious when crops of thousands of flowers are examined. We propose next year to form a more elaborate scale of colouring and continue the experiments especially on the colour side.

Meanwhile the only comparable data we can use occur in the Oxford crop. Here the colours were observed on the first flowers of 319 plants distributed among 24 parentages. Owing however to a different colour classification to W. R. Macdonell's, only 11 of these parentages can be used,—the colour scale not extending over three classes in the others. The mean standard deviation of these 11 parentages—containing only 153 flowers, however,—bears the ratio of ·8479 to the standard deviation of the whole set. Hence we find parental correlation ·5302. This result is not very reliable, of course, the data being so few; yet it is in accordance with the higher parental inheritance values obtained generally for the Oxford crop.

(11) *Fraternal Correlation.* While for resemblance of offspring to parent we have compared an array of offspring plants with the mother plant the seed being taken from one or all capsules of the mother plant, we have a less definite conception in the case of an array of brother plants. All members of the array have, of course, a common mother plant, they are co-ovarial or bi-ovarial brethren, but the proportions of synanthic, dianthic and dysanthic crossing are quite unknown. Hence it is difficult to guess *a priori* at the amount of common pollen parentage in pairs of brother plants. We can hardly, however, suppose that all the seed in the same capsule has the same pollen parentage, but, except in the case of self-fertilisation, we should expect seed from the same capsule to give plants more alike than seed from different capsules. Taking the best results for fraternities of common parentage in man—mean of about 30 results—we should expect the resemblance of brothers to be about ·5. This might be reduced to anything down to ·25, i.e. case of half-brothers, by the degree of diversity in the pollen, from which the arrays of brothers are due. Hence we must be prepared for fraternal correlation in poppies lying between ·25 and ·5, approaching the lower limit, if there has been the freest possible cross-fertilisation, i.e. a great variety of pollen carried not only to the same mother plant, but even to individual ovaries on that plant.

Before we consider actual numerical results we must, however, modify them for homotyposis, remembering that our tables have been formed by comparing fraternal capsules and not fraternal plants,—at least in the case of stigmata as character.

Let r = fraternal correlation of plant means and R = correlation of capsules on brother plants, ρ = homotypic correlation. Let Σ = a summation within a fraternity and S throughout a population.

Let z = number of stigmata on a capsule, and m the mean number of stigmata on a plant of n capsules; let a bar over a letter denote the mean value of that quantity for the whole population and subscripts 1 and 2 refer to a pair of brother plants. Then

$$z = m + x$$

$$\left.\begin{array}{l}\Sigma(z) = nm \\ S(z) = S(nm)\end{array}\right\} \text{ and } \bar{z} = \bar{m},$$

where $\bar{m}$ is the weighted mean of the plant means.

$$z^2 = m^2 + 2mx + x^2,$$

$$\Sigma(z^2) = nm^2 + \Sigma(x^2) = nm^2 + n\sigma_x^2,$$

if σ_x^2 = standard deviation of an array of capsules on the same plant.

Hence σ_z being the standard deviation of all capsules

$$\sigma_z^2 = \frac{S(z^2)}{S(n)} - \bar{z}^2 = \frac{S(nm^2)}{S(n)} - \bar{m}^2 + \frac{S(n\sigma_x^2)}{S(n)}.$$

The first two terms on the right-hand side form σ_m^2, the standard deviation squared of the means, the third term is the mean square of the standard deviations of the arrays of capsules on the same plant $= \sigma_z^2(1 - \rho^2)$, if ρ be the homotypic correlation. Hence we have

$$\rho^2\sigma_z^2 = \sigma_m^2.$$

Next

$$\begin{aligned} R\sigma_z^2 &= \frac{S(z_1 z_2)}{S(n_1 n_2)} - z^2 \\ &= \frac{S\{n_1 n_2 m_1 m_2 + m_2 n_1 \Sigma(x_1) + m_1 n_2 \Sigma(x_2) + \Sigma(x_1)\Sigma(x_2)\}}{S(n_1 n_2)} - m^2 \\ &= \frac{S(n_1 n_2 m_1 m_2)}{S(n_1 n_2)} - m^2 \\ &= r\sigma_m^2. \end{aligned}$$

Therefore

$$r = R\,\rho^2.$$

Here of course

$$\bar{z} = \frac{S(n_2 z_1)}{S(n_1 n_2)} = \frac{S(n_1 z_2)}{S(n_1 n_2)},$$

$$\bar{m} = \frac{S(n_1 n_2 m_1)}{S(n_1 n_2)} = \frac{S(n_1 n_2 m_2)}{S(n_1 n_2)},$$

and it is assumed that the standard deviations about these weighted means of the capsules weighted with the number of their brother capsules, will be closely equal to the standard deviation of the capsules weighted only with number of capsules in the plant. This will be closely true if the number of capsules per plant, and the number of plants in the fraternity are not widely divergent as for example they were in the case of the Bookham crop*.

TABLE X.

Stigmatic Bands. Fraternal Correlation.

Crop	Number of Plants	Number of Capsule Pairs	Capsule Correlation	Homotyposis	Fraternal Correlation
Highgate ...	606	46,490	·2513	—	·2513
Oxford ...	861	338,276	·0898	·5573	·2891
Enfield I ...	907	706,652	·0496	·5095	·1912
Kidderminster	1618	3,813,832	·0740	·4025	·4568
Mean ...	—	—	—	—	·2971

The Highgate crop had with rare exceptions one flower only per plant. Homotyposis could not therefore be considered. We are really comparing as fraternal character the first or apical flower on each plant. This was also done, as will be seen later, at Oxford with the result that the correlation was ·2561, a result agreeing excellently with that from Highgate. The most divergent result is that for the Kidderminster crop which was also the most anomalous in the parental relationship†. While the fraternal relationship is over-emphasised, the parental was diminished—these results are precisely what we might expect if the plants due to different parentages received differential treatment, i.e. if the environment varied somewhat from sub-crop to sub-crop. In this case the fraternal correlation would be emphasised by the differential environment, which would at the same time tend to distort, and so diminish the parental regression. The Kidderminster crop was such a large one—there were 100 separate parentages, that it would be almost impossible to avoid unrecognisable differentiation.

The average of the above results is of course very satisfactory. The deduction, however, indirectly of the fraternal correlation by aid of the homotypic coefficient may be objected to. Especially is this the case, when we take crops in which there is great variability in the number of capsules to the plant and of brother plants to the family. We here reach the same objection ultimately as arises from endeavouring to form the capsule mean for each plant, this mean having to be based on sometimes 1 or 2 capsules and on other occasions on 30 to 40 capsules. So that unless one weights individual means, one may reach very discordant results. The weighting of 30,000 to 50,000 pairs of brother plants is not, however, a task to be lightly undertaken*.

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* The Bookham crop could not be considered for the fraternal data, because some of the plants being divided into two, we could not distinguish homotypic from fraternal relationship.

† Possibly the Enfield I crop was largely self-fertilised.

Accordingly it is of interest to be able to turn to W. F. R. Weldon's Oxford observations on the *first* flowers of 320 odd plants. He observed the following characters on these first or principal flowers:

(*a*) the number of stigmatic bands,

(*b*) the number of petals,

(*c*) the petal length,

(*d*) the breadth of the margin, when coloured differently from the body of the petal,

(*e*) the extent of basal patch,

(*f*) the intensity of wrinkling on the petal,

(*g*) the colouring on the middle part of petal.

For (*b*), (*d*), (*e*), (*f*) and (*g*) Alice Lee worked out the fraternal correlation by the methods of the memoir on the inheritance of characters not quantitatively measurable†. (*a*) and (*c*) were worked out in the usual way by the long form of correlation table.

Under (*b*) the classes were four petals, four petals + one or more petaloid stamens and more than four petals. The ultimate division of the table for purposes of calculation being made at the normal four petals, and the second group containing everything from a single additional petaloid stamen up to cases of 20 to 28 petals. The classes under breadth of margin were: broad (*b*); broad-slight (*bs*),—one petal pair broad, the other slight, one case only; slight (*s*); slight-none (*ns*), one petal pair slight, the other none, five cases only; and none (*n*). The resulting correlation table was worked out for two divisions and the average taken.

The extent of basal patch was perhaps a less satisfactory character to estimate; the groupings were: none (*n*); none to slight (*ns*); slight (*s*); slight to well-defined (*sd*): well-defined (*d*); well-defined to large (*dl*); and large (*l*), understanding by this last category a large indefinite patch passing right up into the body of the petal. The wrinkling of the petals judged about the same period in each case after opening, twelve hours, was divided into: frilled (*f*); wrinkled (*w*); slightly wrinkled (*sw*); slight to no wrinkling (*sw* to *nw*); and no wrinkling (*nw*), the intermediate class being chiefly due to flowers with diversity between the two pairs of petals. Lastly the colour appreciation was based on the middle third of the petal and the classes adopted in grouping W. F. R. Weldon's much more detailed descriptions were: red (*r*): red-pink (*rp*); pink (*p*); pink-white (*pw*); pink-white-white (*pw. w*) and white (*w*), the intermediate classes (*rp*) and (*pw. w*) having only 7 and 6 representatives respectively and being due either to diversity in individual petals, or to actual transition cases, which could be only thus classified. In all these cases the actual continuity of the character observed was easy to demonstrate, and there was no hesitation in applying the methods of continuous variation. The determination of the dividing lines for purposes of calculation was made in general for 2 or 3 different groupings and the mean result appears in Table XI.

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* The Enfield I crop gave 34,486 pairs of brother-plants and the Kidderminster crop must have had between 50,000 and 60,000 pairs.

† *Phil. Trans.* Vol. 195, A, p. 1.

TABLE XI.

First Flower on Oxford Poppies. Fraternal Correlation.

Character	Number of Fraternal Pairs	Correlation
Stigmatic Bands	4716	·2561
Number of Petals	4692	·2256
Petal Length	4480	·2767
Breadth of Margin	4524	·1876
Extent of Basal Patch ...	4716	·2215
Intensity of Wrinkling ...	3912	·2149
Colour of Middle Third ...	4602	·3401
Mean	—	·2404

If we add to these the Oxford result of the previous table for the stigmatic bands of all capsules, i.e. ·2891, we have for the average relationship of pairs of brothers in the Oxford crop ·2514,—a result in excellent accordance with what we might expect from the case of man (·5), if we allow for most complete cross-fertilisation in the original Hampden crop*.

Lastly W. F. R. Weldon made two other measurements on some 600 Oxford plants, (i) the height above ground of the node from which the terminal flower was given off, (ii) the height of the terminal seed-capsule from the ground. A. Lee has found the fraternal correlation for these two characters, which are the only plant as distinguished from pure flower characters dealt with.

TABLE XII.

Oxford Crop. Fraternal Plant Correlations.

Character	Mean Value	S. D.	Number of Brother Pairs	Correlation
Nodal Height ...	27·53 cms.	9·353 cms.	13,800	·4123
Capsule Height ...	51·58 cms.	12·644 cms.	13,800	·3782
Mean	—	—	—	·3952

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* 1899 was a remarkably fine dry summer and the bees were at work on the Hampden poppy crop all day long, day by day.

These results are high, approaching more nearly the result of the Kidderminster crop. But plant height is a character peculiarly influenced by differential environment, and in some fraternities the proportion of secondary poppies in the pots was far greater than in others. Hence we can hardly lay much stress on these results, which may be largely emphasised by the overcrowding of some of the fraternities as compared with others.

Capsule Fraternities, or Co-ovarial Brotherhood.

W. R. Macdonell's observations in the second Enfield crop, in which seed from the same capsule only was used for fraternal arrays, enable us also to form another fraternal correlation based not on common plant, but on common capsule parentage. The mean of the capsules was 12·39 and their S. D. 1·782, and we have the following results.

TABLE XIII.

Brothers from same Capsule and from same Plant.

Crop	Capsule Correlation	Homotyposis	Plant Correlation
Enfield I ...	·0496	·5093	·1912
Enfield II ...	·1764	·6831	·3780

We thus reach the conclusion that plants from seed of the same capsule will be nearly twice as closely related to each other as plants from seed of all capsules on the same plant. But on the other hand plants from the same capsule will be less like the parent plant, than plants from all capsules of the parent plant. The explanation of this apparent paradox (in the case of cross-fertilisation) is quite easy. A single capsule does not represent by its stigmata the individuality of the plant, i.e. its mean number of stigmata; on the other hand in seed from the same capsule fewer pollen-parents are represented than in the seed from all capsules of the plant.

(12) *Conclusions.*

(i) The character, stigmatic band number, was selected because it allowed of the crops being left pretty much to themselves, and the record of the character being formed after the crop had been harvested as a whole. Characters on the individual flower can only be observed when it is possible to isolate the individual during growth, and examine it continuously. The success of this latter method at Oxford fully justifies the time and energy given to it, but under the conditions of most of the crops here recorded full data for individual plants in the great numbers required could not be expected of those who kindly provided ground and superintended the harvesting. Further experiments are in progress which will deal more fully with other, especially colour, characters.

(ii) Local environment immensely influenced the variability and mean of the local crops. This has, however, little or no influence on the determination of heredity. What does influence heredity is differentiation in the local environment of a crop. In such a case when few parents are selected to start with a differential treatment of those individual parents may much modify results, or again a differential treatment of the sub-crops may be very detrimental to consistent results. Something of this kind seems to have affected the Kidderminster crop, for parental heredity is weakened and fraternal exaggerated—just what we should expect from the latter form of local environment differentiation.

(iii) Most plant organs being multiple in appearance, we have to apply special methods to deduce the intensity of heredity from multiple observations on the individual. It will probably be better in future experiments to confine attention to the first, or principal flower, instead of using the indirect method of homotyposis, but this will involve the observation (previous to harvesting) of individual plants in large series—500 to 1000—and much increase the labour of superintendence and observation.

(iv) Notwithstanding the difficulties referred to above we find that for a variety of plant characters in the Shirley Poppy the values of hereditary influence found are on the whole in fair agreement with the like values for man.

Undoubtedly the most self-accordant results are from Weldon's Oxford crop, where growing in pots, although it tended to produce starvelings, gave owing to the careful mixture of soil and administration of water, etc., a probably greater equality of individual environment.

Here the parental heredity was ·4 and fraternal heredity ·25, results in good accord with those for man. The Highgate results, on incomparably more meagre data, of about ·5 and ·25 respectively are also in good agreement. Enfield and Kidderminster are by no means so satisfactory. Yet when we pass from stigmatic band to colour inheritance at Enfield, we reach a good value, of ·41, for parental inheritance, and for all cases our parental means (·33 to ·34) are not below Francis Galton's first determination of heredity in man.

Hence these, the first observations on large series of the laws of inheritance in the continuously varying characters of plants, show numerical results generally in accordance with those already found for animals and insects.

The great influence of environment, and of local differences of environment, the probable stringent selection of seed and seedlings, the fewness of the original mother-plants, are all factors tending to modify and obscure the numerical results; they make the plant-problem much harder than the animal or insect problem. But the present investigation shows that there is nothing, which would lead us to suppose that the methods and results already found sufficient to describe hereditary influence in man and animals will not suffice to describe the like results in the continuously varying characters of plant life.

Further investigations on the more stable colour characters already planned will, to judge from the data here given, provide more consistent numerical results.

(v) Our experiments show that the plant, not the capsule, must be looked upon as the parent individual; closer resemblance of brother plants from seed of the same capsule being in all probability due to the restricted variety of pollen fertilising the seed of the same capsule compared with the diversity of pollen which fertilises all the seed of all capsules on the same plant.

(vi) On the completion of a long series of observations one invariably discovers much that one ought to have done and to have left undone; we reach a knowledge which *a priori* was impossible, however desirable. We have learnt to appreciate the importance of (*a*) a far greater variety in initial parentages, (*b*) the selection of characters like colour and petal form less subject than stigmata to environmental differences, and (*c*) the difficulty of avoiding differential environment when the plants are grown in ordinary garden ground. We feel that secular experiments on large series of Shirley poppies would throw much light on the laws of plant heredity, but to conduct them properly wants not only the space but the individual observation and record of each plant, which we believe can only be easily and successfully provided when the much needed Biometric Farm has been established *.

APPENDIX OF STATISTICAL TABLES.

A. Oxford Homotyposis in Stigmata.
B. Crockham " " "
C. Bookham " " "
D. Enfield I " " "
E. Enfield II " " "
F. Kidderminster " "
G. Highgate Parental Plant Mean and Offspring Capsule.
H. Oxford " " " "
I. Bookham " " " "
J. Enfield I " " " "
K. Kidderminster " " "
L. Enfield I. Parental Plant Mean and Offspring Plant Mean.
M. Oxford. Lateness of Flowering and Number of Stigmata.
N. Oxford. Parental Plant Mean and Stigmata of First Flower of Offspring.

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* To any of our readers willing to assist in further observations on "first flowers," the Editors will most gladly send seed of pedigree poppies with suggestions for further work.

O. Oxford. Offspring Capsule and Parent Capsule.
P. Crockham. ,, ,, ,,
Q. Enfield I. ,, ,, ,,
R. Enfield II. ,, ,, ,,
S. Enfield II. Grandparent Plant Mean and Offspring Capsule.
T. Kidderminster ,, ,, ,, ,,
U. Highgate. Fraternal Correlation First Flowers.
V. Oxford. Correlation of Capsules on Brother Plants.
W. Enfield I. ,, ,, ,, ,,
X. Kidderminster. ,, ,, ,, ,,
Y. Oxford. Nodal Height of Brother Plants.
Z. Oxford. Capsule Height of Brother Plants.
α. Oxford. Fraternal Correlation, Stigmata of First Flower.
β. Oxford. ,, ,, Number of Petals.
γ. Oxford. ,, ,, Petal Length.
δ. Oxford. ,, ,, Breadth of Margin.
ε. Oxford. ,, ,, Extent of Basal Patch.
ζ. Oxford. ,, ,, Intensity of Wrinkling.
η. Oxford. ,, ,, Colour of Middle Third.
θ. Enfield II. ,, ,, Stigmata of Capsules on Plants from same Capsule.

A. *Oxford. Homotyposis.*

Number of Stigmata on First Capsule.

Number of Stigmata on Second Capsule.	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	Totals
4	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	2
5	1	6	3	4	2	2	1	—	—	—	—	—	—	—	—	—	19
6	—	3	35	31	35	25	17	15	9	4	—	—	2	—	—	—	176
7	—	4	31	63	73	54	43	13	12	10	2	—	—	—	—	—	305
8	1	2	35	73	116	139	130	58	63	39	18	6	—	2	—	—	682
9	—	2	25	54	139	194	242	163	101	68	33	14	2	1	—	—	1038
10	—	1	17	43	130	242	364	322	246	141	69	22	4	2	1	—	1604
11	—	—	15	13	58	163	322	483	405	294	155	46	4	3	4	1	1966
12	—	—	9	12	63	101	246	405	534	388	198	86	10	—	—	—	2052
13	—	—	4	10	39	68	141	294	388	414	328	106	9	—	1	—	1802
14	—	—	—	2	18	33	69	155	198	328	296	137	14	2	2	—	1254
15	—	—	—	—	6	14	22	46	86	106	137	66	23	5	1	—	512
16	—	—	2	—	—	2	4	4	10	9	14	23	15	10	6	3	102
17	—	—	—	—	2	1	2	3	—	—	2	5	10	6	6	3	40
18	—	—	—	—	—	—	1	4	—	1	2	1	6	6	2	2	25
19	—	—	—	—	—	—	—	1	—	—	—	—	3	3	2	—	9
Totals	2	19	176	305	682	1038	1604	1966	2052	1802	1254	512	102	40	25	9	11588

B. *Crockham. Homotyposis.*

Number of Stigmata on First Capsule.

Number of Stigmata on Second Capsule.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	Totals
6	2	4	—	1	—	2	18	34	16	4	—	—	—	—	—	—	—	—	84
7	4	2	—	9	8	8	24	49	27	9	3	1	—	—	—	—	—	—	144
8	—	—	4	18	29	46	31	40	32	8	6	4	—	—	—	—	—	—	218
9	4	9	18	70	143	239	226	270	219	67	35	25	9	1	3	—	—	3	1341
10	—	8	29	143	300	452	194	622	566	261	137	107	18	2	6	—	—	2	3147
11	2	8	46	239	452	826	985	1155	1027	391	151	78	27	2	8	—	—	1	5398
12	18	24	31	226	494	985	1414	2050	1678	745	309	190	66	15	17	—	—	4	8266
13	34	49	40	270	622	1155	2050	3714	3379	1284	474	228	69	13	12	—	—	11	13404
14	16	27	32	219	566	1027	1678	3379	3406	1727	729	377	112	26	13	—	—	10	13344
15	4	9	8	67	261	391	745	1284	1727	1476	925	485	165	44	31	—	—	1	7623
16	—	3	6	35	137	151	309	474	729	925	610	451	130	43	20	—	—	—	4026
17	—	1	4	25	107	78	190	228	377	485	454	506	207	57	41	—	—	—	2760
18	—	—	—	9	18	27	66	69	112	165	130	207	160	58	37	—	—	—	1058
19	—	—	—	1	2	2	15	13	26	44	43	57	58	20	6	—	—	—	287
20	—	—	—	3	6	8	17	12	13	31	20	41	37	6	8		—	—	202
21	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
22	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
23	—	—	—	3	2	1	4	11	10	1	—	—	—	—	—	—	—	—	32
Totals	84	144	218	1341	3147	5398	8266	13401	13344	7623	4026	2760	1058	287	202	—	—	32	61334

C. *Bookham. Homotyposis.*

Number of Stigmata on First Capsule.

Number of Stigmata on Second Capsule.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	Totals
6	—	—	—	1	1	—	1	2	1	—	—	—	—	—	—	6
7	—	—	2	1	4	3	3	1	2	1	—	—	—	—	—	17
8	—	2	8	12	12	8	13	12	17	15	3	2	—	—	—	104
9	1	1	12	30	16	20	32	28	13	27	6	—	1	—	—	187
10	1	4	12	16	44	55	42	45	24	23	3	7	1	—	—	277
11	—	3	8	20	55	40	100	55	53	42	20	20	6	4	—	426
12	1	3	13	32	42	100	134	127	97	63	28	20	2	—	—	662
13	2	1	12	28	45	55	127	126	129	85	31	22	5	—	—	668
14	1	2	17	13	24	53	97	129	144	121	47	25	6	2	—	684
15	—	1	15	27	23	42	63	85	124	146	89	37	12	6	—	670
16	—	—	3	6	3	20	28	31	47	89	78	26	8	5	—	344
17	—	—	2	—	7	20	20	22	25	37	26	24	3	7	1	204
18	—	—	—	1	1	6	2	5	6	12	8	13	2	3	—	59
19	—	—	—	—	—	4	—	—	2	6	5	7	3	2	—	29
20	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	6	17	104	187	277	426	662	668	684	670	344	204	59	29	1	4338

D. *Enfield I. Homotyposis.*

Number of Stigmata on First Capsule.

No. of Stigmata on Second Capsule.	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	Totals
9	6	—	—	2	4	3	3	—	—	—	—	—	18
10	—	4	18	24	13	9	—	—	—	—	—	—	68
11	—	18	106	151	179	96	31	10	—	1	—	—	592
12	2	24	151	368	465	485	180	47	9	—	—	—	1731
13	4	13	179	465	1026	1191	625	189	49	8	2	—	3751
14	3	9	96	485	1191	1642	1158	462	183	25	2	—	5256
15	3	—	31	180	625	1158	1140	678	309	57	8	7	4196
16	—	—	10	47	189	462	678	590	277	88	20	4	2365
17	—	—	—	9	49	183	309	277	138	55	11	7	1038
18	—	—	1	—	8	25	57	88	55	34	9	2	279
19	—	—	—	—	2	2	8	20	11	9	2	1	55
20	—	—	—	—	—	—	7	4	7	2	1	—	21
Totals	18	68	592	1731	3751	5256	4196	2365	1038	279	55	21	19370

E. *Enfield II. Homotyposis.*

Number of Stigmata on First Capsule.

No. of Stigmata on Second Capsule.	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	Totals
7	—	—	1	—	—	—	—	—	—	—	—	—	1
8	—	—	3	3	1	—	—	—	—	—	—	—	7
9	1	3	6	6	9	2	—	—	—	—	—	—	27
10	—	3	6	14	31	22	10	1	1	—	—	—	88
11	—	1	9	31	50	52	29	7	2	1	—	—	182
12	—	—	2	22	52	72	64	32	10	—	—	—	254
13	—	—	—	10	29	64	74	67	30	3	1	—	278
14	—	—	—	1	7	32	67	64	45	4	2	—	222
15	—	—	—	1	2	10	30	45	20	12	3	1	124
16	—	—	—	—	1	—	3	4	12	14	5	2	41
17	—	—	—	—	—	—	1	2	3	5	2	2	15
18	—	—	—	—	—	—	—	—	1	2	2	—	5
Totals	1	7	27	88	182	254	278	222	124	41	15	5	1244

F. *Kidderminster. Homotyposis.*

Number of Stigmata on First Capsule.

Number of Stigmata on Second Capsule.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	Totals
6	—	—	1	—	4	2	1	1	1	—	—	—	—	—	—	13
7	—	2	4	16	11	25	34	26	22	15	10	1	1	—	—	167
8	1	4	10	51	91	134	136	117	80	37	7	3	1	—	—	672
9	—	16	51	192	486	692	738	558	266	129	31	—	—	—	—	3159
10	4	11	91	486	1922	2962	3450	2884	1732	761	223	44	3	3	3	14582
11	2	25	134	692	2962	6686	8886	7958	1791	2290	687	176	12	1	—	35302
12	1	31	136	738	3450	8886	14740	15598	10350	5065	1753	409	52	13	1	61226
13	4	26	117	558	2884	7958	15598	20082	16128	8453	2945	653	141	31	4	75582
14	1	22	80	266	1732	4791	10350	16128	16722	10722	4588	1233	276	76	10	66997
15	—	15	37	129	761	2290	5065	8453	10722	9056	4869	1615	396	132	11	43554
16	—	10	7	31	223	687	1753	2945	4588	4869	3048	1288	357	123	7	19936
17	—	1	3		44	176	409	653	1233	1615	1288	678	248	77	8	6433
18	—	1	1	—	3	12	52	141	276	396	357	248	86	44	9	1626
19	—	—	—	—	3	1	13	31	76	132	123	77	44	28	5	533
20	—	—	—	—	3	—	1	4	10	11	7	8	9	5	—	58
Totals	13	167	672	3159	14582	35302	61226	75582	66997	13554	19936	6433	1626	533	58	329840

G. *Highgate. Parent and Offspring.*

Offspring Plant. Number of Stigmata on Capsule.

Parent Plant. Mean Number of Stigmatic Bands.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	Totals
9·5—10·5	—	—	2	—	1	1	1	1	—	—	—	—	6
10·5—11·5	1	—	3	4	2	—	1	—	—	—	—	—	11
11·5—12·5	—	2	4	4	12	30	37	38	15	6	—	—	148
12·5—13·5	2	1	8	26	12	43	19	34	20	13	2	—	240
13·5—14·5	—	—	1	2	1	5	13	31	21	7	3	1	88
14·5—15·5	—	—	1	5	2	7	17	35	32	13	—	1	113
Totals	3	3	19	41	63	86	118	139	88	39	5	2	606

H. *Oxford. Parent and Offspring.*

Offspring Plant. Number of Stigmata on Capsule.

Parent Plant. Mean Number of Stigmata.	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	Totals
10·099—10·599	—	1	15	14	16	22	19	14	13	2	1	1	—	—	—	—	118
10·599—11·099	—	1	1	5	8	9	17	21	16	16	9	3	—	—	—	—	106
11·099—11·599	—	—	5	11	19	35	32	43	48	36	10	3	—	—	—	—	242
11·599—12·099	5	13	28	35	53	63	94	81	85	53	31	12	4	3	2	1	563
12·099—12·599	—	10	20	40	59	97	111	122	98	55	25	6	2	—	1	—	646
12·599—13·099	—	1	6	12	23	49	87	120	127	101	72	22	6	1	—	—	627
13·099—13·599	—	—	2	1	4	9	10	17	10	20	13	6	3	1	—	—	99
13·599—14·099	—	—	1	10	5	15	22	37	50	34	12	6	2	1	—	—	195
14·099—14·599	—	—	5	12	8	10	16	14	22	14	16	12	3	—	—	—	132
Totals	5	26	83	143	195	309	408	469	469	331	189	71	20	6	3	1	2728

I. *Bookham. Parent and Offspring.*

Offspring Plant. Number of Stigmata on Capsule.

Parent Plant. Mean Number of Stigmata.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	Totals
9·75—10·25	2	2	4	15	18	19	26	23	13	5	1	—	—	—	—	128
10·25—10·75	—	1	4	11	13	18	18	33	28	19	12	3	2	—	—	162
10·75—11·25	—	1	1	4	4	4	2	4	—	—	—	—	1	—	—	21
11·25—11·75	—	1	3	13	23	27	37	35	25	18	7	1	—	—	—	190
11·75—12·25	1	2	6	16	25	40	50	46	45	20	8	5	2	1	—	267
12·25—12·75	—	1	7	23	30	38	35	29	19	20	9	1	—	—	—	212
12·75—13·25	—	2	11	10	22	45	81	83	108	92	49	23	4	—	1	531
13·25—13·75	—	—	—	1	1	4	4	6	7	4	8	1	1	—	—	37
13·75—14·25	—	—	—	1	3	5	3	11	7	12	11	3	2	—	—	58
14·25—14·75	—	—	1	4	3	15	28	20	20	22	11	11	3	2	—	140
14·75—15·25	—	1	1	8	10	23	21	25	30	22	12	2	4	—	—	159
Totals	3	11	38	106	152	238	305	315	302	234	128	50	19	3	1	1905

J. *Enfield I. Parent and Offspring.*

Offspring Plant. Number of Stigmata on Capsule.

Parent Plant. Mean Number of Stigmata.	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	Totals
9·75—10·25	—	—	7	20	37	71	52	28	16	5	—	—	—	236
10·25—10·75	—	—	—	—	—	—	—	—	—	—	—	—	—	0
10·75—11·25	—	—	4	8	12	37	34	19	5	2	2	—	—	123
11·25—11·75	1	3	3	26	33	75	84	61	19	11	1	—	—	317
11·75—12·25	1	4	14	68	127	186	200	111	58	15	4	—	—	788
12·25—12·75	—	1	3	13	45	64	73	43	23	5	2	—	—	272
12·75—13·25	—	3	4	34	109	219	308	274	186	74	27	6	1	1245
13·25—13·75	—	1	4	26	52	96	120	107	55	25	11	2	—	499
13·75—14·25	—	—	1	8	14	44	53	51	17	10	2	1	—	201
14·25—14·75	1	—	—	15	22	63	85	72	42	15	5	1	1	322
14·75—15·25	—	—	8	4	20	30	45	44	30	13	6	1	—	201
Totals	3	12	48	222	471	885	1054	810	451	175	60	11	2	4204

K. *Kidderminster. Parent and Offspring.*

Parent Plant. Mean Number of Stigmata on Capsule.

Offspring Plant. Number of Stigmata on Capsule.	*11·5–12·0*	*12·0–12·5*	*12·5–13·0*	*13·0–13·5*	*13·5–14·0*	*14·0–14·5*	*14·5–15·0*	*15·0–15·5*	*15·5–16·0*	*16·0–16·5*	*16·5–17·0*	*17·0–17·5*	Totals
6	—	—	—	—	—	—	—	—	—	1	—	—	1
7	—	—	1	3	—	—	1	2	—	1	1	—	9
8	—	4	1	11	5	13	3	5	2	2	3	1	50
9	4	14	14	52	15	66	11	18	10	3	3	2	212
10	22	52	52	289	63	175	62	67	41	27	27	20	897
11	62	151	118	603	151	357	179	202	82	75	54	51	2085
12	75	233	172	995	250	586	384	380	202	148	116	80	3621
13	91	294	209	1133	326	588	470	441	266	261	162	116	4360
14	60	260	184	825	339	469	409	396	297	263	129	122	3753
15	43	135	155	467	228	235	284	286	228	181	86	74	2402
16	17	55	76	214	108	89	133	130	112	99	45	53	1131
17	3	12	33	69	32	20	50	46	23	29	21	37	375
18	—	3	7	18	6	—	6	23	6	9	7	13	98
19	—	—	2	2	1	—	4	5	5	7	—	2	28
20	—	—	—	1	—	—	—	3	—	—	—	1	5
Totals	377	1213	1024	4682	1524	2598	1996	2007	1274	1106	654	572	19027

L. *Enfield I. Parent and Offspring Means.*

Offspring Plant. Mean Number of Stigmata on Capsules.

Parent Plant. Mean Number of Stigmata on Capsules.	*8·505–9·005*	*9·005–9·505*	*9·505–10·005*	*10·005–10·505*	*10·505–11·005*	*11·005–11·505*	*11·505–12·005*	*12·005–12·505*	*12·505–13·005*	*13·005–13·505*	*13·505–14·005*	*14·005–14·505*	*14·505–15·005*	*15·005–15·505*	*15·505–16·005*	*16·005–16·505*	*16·505–17·005*	*17·005–17·505*	*17·505–18·005*	Totals
10·005—10·505	—	—	—	—	1	3	5	3	10	6	8	1	4	1	2	—	—	—	—	44
10·505—11·005	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
11·005—11·505	1	—	—	—	2	3	8	13	12	10	21	12	9	—	5	—	1	—	—	97
11·505—12·005	—	—	—	1	4	9	5	12	25	20	26	15	15	8	9	1	3	1	—	154
12·005—12·505	—	—	1	—	—	1	7	6	12	4	10	9	4	5	2	2	—	—	—	63
12·505—13·005	—	—	—	—	1	1	6	10	31	24	40	26	39	23	17	10	5	3	1	237
13·005—13·505	—	—	1	2	5	1	9	14	16	21	24	18	18	7	4	3	4	1	—	148
13·505—14·005	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
14·005—14·505	—	—	—	—	1	2	6	3	4	5	19	6	13	5	5	4	1	—	—	71
14·505—15·005	—	—	—	—	—	—	3	—	4	7	8	5	3	3	3	1	—	—	—	37
15·005—15·505	—	—	—	—	2	1	3	1	3	4	9	8	7	4	3	2	1	1	—	49
Totals	1	—	2	3	16	21	52	62	117	101	165	100	112	56	50	23	15	6	1	903

M. *Oxford. Stigmata and Lateness of Flowering.*

Number of Stigmatic Bands on Capsule of First Flower of Plant.

	6	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	Totals
Early Flowering...	3	6	11	26	30	28	37	14	5	1	161
Late Flowering ...	1	1	7	12	21	41	28	30	18	2	161
Totals	4	7	18	38	51	69	65	44	23	3	322

N. *Oxford. Parent and Offspring's First Flower.*

Number of Stigmata on First Flower of Offspring Plant.

Parent Plant. Mean Number of Stigmata on Capsules.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	Totals
15·099—15·599	1	—	—	1	3	5	2	—	—	1	13
14·599—15·099	—	—	—	2	1	3	3	4	—	—	13
14·099—14·599	—	—	2	4	5	8	5	4	3	—	31
13·599—14·099	—	—	—	—	—	—	—	—	—	—	0
13·099—13·599	—	1	2	2	11	14	16	12	10	1	69
12·599—13·099	—	1	1	4	6	17	13	9	5	—	56
12·099—12·599	—	1	3	10	10	9	9	5	3	—	50
11·599—12·099	—	—	3	5	11	10	1	5	3	—	38
11·099—11·599	—	—	1	5	2	7	6	6	1	2	30
10·599—11·099	—	—	—	—	—	—	—	—	—	—	0
10·099—10·599	2	4	7	6	4	2	2	—	—	—	27
Totals	3	7	19	39	53	75	57	45	25	4	327

O. *Oxford Parent and Offspring Capsules.*

Offspring Capsule. Number of Stigmata.

Parent Capsule. Number of Stigmata.	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	Totals
7	—	—	—	1	4	1	5	9	4	7	1	—	—	—	—	32
8	—	1	7	8	7	19	14	3	6	1	1	1	—	—	—	68
9	—	1	5	10	16	14	17	17	7	9	—	3	—	—	—	99
10	—	2	7	2	16	19	25	14	9	3	1	—	—	—	—	98
11	3	2	7	9	12	25	13	20	17	9	5	3	—	—	—	125
12	—	3	12	18	21	15	24	16	4	4	3	1	—	—	—	121
13	—	—	7	1	19	19	20	18	15	2	4	1	—	—	—	106
14	—	2	14	16	8	20	13	12	7	4	3	2	—	1	—	102
15	—	1	3	5	4	8	15	17	21	16	3	1	1	1	—	96
16	—	1	10	11	9	17	11	14	7	7	2	—	—	—	—	89
17	—	1	1	4	14	16	9	33	21	15	2	1	—	—	—	117
18	—	—	—	9	9	8	18	18	10	6	4	1	2	1	1	87
Totals	3	14	73	94	139	181	184	191	128	83	29	14	3	3	1	1140

P. *Crockham. Parent and Offspring Capsules.*

Offspring Capsule. Number of Stigmata.

Parent Capsule. Number of Stigmata.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	Totals
7	—	—	2	7	19	26	44	70	56	18	5	3	—	—	—	—	—	—	250
8	—	1	3	7	20	34	42	50	55	25	4	1	1	—	—	—	—	—	243
9	2	2	1	10	15	26	51	72	52	26	13	9	1	—	—	—	—	—	280
10	—	2	3	8	30	40	55	72	62	72	49	26	11	2	3	—	—	—	438
11	—	—	3	6	7	22	52	52	52	34	16	9	6	1	—	—	—	—	260
12	—	—	1	2	2	3	7	11	14	15	5	2	—	—	—	—	—	—	62
13	—	—	—	4	9	22	53	41	51	40	29	21	9	4	2	—	—	—	288
14	—	—	—	4	6	6	14	29	26	14	12	5	—	1	1	—	—	1	119
15	—	2	—	6	11	36	50	84	72	35	25	11	5	2	—	—	—	—	342
16	—	—	—	1	—	9	23	33	32	36	22	6	7	1	—	—	—	—	170
17	—	—	1	5	7	26	32	36	60	52	32	10	—	—	—	—	—	—	261
18	—	—	2	8	16	16	23	31	35	42	42	39	15	7	2	—	—	—	278
Totals	2	7	16	68	145	266	446	581	567	409	254	145	58	18	8	—	—	1	2991

Q. *Enfield I. Parent and Offspring Capsules.*

Offspring Capsule. Number of Stigmata.

Parent Capsule. Number of Stigmata.	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	Totals
7	—	—	—	3	8	12	23	35	31	30	18	7	2	1	—	170
8	—	—	1	1	10	19	45	49	37	19	8	1	—	—	—	190
9	—	—	—	1	8	23	29	40	29	22	9	6	2	2	2	173
10	—	—	1	5	7	20	29	39	33	17	7	3	1	1	—	163
11	—	2	1	6	16	29	37	49	33	18	4	3	—	—	—	198
12	—	1	2	9	13	31	45	29	38	14	11	3	1	—	—	197
13	—	2	2	9	18	21	32	40	28	23	8	1	1	2	—	187
14	—	—	—	1	5	14	18	36	35	32	25	7	6	—	—	179
15	1	3	1	13	6	18	26	31	41	28	15	12	3	2	—	203
16	—	—	1	—	9	12	15	27	17	17	13	6	3	—	—	120
17	—	—	2	3	3	10	23	22	24	28	14	10	3	—	—	142
18	—	—	—	—	—	1	7	19	39	36	21	14	5	2	—	144
Totals	1	8	11	51	103	210	329	416	388	284	153	73	27	10	2	2066

R. *Enfield II. Parent and Offspring Capsules.*

Offspring Plant. Number of Stigmata on Capsule.

Parent Capsule. Number of Stigmata.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	Totals
10	—	—	—	—	—	—	1	1	—	—	—	—	—	2
11	1	—	2	3	14	17	7	13	2	1	—	—	—	60
12	—	—	—	2	1	1	8	3	—	—	—	—	—	15
13	—	—	—	5	10	16	13	15	11	2	—	—	—	72
14	—	1	4	8	22	43	47	51	36	17	3	1	—	233
15	—	—	4	5	9	21	14	8	9	5	—	—	—	78
16	—	—	—	—	—	5	9	2	3	2	2	2	1	26
17	—	—	—	—	1	1	6	2	5	—	—	—	—	15
18	—	—	—	—	—	—	—	—	—	—	—	—	—	—
19	—	—	—	1	—	2	7	14	9	6	4	—	—	43
20	—	—	—	—	1	2	5	7	3	4	—	—	—	22
Totals	1	1	10	24	58	111	117	116	78	37	9	3	1	566

S. *Enfield II. Grandparent and Offspring.*

Stigmata on Capsule of Grandchild Plant.

Grandparent Plant. Stigmatic Band Mean.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	Totals
10·75—11·25	—	—	3	1	4	2	4	4	2	—	—	—	—	20
11·25—11·75	—	—	—	—	—	1	2	1	—	—	—	—	—	4
11·75—12·25	1	—	—	—	6	5	11	16	11	8	1	1	—	60
12·25—12·75	—	—	—	—	4	14	15	5	7	4	2	2	1	54
12·75—13·25	—	—	5	11	23	49	41	43	27	11	4	—	—	214
13·25—13·75	—	1	2	10	17	27	28	26	19	9	1	—	—	140
13·75—14·25	—	—	—	2	3	8	10	16	9	5	—	—	—	53
14·25—14·75	—	—	—	—	—	—	—	—	—	—	—	—	—	—
14·75—15·25	—	—	—	—	1	5	6	5	3	—	1	—	—	21
Totals	1	1	10	24	58	111	117	116	78	37	9	3	1	566

T. *Kidderminster. Grandparent and Offspring.*

Stigmata on Capsule of Grandchild Plant.

Grandparent Plant. Stigmatic Band Mean.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	Totals
10·005—10·505	—	—	6	16	43	107	149	170	157	113	64	19	3	—	—	847
10·505—11·005	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
11·005—11·505	—	2	7	56	173	354	572	630	432	247	103	17	5	1	—	2599
11·505—12·005	—	1	4	30	130	303	516	527	423	291	172	65	16	3	1	2482
12·005—12·505	—	2	1	11	58	187	286	371	283	173	61	27	3	1	—	1470
12·505—13·005	1	2	13	45	298	605	1019	1416	1348	880	415	141	44	9	1	6240
13·005—13·505	—	2	12	54	136	330	533	548	452	308	109	38	14	4	2	2542
13·505—14·005	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
14·005—14·505	—	—	8	10	36	132	265	336	356	214	123	41	8	5	1	1535
14·505—15·005	—	—	—	2	31	66	123	115	97	62	25	5	—	—	—	556
15·005—15·505	—	—	—	4	33	89	210	222	196	104	48	16	6	5	—	933
Totals	1	9	51	228	938	2173	3673	4368	3714	2392	1123	372	99	28	5	19204

U. *Highgate. Brother Plants.*

First Brother Plant. Number of Stigmata on Capsule.

Second Brother Plant. Number of Stigmata on Capsule.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	Totals
6	—	1	6	8	9	11	7	4	5	9	1	—	61
7	1	2	7	5	10	11	6	4	5	9	1	—	61
8	6	7	18	45	52	55	52	58	44	35	6	1	379
9	8	5	45	186	404	425	562	490	260	126	13	5	2529
10	9	10	52	404	120	655	990	1353	738	287	48	6	4672
11	11	11	55	425	655	874	1593	1586	695	305	29	10	6249
12	7	6	52	562	990	1593	2158	2331	1227	471	49	28	9474
13	4	4	58	490	1353	1586	2331	3230	2116	809	106	63	12150
14	5	5	44	260	738	695	1227	2116	1478	640	75	51	7334
15	9	9	35	126	287	305	471	809	640	284	31	20	3026
16	1	1	6	13	48	29	49	106	75	31	6	3	368
17	—	—	1	5	6	10	28	63	51	20	3	0	187
Totals	61	61	379	2529	4672	6249	9474	12150	7334	3026	368	187	46490

V. *Oxford. Brother Plant.*

First Brother Plant. Number of Stigmata on Capsule.

Second Brother Plant. Number of Stigmata on Capsule.

	4	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	Totals
4	6	29	50	63	68	86	107	95	83	52	36	15	4	3	5	1	703
5	29	88	210	278	330	422	520	513	383	239	162	55	13	10	15	3	3270
6	50	210	541	774	1001	1364	1631	1567	1402	825	536	232	62	30	31	9	10265
7	63	278	774	1051	1411	2124	2560	2784	2547	1649	1023	459	124	34	33	8	16922
8	68	330	1001	1411	1900	2949	3730	3942	3622	2314	1347	541	168	55	47	16	23441
9	86	422	1364	2124	2949	4362	5680	6345	5914	3894	2236	891	261	62	46	14	36650
10	107	520	1631	2560	3730	5680	7604	8575	8515	5694	3367	1337	412	106	70	27	49935
11	95	513	1567	2781	3942	6345	8575	9789	9693	6734	4069	1641	473	96	49	16	56381
12	83	383	1402	2547	3622	5914	8515	9693	9916	7108	4065	1541	470	149	67	30	55505
13	52	239	825	1649	2314	3894	5694	6734	7108	4906	2953	1105	345	105	33	14	37970
14	36	162	536	1023	1347	2236	3367	4069	4065	2953	1844	748	225	50	14	5	22680
15	15	55	232	459	541	891	1337	1641	1541	1105	748	306	107	13	6	2	8999
16	4	13	62	124	168	261	412	473	470	345	225	107	21	4	—	—	2689
17	3	10	30	34	55	62	106	96	149	105	50	13	4	—	—	—	717
18	5	15	31	33	47	46	70	49	67	33	14	6	—	—	—	—	416
19	1	3	9	8	16	14	27	16	30	14	5	2	—	—	—	—	145
Totals	703	3270	10265	16922	23441	36650	49935	56381	55505	37970	22680	8999	2689	717	416	145	326688

W. *Enfield I. Brother Plants.*

First Brother Plant. Number of Stigmata on Capsule.

Second Brother Plant. Number of Stigmata on Capsule.

	8	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	Totals
8		6	5	44	59	121	132	92	43	15	7	1	1	526
9	6	8	27	160	285	506	577	374	201	56	26	1	—	2227
10	5	27	144	430	971	1839	2036	1410	772	288	116	14	1	8053
11	44	160	430	1884	4014	7915	8671	6148	3276	1276	445	62	12	34337
12	59	285	971	4014	8636	17042	19309	13862	7958	2941	1057	158	12	76304
13	121	506	1839	7915	17042	32776	38389	28060	15712	6102	2061	301	42	150866
14	132	577	2036	8671	19309	38389	45082	34343	19427	7550	2680	386	63	178645
15	92	374	1410	6148	13862	28060	34343	26808	15581	6256	2172	393	60	135559
16	43	201	772	3276	7958	15712	19427	15581	9108	3597	1363	259	29	77326
17	15	56	288	1276	2941	6102	7550	6256	3597	1398	532	104	16	30131
18	7	26	116	445	1057	2061	2680	2172	1363	532	198	42	5	10704
19	1	1	14	62	158	301	386	393	259	104	42	10	1	1732
20	1	—	1	12	12	42	63	60	29	16	5	1	—	242
Totals	526	2227	8053	34337	76304	150866	178645	135559	77326	30131	10704	1732	242	706652

X. *Kidderminster. Brother Plants.*

Number of Stigmata on Capsule of First Brother Plant.

Number of Stigmata on Capsule of Second Brother Plant.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	Totals
6	—	—	1	1	5	8	10	31	29	24	18	5	1	—	—	136
7	—	—	4	13	98	231	343	468	361	216	73	24	5	—	—	1839
8	1	4	58	228	632	1300	2010	2192	1782	1037	429	112	31	2	—	9818
9	1	13	228	822	2709	5339	8569	9330	7333	4275	1687	553	108	17	3	40987
10	5	98	632	2709	10668	23028	37310	13163	33885	19733	8836	2735	784	153	26	183765
11	8	231	1300	5339	23028	50146	85615	98062	80399	48752	21743	6841	1742	378	61	423948
12	10	343	2010	8569	37310	85615	146104	168554	140664	85882	38603	12099	2772	734	100	729369
13	34	468	2192	9330	13163	98062	168551	198452	170465	106716	49780	16708	4250	1063	179	869416
14	29	364	1782	7333	33885	80399	140664	170465	153152	98284	46342	15892	4135	1117	210	754053
15	24	216	1037	4275	19733	48752	85882	106716	98281	64511	31968	11064	2914	881	188	476478
16	18	73	429	1687	8836	21743	38603	49780	46342	31968	15818	5820	1620	497	122	223386
17	5	24	112	553	2735	6844	12099	16708	15892	11064	5820	2320	695	200	66	75137
18	1	5	31	108	784	1742	2772	1250	4135	2914	1620	695	200	67	20	19374
19	—	—	2	17	153	378	734	1063	1117	881	497	200	67	26	7	5142
20	—	—	—	3	26	61	100	179	210	188	122	66	20	7	2	984
Totals	136	1839	9818	40987	183765	423948	729369	869416	754053	476478	223386	75137	19374	5142	984	3813832

Y. *Oxford. Brother Plants.*

First Brother Plant. Nodal Height in cms.

Second Brother Plant. Nodal Height in cms.	*5·25–10·25*	*10·25–15·25*	*15·25–20·25*	*20·25–25·25*	*25·25–30·25*	*30·25–35·25*	*35·25–40·25*	*40·25–45·25*	*45·25–50·25*	*50·25–55·25*	*55·25–60·25*	*60·25–65·25*	*65·25–70·25*	Totals
5·25—10·25	—	—	3	5	8	6	1	—	1	—	—	—	—	24
10·25—15·25	—	170	207	242	111	60	16	8	—	—	—	—	2	816
15·25—20·25	3	207	584	762	139	213	86	23	6	—	—	—	5	2328
20·25—25·25	5	212	762	888	626	360	186	77	28	5	—	2	11	3192
25·25—30·25	8	111	439	626	540	436	231	124	66	28	—	2	5	2616
30·25—35·25	6	60	213	360	436	468	296	218	98	58	—	8	11	2232
35·25—40·25	1	16	86	186	231	296	166	135	59	32	—	7	9	1224
40·25—45·25	—	8	23	77	121	218	135	128	46	31	—	1	1	792
45·25—50·25	1	—	6	28	66	98	59	46	18	8	—	3	3	336
50·25—55·25	—	—	—	5	28	58	32	31	8	6	—	—	—	168
55·25—60·25	—	—	—	—	—	—	—	—	—	—	—	—	—	0
60·25—65·25	—	—	—	2	2	8	7	1	3	—	—	—	1	24
65·25—70·25	—	2	5	11	5	11	9	1	3	—	—	1	—	48
Totals	24	816	2328	3192	2616	2232	1224	792	336	168	0	24	48	13800

Z. Oxford. Brother Plants.

First Brother Plant. Capsule Height in cms.

Second Brother Plant. Capsule Height in cms.	*18·25–23·25*	*23·25–28·25*	*28·25–33·25*	*33·25–38·25*	*38·25–43·25*	*43·25–48·25*	*48·25–53·25*	*53·25–58·25*	*58·25–63·25*	*63·25–68·25*	*68·25–73·25*	*73·25–78·25*	*78·25–83·25*	*83·25–88·25*	*88·25–93·25*	*93·25–98·25*	*98·25–103·25*	Totals
18·25— 23·25	—	—	—	—	3	2	5	4	7	1	1	—	1	—	—	—	—	24
23·25— 28·25	—	—	8	15	20	24	14	3	7	1	1	2	—	—	—	1	—	96
28·25— 33·25	—	8	24	85	88	107	72	29	28	12	1	—	—	—	—	2	—	456
33·25— 38·25	—	15	85	244	255	270	223	90	64	40	4	4	—	—	—	2	—	1296
38·25— 43·25	3	20	88	255	354	424	316	145	144	93	21	14	7	3	—	8	1	1896
43·25— 48·25	2	24	107	270	424	580	376	219	206	119	35	34	12	10	1	4	1	2424
48·25— 53·25	5	14	72	223	316	376	314	205	228	150	36	19	26	19	2	8	3	2016
53·25— 58·25	4	3	29	90	145	219	205	226	254	210	75	54	45	35	2	6	6	1608
58·25— 63·25	7	7	28	64	144	206	228	254	240	200	82	48	53	28	5	9	5	1608
63·25— 68·25	1	1	12	40	93	119	150	210	200	172	67	48	36	40	7	2	2	1200
68·25— 73·25	1	1	1	4	21	35	36	75	82	67	26	27	14	9	3	3	3	408
73·25— 78·25	—	2	—	4	14	34	19	54	48	48	27	14	8	10	2	2	2	288
78·25— 83·25	1	—	—	—	7	12	26	45	53	36	14	8	8	6	—	—	—	216
83·25— 88·25	—	—	—	—	3	10	19	35	28	40	9	10	6	6	2	—	—	168
88·25— 93·25	—	—	—	—	—	1	2	2	5	7	3	2	—	2	—	—	—	24
93·25— 98·25	—	1	2	2	8	4	8	6	9	2	3	2	—	—	—	—	1	48
98·25—103·25	—	—	—	—	1	1	3	6	5	2	3	2	—	—	—	1	—	24
Totals	24	96	456	1296	1896	2424	2016	1608	1608	1200	408	288	216	168	24	48	24	13800

a. Oxford. Brother Plants. First Flower.

First Brother Plant. Stigmata of First Flower.

Second Brother Plant. Stigmata of First Flower.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	Totals
6	2	9	15	14	17	11	10	1	2	—	81
7	9	12	30	30	26	14	13	3	4	—	141
8	15	30	44	62	58	50	52	22	9	2	344
9	14	30	62	72	101	106	103	58	29	5	580
10	17	26	58	101	120	156	159	75	50	3	765
11	11	14	50	106	156	166	207	129	67	10	916
12	10	13	52	103	159	207	174	146	70	9	943
13	1	3	22	58	75	129	146	88	52	11	585
14	2	4	9	29	50	67	70	52	36	1	320
15	—	—	2	5	3	10	9	11	1	—	41
Totals	81	141	344	580	765	916	943	585	320	41	4716

β. Oxford. Brother Plants. First Flower.

First Brother Plant. Number of Petals on First Flower.

Second Brother Plant. Petals on First Flower.	Four Petals	Four Petals and Petaloid Stamens	Five and more Petals	Totals
Four Petals	3086	118	549	3753
Four Petals and Petaloid Stamens	118	2	29	149
Five and more Petals	549	29	212	790
Totals	3753	149	790	4692

γ. *Oxford. Brother Plants.*

First Brother Plant. Length of Petal on Principal Flower in millimetres.

Second Brother Plant. Length of Petal on Principal Flower in millimetres.

	15	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	*41*	*42*	*43*	*44*	*45*	*46*	*47*	*48*	*49*	*50*	*51*	Totals
15	—	—	—	—	—	1	1	2	2	2	—	1	3	1	—	2	3	1	—	1	2	1	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	25
16	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
17	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	3	—	2	1	—	2	3	1	2	1	—	1	—	1	—	—	—	19
18	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
19	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
20	1	—	—	—	—	—	1	2	2	3	—	1	1	3	—	3	4	4	—	2	2	1	1	—	1	—	—	—	2	—	1	—	—	1	—	—	—	36
21	1	—	—	—	—	1	—	2	2	2	—	1	3	1	—	3	5	1	1	2	5	2	2	1	1	1	—	2	3	—	—	—	—	1	—	—	—	43
22	2	—	—	—	—	2	2	2	4	4	—	2	6	2	—	4	6	2	—	2	4	2	—	—	—	—	—	—	2	—	—	—	—	2	—	—	—	50
23	2	—	—	—	—	2	2	4	2	4	—	2	6	2	—	4	6	2	—	2	4	2	—	—	—	—	—	—	2	—	—	—	—	2	—	—	—	50
24	2	—	—	—	—	3	2	4	4	2	1	2	9	5	1	6	8	5	2	8	7	4	4	1	3	2	—	—	5	—	2	—	—	2	—	—	—	94
25	—	—	—	—	—	—	—	—	—	1	—	1	—	1	1	1	—	2	1	1	1	—	1	3	1	2	—	1	1	—	1	—	—	—	—	—	—	20
26	1	—	—	—	—	1	1	2	2	2	1	—	3	1	1	2	3	2	1	2	2	1	—	2	1	1	—	1	1	—	—	—	—	1	—	—	—	35
27	3	—	—	—	—	4	3	6	6	9	—	3	8	5	2	8	12	8	6	11	12	5	8	4	6	6	2	1	5	—	1	1	1	3	—	—	—	149
28	1	—	—	—	—	3	1	2	2	5	1	1	5	4	—	5	5	5	4	7	6	4	11	4	5	4	6	2	4	2	3	—	—	1	—	—	—	103
29	—	—	—	—	—	—	—	—	—	1	1	1	2	—	6	1	1	5	3	5	8	6	2	2	5	3	7	7	—	3	3	—	—	—	—	—	—	72
30	2	—	—	—	—	3	3	4	4	6	1	2	8	5	1	2	9	4	2	7	12	4	5	5	6	7	4	4	8	1	2	1	1	2	—	—	—	125
31	3	—	—	—	—	4	5	6	6	8	—	3	12	5	1	9	8	5	5	9	15	6	7	3	6	4	2	5	9	—	1	1	—	4	—	—	—	152
32	1	—	1	—	—	1	1	2	2	5	2	2	8	5	5	4	5	4	14	13	26	13	21	14	9	14	17	11	8	7	4	4	1	3	—	—	—	227
33	—	—	—	—	—	—	1	—	—	2	1	1	6	4	3	2	5	14	6	12	17	10	21	9	11	13	13	13	6	3	2	4	1	1	—	—	—	181
34	1	—	1	—	—	2	2	2	2	8	1	2	11	7	5	7	9	13	12	10	19	7	19	10	16	16	13	7	10	7	2	2	2	2	—	—	—	227
35	2	—	3	—	—	2	5	4	4	7	1	2	12	6	8	12	15	26	17	19	28	21	33	26	20	25	30	22	22	11	6	7	3	7	—	—	—	406
36	1	—	—	—	—	1	2	2	2	4	—	1	5	4	6	4	6	13	10	7	21	20	15	17	9	14	18	31	16	3	5	6	—	2	2	—	2	249
37	—	—	2	—	—	1	2	—	—	4	1	—	8	11	2	5	7	21	21	19	33	15	22	15	17	16	20	18	12	10	4	3	1	2	—	—	—	292
38	—	—	1	—	—	—	1	—	—	1	3	2	4	4	2	5	3	14	9	10	26	17	15	20	20	31	21	17	15	5	5	7	5	2	2	—	2	269
39	—	—	—	—	—	1	1	—	—	3	1	1	6	5	5	6	6	9	11	16	20	9	17	20	8	16	14	12	6	5	4	2	3	1	—	—	—	208
40	—	—	2	—	—	—	1	—	—	2	2	1	6	4	3	7	4	14	13	16	25	14	16	31	16	20	27	16	22	6	3	11	5	3	3	—	3	296
41	—	—	3	—	—	—	—	—	—	—	—	—	2	6	7	4	2	17	13	13	30	18	20	21	14	27	16	23	16	11	6	10	3	5	1	—	1	289
42	—	—	1	—	—	—	2	—	—	—	1	1	1	2	7	4	5	11	13	7	22	31	18	17	12	16	23	20	17	7	8	7	2	2	1	—	1	259
43	1	—	2	—	—	2	3	2	2	5	1	1	5	4	—	8	9	8	6	10	22	16	12	15	6	22	16	17	10	4	4	8	2	4	3	—	3	233
44	—	—	1	—	—	—	—	—	—	—	—	—	—	2	3	1	—	7	3	7	11	3	10	5	5	6	11	7	4	2	3	1	2	1	—	—	—	95
45	—	—	—	—	—	1	—	—	—	2	1	—	1	3	3	2	1	4	2	2	6	5	4	5	4	3	6	8	4	3	—	1	2	—	—	—	—	73
46	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	1	1	4	4	2	7	6	3	7	2	11	10	7	8	1	1	—	1	2	1	—	1	82
47	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	1	1	2	3	—	1	5	3	5	3	2	2	2	2	1	—	—	—	—	—	35
48	1	—	1	—	—	1	1	2	2	2	—	1	3	1	—	2	4	3	1	2	7	2	2	2	1	3	5	2	4	1	—	2	—	—	—	—	—	58
49	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	2	—	3	1	1	3	—	—	1	—	—	—	—	1	14
50	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
51	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	2	—	3	1	1	3	—	—	1	—	—	1	—	—	14
Totals	25	0	19	0	0	36	43	50	50	94	20	35	149	103	72	125	152	227	181	227	406	249	292	269	208	296	289	259	233	95	73	82	35	58	14	0	14	4480

δ. *Oxford. Brother Plants. First Flower.*

First Brother Plant. Breadth of Margin of First Flower.

Second Brother Plant. Breadth of Margin of First Flower.	Broad	Broad-Slight	Slight	Slight-None	None	Totals
Broad	122	3	108	21	411	665
Broad to Slight	3	—	4	—	6	13
Slight	108	4	86	15	381	594
Slight to None...	21	—	15	2	46	84
None	411	6	381	46	2324	3168
Totals ...	665	13	594	84	3168	4524

ε. *Oxford. Brother Plants. First Flower.*

First Brother Plant. Extent of Basal Patch of First Flower.

Second Brother Plant. Extent of Basal Patch, First Flower.	None	None-Slight	Slight	Slight-Definite	Definite	Definite-Large	Large	Totals
None	24	21	44	1	62	—	128	280
None to Slight ...	21	12	22	—	21	—	46	122
Slight	44	22	86	3	159	6	224	544
Slight to Definite	1	—	3	—	3	—	19	26
Definite	62	21	159	3	858	19	582	1704
Definite to Large	—	—	6	—	19	—	8	33
Large	128	46	224	19	582	8	1000	2007
Totals ...	280	122	544	26	1704	33	2007	4716

ζ. *Oxford. Brother Plants. First Flower.*

First Brother Plant. Intensity of Wrinkling on First Flower.

Second Brother Plant. Intensity of Wrinkling on First Flower.	Frilled	Wrinkled	Slightly-Wrinkled	Slightly-Wrinkled Not-Wrinkled	Not-Wrinkled	Totals
Frilled	10	89	75	1	22	197
Wrinkled	89	566	569	17	145	1386
Slightly-Wrinkled	75	569	718	26	275	1663
Slightly-Wrinkled to Not-Wrinkled	1	17	26	2	12	58
Not-Wrinkled	22	145	275	12	154	608
Totals	197	1386	1663	58	608	3912

η. Oxford. Brother Plants. First Flower.

First Brother Plant. Colour of Middle Third of First Flower.

Second Brother Plant. Colour of Middle Third of First Flower.

	Red	Red-Pink	Pink	Pink-White	White Pink-White	White	Totals
Red	1032	55	387	154	40	11	1679
Red-Pink	55	4	49	6	1	—	115
Pink	387	49	1078	312	26	46	1928
Pink-White	154	6	342	182	10	18	712
White to Pink-White	40	1	26	10	2	6	85
White	11	—	46	18	6	2	83
Totals ...	1679	115	1928	712	85	83	4602

θ. Enfield II. Plants from seed of same Capsule.

Stigmata on Capsules of First Brother Plant.

Stigmata on Capsules of Second Brother Plant.

	6	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	Totals
6	—	—	—	—	2	1	2	4	—	—	—	—	—	9
7	—	—	—	1	—	4	8	5	7	3	1	—	—	29
8	—	—	4	10	18	37	21	23	14	4	—	—	—	131
9	—	1	10	24	38	92	68	70	50	26	6	—	—	385
10	2	—	18	38	124	270	158	137	96	37	2	1	—	883
11	1	4	37	92	270	472	358	321	259	119	26	13	5	1977
12	2	8	21	68	158	358	374	415	340	186	65	26	9	2030
13	4	5	23	70	137	321	415	492	379	228	77	16	2	2169
14	—	7	14	50	96	259	340	379	250	167	58	15	3	1638
15	—	3	4	26	37	119	186	228	167	98	27	9	1	905
16	—	1	—	6	2	26	65	77	58	27	—	—	—	262
17	—	—	—		1	13	26	16	15	9	—	—	—	80
18	—	—	—	—	—	5	9	2	3	1	—	—	—	20
Totals	9	29	131	385	883	1977	2030	2169	1638	905	262	80	20	10518

MISCELLANEA.

I. Note on the Results of Crossing Japanese Waltzing Mice with European Albino Races.

By A. D. DARBISHIRE.

THE breeding experiments, of which this is only a preliminary account, were undertaken at Professor Weldon's suggestion with the object of throwing some light on the problem of Heredity, and especially on the Laws of Mendel.

For this purpose crosses were made between the Japanese "waltzing" mouse and the common albino mouse. These animals were used because they have already been made the subject of similar experiments, the results of which have been recorded first by Haacke*, who gives no numerical statements, but says that when a Japanese mouse is crossed with an albino the offspring is wild-coloured or black, sometimes with a white spot on the forehead or belly, and secondly by von Guaita † who gives elaborate tables, showing that the first cross-bred generation has always the colour of wild mice, while individuals of subsequent generations are white, black, brown or piebald. The cross-bred forms of the first generation did not exhibit the "dancing" movements of the Japanese mice, while a certain proportion of individuals in subsequent generations did so. Bateson ‡ has suggested that the results obtained by these observers, and by others, "show an essential harmony in the fact that both found *albino* an obvious recessive, pure almost without exception, while the coloured forms show various phenomena of dominance." Lack of uniformity in the characters of the first cross-bred generation is elsewhere attributed by Bateson to impurity in the stock used §.

The experiments were made, then, in order to answer two simple questions:

(1) Does the first generation of cross-bred individuals, produced by pairing albino and Japanese waltzing mice, exhibit such a uniformity of colour as will justify us in considering that Mendel's Law of Dominance applies to it?

(2) Does the Mendelian Law of Segregation apply to the offspring of hybrids resulting from such crosses?

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* "Über Wesen, Ursachen und Vererbung von Albinismus, etc.," *Biologisches Centralblatt*, xv. 1895, pp. 44 *et seq.*

† "Versuche mit Kreuzungen von verschiedenen Rassen des Hausmaus," *Berichte d. naturforsch. Gesellsch. Freiburg*, x. (1898) pp. 317–332, and xi. (1900) pp. 131–138.

‡ *Mendel's Principles of Heredity*, Cambridge, 1902, p. 174.

§ *Report to the Evolution Committee of the Royal Society*, 1902, p. 145.

The answer to the first question has been obtained, and it is in the negative: the answer to the second has not, as the hybrids are not yet old enough to be paired. The experiments are as yet only beginning; but the evidence concerning the characters of the first generation is already greater than that obtained by von Guaita, who only observed the result of crossing four pairs of mice, while the litters already obtained by me are the result of nine crosses.

The Parent forms.

(*a*) *The Japanese Waltzing Mice.*

The average size of these mice is slightly smaller than that of the common house mouse: they are characterized by their faculty of spinning round, which is due to abnormality of the semicircular canals; and by the restlessness of their demeanour when not dancing. The ground colour of their coat is pure white; but there is always a variable amount of pale fawn on the cheeks, shoulders and rump; the arrangement of the colour on the mouse is seen in Fig. 1. They have pink eyes.

Our stock of waltzing mice arrived in December last: from that time till August they bred freely; and in all cases the offspring of two waltzing mice were indistinguishable from their parents, except for slight differences in the distribution of the fawn colour on the body: that is to say the original stock are shown by the character of their offspring to have been pure-bred. This fact is emphasized in order to remove any suspicion which may arise in the mind of some careful critic that the waltzing mice dealt with may have been dominant hybrids.

(*b*) *The Albino Mice.*

The mice used were the true albinos with pink eyes which are familiar to everyone: they may be roughly divided into two categories:—

(i) Pure-bred Albinos from the well-known mouse breeder Mr Steer and others.

(ii) Cross-bred Albinos which have appeared from time to time in the litters of piebald mice kept in the Oxford Laboratory for embryological purposes.

The cross-bred albinos were used advisedly: for albinism is said to be a recessive character; and this being so any albino is perfectly pure regardless of ancestry: that is to say, on the Mendelian hypothesis it makes no difference, as far as its offspring is concerned, whether a certain albino is the child of piebald parents or whether its parents have been pure white for many generations.

On the Mendelian hypothesis the ancestry of the albinos should make no difference: we shall see that, as a matter of fact, it probably does.

The Hybrids.

The number of hybrid families in which the colours can be seen is at present nine. The coloration of the hybrid differs from that of either parent in the fact that with two exceptions there are patches of colour which it is difficult to distinguish from that of the common house mouse.

The hybrids can be roughly classified according to the distribution of this colour on their bodies.

(*a*) Mice in which the distribution of the gray colour corresponds roughly with the distribution of fawn colour in the waltzing mouse. Fig. 2.

(*b*) Mice in which the gray colour covers much more of the body. Fig. 3.

(*c*) Mice which are all gray except on the belly and tail which are always nearly white. At a casual glance this type might easily be mistaken for a house mouse; but inspection of its belly reveals its hybrid nature. Fig. 4.

(*d*) Mice which are all fawn coloured except on the belly. The colour is like that of the markings on the Japanese waltzers: its distribution corresponds to that of the gray in (*c*).

a and *b* merge into one another, and the standard of separation is more or less arbitrary; but the line dividing *b* or *a* from *c* is at present quite sharp.

The following is a record of the crosses:

Cross 1. ♀ cross-bred albino.
♂ Japanese waltzer.
Young. 4 *a*.
2 *b*.

Cross 6. ♀ cross-bred albino.
♂ Japanese waltzer.
Young. 1 *a*.

Cross 7. ♀ pure-bred albino.
♂ Japanese waltzer.
Young. 3 *a*.
2 *b*.
3 *c*.

Cross 8. ♀ pure-bred albino.
♂ Japanese waltzer.
Young. 5 *c*.

Cross 9. ♀ cross-bred albino.
♂ Japanese waltzer.
Young. 6 *a*.

Cross 12. ♀ pure-bred albino.
♂ Japanese waltzer.
Young. 4 *c*.
2 *d*.

Cross 13. ♀ pure-bred albino.
♂ Japanese waltzer.
Young. 2 *b*.
2 *c*.

Cross 16. ♀ cross-bred albino.
♂ Japanese waltzer.
Young. 6 *b*.

Cross 20. ♀ pure-bred albino.
♂ Japanese waltzer.
Young. 4 *a*.
1 *b*.
1 *c*.

It will be seen by reference to the above figures that there are 18 *a*, 13 *b*, 15 *c* and 2 *d*, that is to say, 31 cases at least out of 48 in which albinism is not recessive: and even in *c*, which is a gray mouse superficially not unlike the house mouse, we by no means find a complete disappearance of whiteness; for as it has already been said, their bellies are nearly white; which is not true of the house mouse: and the bellies of *d* are like those of *c*.

The Influence of Ancestry.

The above tables show that hybrids with the colour of the house mouse all over (except the belly) appear only in the litters of pure-bred albinos, *i.e.* 3*c* in Cross 7, and 5*c* in Cross 8, 4*c* in Cross 12, 2*c* in Cross 13, and 1*c* in Cross 20, whereas they do not appear in the litters of cross-bred mothers. Of course so small a number of trials does not prove a definite rule: but the result observed may be connected with the fact that von Guaita who used albinos in-bred for 29 generations always got hybrids identical with the house mouse. And it looks as if it could be said that the more in-bred an albino is the less power it has of transmitting its whiteness. But many more crosses must be made before any definite statement can be made on this point.

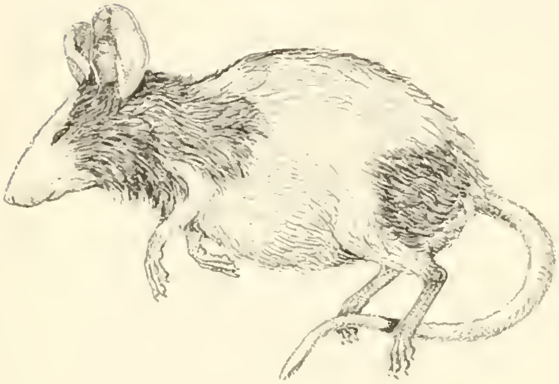

Fig. 1.

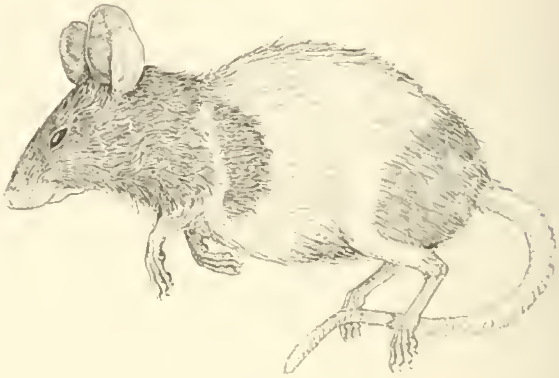

Fig. 2.

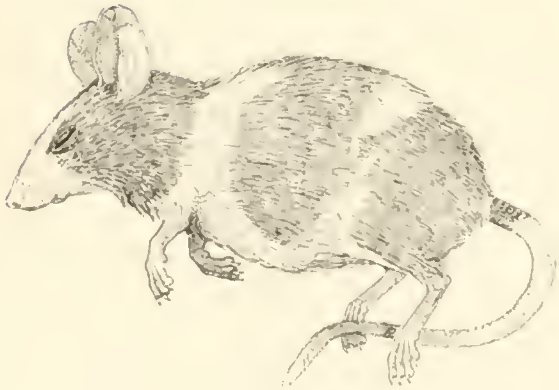

Fig. 3.

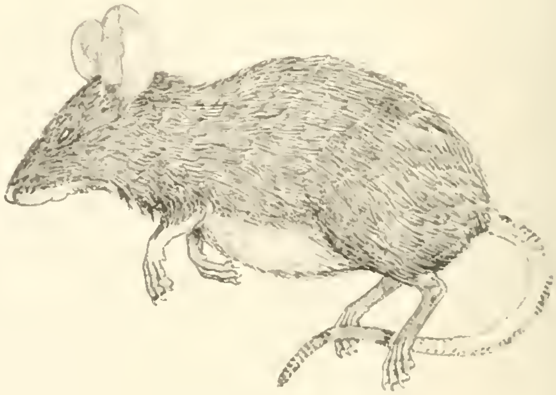

Fig. 4.

II. Interpolation by Finite Differences. (*Two Independent Variables.*)

By W. PALIN ELDERTON.

Let it be required to find $u_{p:r}$ in terms of $u_{0\cdot 0}, u_{0\cdot 1}, \ldots u_{1:0} \ldots u_{-1\cdot 0} \ldots, u_{0:-1} \ldots$, where p and r are both <1.

$$u_{p:r}=(1+\Delta_1)^p\,u_{0:r}=(1+\Delta_1)^p\,(1+\Delta_2)^r\,u_{0\cdot 0}$$

$$=\left\{1+(p\Delta_1+r\Delta_2)+\frac{1}{2\,!}\,(p^{(2)}\,\Delta_1{}^2+2pr\,\Delta_1\Delta_2+r^{(2)}\Delta_2{}^2)\right.$$

$$\left.+\frac{1}{3\,!}\,(p^{(3)}\Delta_1{}^3+3p^{(2)}r\,\Delta_1{}^2\Delta_2+3pr^{(2)}\,\Delta_1\Delta_2{}^2+r^{(3)}\Delta_2{}^3)\ldots\right\}u_{0\cdot 0}\quad\ldots\ldots\ldots(1),$$

where $p^{(n)}=p\,(p-1)\ldots(p+n-1)$ and Δ_1 and Δ_2 represent operations with respect to p and r respectively.

If we use the expression $\{1+(p\Delta_1+r\Delta_2)\}\,u_{0:0}$ for $u_{p:r}$ we require only 3 values of the function, while if we take in the next term in round brackets we require 6 values, but the objection to the formulæ seems to be that the values of the function which we use are not necessarily the nearest values to $u_{p:r}$. For practical purposes it would be better to have the expression in terms of the function rather than in terms of the differences, as the calculation of differences running in three directions is troublesome and the work is likely to contain slips*.

The following scheme shews the form of the problem and gives an idea of which are the best functions to use for interpolation :

$$\begin{array}{cccc}
u_{-1:-1} & u_{-1\cdot 0} & u_{-1:1} & u_{-1:2}\\
u_{0:-1} & u_{0\cdot 0} & u_{0\cdot 1} & u_{0:2}\\
 & \multicolumn{2}{c}{u_{p\cdot r}} & \\
u_{1:-1} & u_{1:0} & u_{1:1} & u_{1:2}\\
u_{2:-1} & u_{2:0} & u_{2:1} & u_{2:2}.
\end{array}$$

Now $\qquad u_{p\cdot 0}=u_{0:0}+p\,(u_{1\cdot 0}-u_{0\cdot 0}),\quad u_{p:1}=u_{0\cdot 1}+p\,(u_{1\cdot 1}-u_{0:1})$

and interpolating between these values we get (when $s=1-r$ and $q=1-p$)

$$u_{p\cdot r}=qsu_{0\cdot 0}+qru_{0\cdot 1}+psu_{1\cdot 0}+pru_{1\cdot 1}\dagger\quad\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2).$$

In most cases occurring in practice the coefficients in (2) can be calculated at sight and the labour of the whole interpolation is very small. The coefficients can easily be remembered by considering the distances of the required function from a given value and bearing in mind that the nearer the position of the given value to that of the required value the larger its coefficient.

For some purposes (2) will not be sufficiently accurate and we must seek for a similar formula involving more terms.

For this purpose Lagrange's Interpolation formula can be used and would be the only one applicable when the intervals between the given values of the function are not all equal. We should first find $u_{p\cdot a}, u_{p:b} \ldots$ and then $u_{p:r}$ by independent interpolations or else by working out coefficients as in formula (2). If we consider Lagrange's formula, viz.

$$u_p=\frac{(p-b)\,(p-c)\ldots}{(a-b)\,(a-c)\ldots}\,u_a+\frac{(p-a)\,(p-c)\ldots}{(b-a)\,(b-c)\ldots}\,u_b+\ldots$$

[* The biometrician has repeatedly to use tables of double entry: e.g. in the cases of skew variation when using the G-integral (*Brit. Assoc. Report*, 1899), in dealing with goodness of fit (*Biometrika*, Vol. I. p. 155), in finding the influence of natural selection on correlation (*Phil. Trans.* A, Vol. 200, p. 64), etc., etc. Hence the importance of a good method of interpolation. Ed.]

† See note by T. G. Ackland, *Journal Institute of Actuaries*, Vol. 32, p. 286.

and put $a=0$, $b=1$, $c=-1$, $d=2$, $e=-2$, etc., we have

$$u_p=\frac{(p-1)(p+1)(p-2)(p+2)\ldots}{(-1)(1)(-2)(2)\ldots}u_0+\frac{p(p+1)(p-2)(p+2)\ldots}{(1)(2)(-1)(3)(-2)\ldots}u_1+\frac{p(p-1)(p-2)(p+2)\ldots}{(-1)(-2)(1)(-3)(2)\ldots}u_{-1}\text{etc.}$$

$$=p_{2n}\left\{c_0\frac{u_0}{p}+c_1\frac{u_1}{p-1}+c_{-1}\frac{u_{-1}}{p+1}+\ldots+\frac{c_t u_t}{p-t}+\ldots\right\}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (3),$$

where it is assumed that $2n$ terms are used, so that

$$p_{2n}=(p+n-1)\ldots(p-n) \text{ and } c_t=(-1)^{n-t}\frac{1}{\underline{|n-t}\,\underline{|n+t-1}}.$$

These two functions, p_{2n} and c_t, can be calculated easily as in the case of a two-variables interpolation n will not be greater than 2 or for a one-variable than 4. Taking $n=4$, $t=2$ as an example

$$c=\frac{1}{\underline{|2}\,\underline{|5}}=\frac{1}{240}.$$

For the two-variables we shall use (3) for obtaining an interpolated value in the following form

$$\left.\begin{aligned}u_{p.r}&=p_{2n}\left\{c_0\frac{u_{0:r}}{p}+c_1\frac{u_{1\ r}}{p-1}+\ldots+\frac{c_t u_{tr}}{p-t}+\ldots\right\}\\ \text{and}\qquad u_{p:r}&=r_{2n}\left\{c_0\frac{u_{p:0}}{r}+c_1\frac{u_{p:1}}{r-1}+\ldots+\frac{c_t u_{pt}}{r-t}+\ldots\right\}\end{aligned}\right\}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4).$$

By this method we see the connection between Professor Everett's Central Difference* and Lagrange's formulæ, for if we write the differences in the former in terms of the function and take p_{2n} outside a bracket we see that the two are alike. In the actual work in the case of a two-variables interpolation $p_{2n}\times r_{2n}$ could easily be obtained and then we could find the coefficients $\div p_{2n}\times r_{2n}$. Such reduced coefficient for $u_{t.s}$ would be

$$\frac{c_t\,c_s}{(p-t)\,(r-s)}.$$

Let us find as an example of the formulæ an interpolated value for ages 51 and 28 from the following values at 3 °/。 interest from the H^M Joint Life Annuity Table†.

Age of Younger Life	Age of Elder Life			
	45	*50*	*55*	*60*
20	14·1936	12·8092	11·2791	9·6503
25	14·0130	12·6787	11·1886	9·5902
30	13·7313	12·4690	11·0378	9·4855
35	13·3625	12·1954	10·8436	9·3536

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* $u_p=qu_0+\frac{(q+1)\,q\,(q-1)}{\underline{|3}}a_2+\frac{(q+2)\ldots(q-2)}{\underline{|5}}a_4+\ldots+pu_1+\frac{(p+1)\,p\,(p-1)}{\underline{|3}}b_2+\frac{(p+2)\ldots(p-2)}{\underline{|5}}b_4+\ldots$

where a_2, $a_4\ldots$ are even central differences of u_0 and b_2, $b_4\ldots$ those of u_1.—See *Journal Institute of Actuaries*, Vol. 35, p. 452.

† H^M is the name given to the mortality table constructed by the Institute of Actuaries (1869) from the experience of healthy male lives assured by English offices.

We obtain the following values

By (1) 1st Differences (3 values)..................12·2549
(1) 2nd ,, (6 values)..................12·2773
(2) 4 values ....12·2619
(4) 16 values ....12·2815
The value given in the Tables is ....12·2811.

In applying formulæ (4) if we use $2n$ terms in each equation we require $4n^2$ terms altogether—thus 4, 16, 36... are the numbers of terms that will occur. Of course if we use an odd number of terms, say $2n+1$, the formula reads

$$u_{p.r}=p_{2n+1}\left\{\frac{c_0u_0}{p}+\ldots+\frac{c_tu_t}{p-t}+\ldots\right\} \text{ and } u_{p.r}=r_{2n+1}\left\{\frac{c_0u_0}{r}+\ldots+\frac{c_tu_t}{r-t}\text{ etc.}\right\}$$

where $p_{2n+1}=(p+n)(p+n-1)\ldots(p+1)p(p-1)\ldots(p-n)$. The formulæ can also be used when m terms are used in one direction and n in the other.

As an example of the full working take the table of joint life annuities given and find the annuity on two lives aged 51·27355 and 28·63984. Here $p=\cdot 25471$, $r=\cdot 72797$.

$\log(p+1)$	$\cdot 0985433$
$\log(2-r)$	$\cdot 1044973$
$\log(2-p)$	$\cdot 2418676$
$\log(r+1)$	$\cdot 2375362$
$\log(p)$	$\bar{1}\cdot 4060460$
$\log(1-r)$	$\bar{1}\cdot 4346168$
$\log(1-p)$	$\bar{1}\cdot 8723253$
$\log(r)$	$\bar{1}\cdot 8621135$
$\log p_4 r_4$	$\bar{1}\cdot 2575460$

$$c_0=\tfrac{1}{2},\quad c_1=-\tfrac{1}{2},\quad c_2=\tfrac{1}{6},\quad c_{-1}=-\tfrac{1}{6},$$
$$\log\tfrac{1}{2}=\bar{1}\cdot 6989700 \text{ and } \log\tfrac{1}{6}=\bar{1}\cdot 2218487,$$

and we get the following expressions corresponding to {...} in (4), where [·2...] represents antilog ·2....

"p" terms $-[\bar{1}\cdot 1233054]\,u_{-1}+[\ \cdot 2929240]\,u_0+[\bar{1}\cdot 8266447]\,u_1-[\bar{2}\cdot 9799811]\,u_2,$

"r" terms $-[\bar{2}\cdot 9843125]\,u_{-1}+[\bar{1}\cdot 8368565]\,u_0+[\ \cdot 2643532]\,u_1-[\bar{1}\cdot 1173514]\,u_2,$

(+) 1·1520924	(−) 1·1075220	(−) 1·0512745	(+) ·9845408
$\bar{2}\cdot 1076179$	$\bar{1}\cdot 2772365$	$\bar{2}\cdot 8109572$	$\bar{3}\cdot 9642936$
(−) 1·1465311	(+) 1·1030747	(+) 1·0487757	(−) ·9818277
$\bar{2}\cdot 9601619$	·1297805	$\bar{1}\cdot 6635012$	$\bar{2}\cdot 8168376$
(−) 1·1377117	(+) 1·0958316	(+) 1·0428827	(−) ·9770602
$\bar{1}\cdot 3876586$	·5572772	·0909979	$\bar{1}\cdot 2443343$
(+) 1·1258878	(−) 1·0861960	(−) 1·0351735	(+) ·9709788
$\bar{2}\cdot 2406568$	$\bar{1}\cdot 4102754$	$\bar{2}\cdot 9439961$	$\bar{2}\cdot 0973325$

The upper line in each case gives the logarithm of the joint life annuity, and the lower line the sum of the logarithms of the coefficients shewn in the two expressions marked "p" terms and "*r*" terms. The two lines are added at sight and the anti-logarithm found. The sign of each expression, given in brackets, is a great help to rapid work.

+	−
·18185	1·27847
·23257	3·35251
17·09135	2·42526
44·98925	3·13669
5·15557	·72984
13·61071	·95317
·08889	·62902
·11703	1·66492
81·47022	14·16988
14·16988	
log 67·30034	= 1·8280173
	$\bar{1}$·2575460
	1·0855633 ... 12·17765

Using the annuities for 51, 28 ; 51, 29 ; 52, 28 and 52, 29 with formula (2) I get 12·1775.

III. **Variation in the Moscatel** (*Adoxa Moschatellina*, L.).

By HENRY WHITEHEAD.

The mode of arrangement of flowers in globose heads in the Moscatel (*Adoxa moschatellina*, L.) is often cited as an example of packing the maximum number of flowers in the minimum of space. An inflorescence of the Moscatel generally consists of five flowers—one terminal and four lateral. The terminal flower has two bracts, four petals, four branched stamens and four carpels ; and each of the lateral flowers possesses three bracts, five petals, five branched stamens and five carpels. This is the most common form of arrangement, but the plant varies considerably and the following figures show that only 55 per cent. of the numbers counted agree with the above description, and 51 other methods of arrangement are noted.

The number of flowers in the inflorescence is a variable quantity as well as the number of parts in the perianth, and for the sake of convenience these variations are considered separately.

The specimens examined were obtained from three localities, Chislehurst in Kent, Caterham in Surrey, and Theydon Garnon in Essex.

I. *Variation in the number of flowers in an inflorescence.*

The number of flowers constituting an inflorescence varies from three to ten. 1071 inflorescences were counted and the following results were obtained :

TABLE I.

Number of flowers	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Actual quantities...	6	71	934	26	24	7	1	2
Percentages	0·6	6·6	87·2	2·4	2·1	0·6	0·1	0·2

Diagram I. represents the above figures in a graphical form.

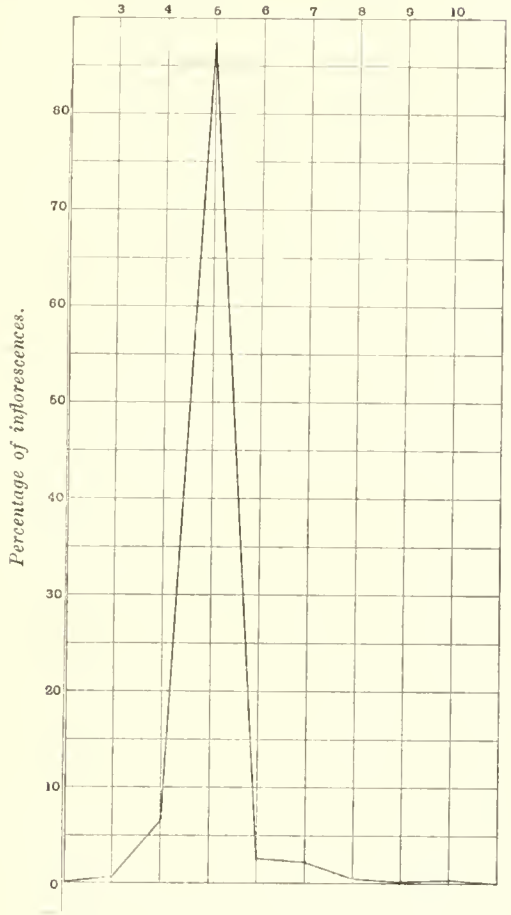

Diagram I. Number of flowers per inflorescence.

In Gerard's *Herbal* (2nd Ed. 1633), p. 1090, there is a note relating to the Moscatel which is interesting. He says "the floures grow clustering on the top of a stalk, commonly five or seven together,...... ; it floures in Aprill and is to be found in divers places amongst bushes at that time, as in Kent about Chiselhurst, especially in Pits his wood and at the further end of Cray heath on the left hand under a hedge among bryers and brambles, which is his proper seat." Gerard gives an illustration of the Moscatel which shows two flowering stems, one of these has a clustered group at the top with one small lateral flower some distance down the stem, and the other a clustered group with two lateral flowers in a lower position. Several plants bearing seven flowers were found near the spot mentioned by Gerard as "Cray heath." It seems strange that Gerard should give the numbers as five to seven when the four-flowered inflorescences occur more than twice as often as either the six-flowered or the seven-flowered.

II. *Variation in the number of petals.*

Before considering the variation in the number of petals it would be as well to say a few words about the mode of arrangement of the parts of a flower of the normal type.

The tips of each of the uppermost petals of the four pentamerous lateral flowers (1, Diagram II) fit into the space between two petals (4) of the terminal and tetramerous flower.

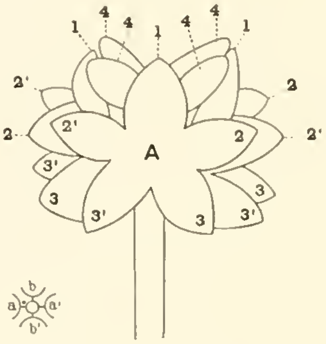

Diagram II. Plan and side view of inflorescence without terminal flower.

The arrangement of the remaining petals is shown in Diagram II, where the petal indicated by 2′ in the expanded lateral flower shown in the diagram touches petal 2 of the flower on the left hand, while petal 2 of flower *A* touches 2′ of its neighbour on the right. The petals marked 3 and 3′ touch in a similar manner.

The order in which the various flowers and petals in an inflorescence generally open is very curious. First, the four petals of the terminal flower expand simultaneously, then two lateral buds which are diametrically opposite (*a* and *a*′) open, and finally the opening of the other pair of buds (*b* and *b*′) takes place. The order in which the petals of the pentamerous flowers expand is 1, 2 and 2′, and 3 and 3′. Mathematical precision is necessary where the flowers are packed in such a neat fashion, otherwise the struggle for space would result in a haphazard arrangement and consequently a loss of space.

The terminal flower is the least variable of all. Out of 763 terminal flowers 754 (about 99 per cent.) were found to be tetramerous; 4 being pentamerous and 5 trimerous. The fixity in the number of parts of the terminal flower is used for the purpose of classifying and tabulating the variations from the normal type as given in Table II.

The numbers given in Table II. lines 16—20 deserve special notice. Line 16 shows that 420 inflorescences with 4 pentamerous lateral flowers (represented by the entry 4 in column 5) were noted. In line 17 where there are 3 "fives" and 1 "four" in the inflorescence the number has dropped to 125. In line 18 where there are 2 "fives" and 2 "fours" the number is 59, and lines 19 and 20 show a further decrease in the totals as the florets with four petals take the places of those with five. The same rule holds in cases where the numbers of flowers in the inflorescences are seven, six and four.

TABLE II.

Variations in the Number of Divisions of the Corolla.

Terminal Flower	Total Number of Flowers per inflorescence	Distribution of Petals in lateral flowers*						Locality			Totals	Number of Row
		8	*7*	*6*	*5*	*4*	*3*	Near Chislehurst, Kent	Caterham, Surrey	Theydon Garnon, Essex		
Trimerous	*4*	—	1	—	2	—	—	1	—	—	1	1
	4	—	—	—	2	1	—	1	—	—	1	2
	4	—	—	—	1	2	—	—	—	1	1	3
	4	—	—	—	—	3	—	—	—	2	2	4
Tetramerous	*3*	1	—	—	1	—	—	1	—	—	1	5
	3	—	—	—	2	—	—	4	—	—	4	6
	4	—	—	—	3	—	—	16	3	2	21	7
	4	—	—	—	2	1	—	4	1	—	5	8
	4	—	—	—	1	2	—	3	—	1	4	9
	4	—	—	—	—	3	—	—	—	1	1	10
	4	—	—	2	—	1	—	1	—	—	1	11
	4	—	—	1	2	—	—	2	—	—	2	12
	4	—	—	1	1	1	—	1	—	—	1	13
	4	—	—	1	—	2	—	1	—	—	1	14
	4	—	—	—	2	—	1	1	—	—	1	15
	5	—	—	—	4	—	—	283	30	107	420	16
	5	—	—	—	3	1	—	91	6	28	125	17
	5	—	—	—	2	2	—	38	10	11	59	18
	5	—	—	—	1	3	—	27	1	4	32	19
	5	—	—	—	—	4	—	10	2	2	14	20
	5	—	—	3	—	1	—	—	1	—	1	21
	5	—	—	2	2	—	—	1	1	—	2	22
	5	—	—	2	1	1	—	1	—	—	1	23
	5	—	—	1	3	—	—	4	3	—	7	24
	5	—	—	1	2	1	—	3	1	1	5	25
	5	—	—	1	1	2	—	1	1	—	2	26
	5	—	—	—	3	—	1	1	—	—	1	27
	5	—	—	—	2	1	1	1	—	—	1	28
	5	—	—	—	1	2	1	7	—	—	7	29
	5	—	—	—	1	1	2	—	—	1	1	30
	5	—	—	—	—	3	1	2	—	—	2	31
	6	—	—	1	4	—	—	—	1	—	1	32
	6	—	—	—	5	—	—	5	—	1	6	33
	6	—	—	—	4	1	—	1	—	—	1	34
	6	—	—	—	3	2	—	1	—	—	1	35
	6	—	—	—	2	3	—	1	—	—	1	36
	6	—	—	—	2	2	1	—	2	—	2	37
	6	—	—	—	1	4	—	2	2	—	4	38
	7	—	—	—	6	—	—	5	—	—	5	39
	7	—	—	—	5	1	—	1	1	1	3	40
	7	—	—	—	4	2	—	—	1	—	1	41
	7	—	—	—	4	1	1	1	—	—	1	42
	7	—	—	—	3	3	—	—	1	—	1	43
	7	—	—	—	3	2	1	1	—	—	1	44
	7	—	—	—	—	6	—	1	—	—	1	45
	7	—	—	—	—	3	3	2	—	—	2	46
	8	—	—	—	6	1	—	1	—	—	1	47
Pentamerous	*5*	—	—	—	4	—	—	1	—	—	1	48
	5	—	—	—	2	2	—	1	—	—	1	49
	5	—	—	—	1	2	1	1	—	—	1	50
	6	—	—	—	2	3	—	—	1	—	1	51

* The figures at the top of this column denote the number of petals per flower. The figures in the vertical columns denote the number of flowers of each type.

It will be seen that the corollas of the *Adoxa* are 8-, 7-, 6-, 5-, 4- and 3-partite, and the actual number of each of these types of corolla is shown in Table III, while Diagram III gives the results in a graphical form.

TABLE III.

	3	*4*	*5*	*6*	*7*	*8*
Number of flowers not including terminal flowers Number of terminal flowers	25 5	514 754	2494 4	30 —	1 —	1 —
Totals	30	1268	2498	30	1	1
Percentages of totals (to nearest 10th) ...	0·8	33·1	65·3	0·8	—	—
Assuming that all inflorescences were normal then percentages of totals would be	—	20	80	—	—	—

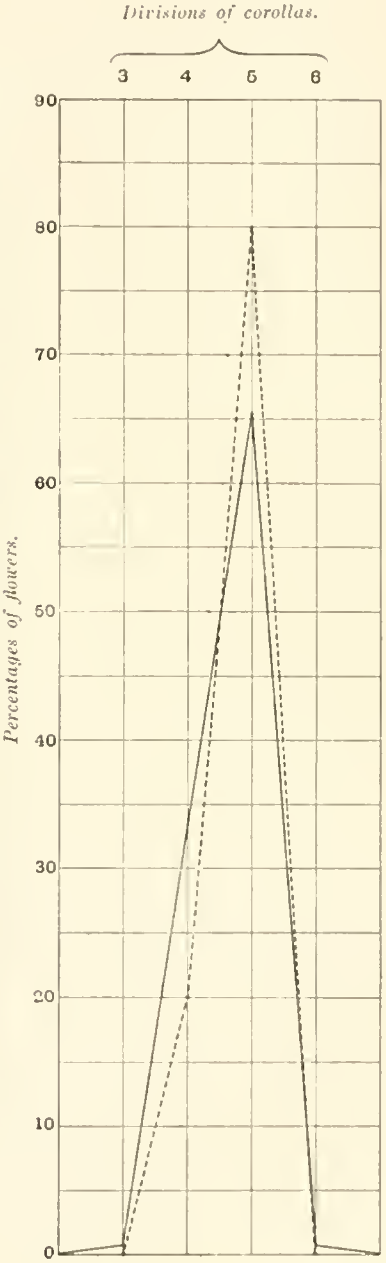

DIAGRAM III.

——— Curve plotted from actual numbers.
- - - - - Curve if all the inflorescences were normal.

The statistics presented in this paper show that the inflorescences of the Moscatel vary considerably from the normal type. A normal inflorescence consisting of five flowers has one tetramerous and four pentamerous, that is, a percentage ratio of 20 to 80, but the ratio of the observed percentages is as 33·1 to 65·3. The variation from the normal type therefore favours the tetramerous type more than the pentamerous type, while sports occur in the form of 3-, 6-, 7-, and 8-partite corollas though these form less than 2 per cent. of the total.

NOTE.

At the suggestion of Prof. Karl Pearson several old herbaria at Kew, the British Museum and at Cambridge were examined in order to see whether any of the specimens agree with the figure given in Gerard's *Herbal*, 2nd Ed., p. 1091 (edited by Thos. Johnson). None were found exhibiting the peculiar racemose arrangement which is so marked in his drawing. The first edition (published 1597) has an entirely different figure of *Adoxa*, which has evidently been borrowed from the *Kräuter Buch* of Tabernæmontanus. In this case all the lateral flowers are tetramerous.

I am indebted to Miss K. M. Hall, Curator of the Stepney Borough Museum, for much help and many useful suggestions.

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IV. Seasonal Change in the Characters of Aster prenanthoides, Muhl.

IN the last number of *Biometrika* (pp. 304—315) a collection of evidence was published, showing the existence of periodic changes in the mean and modal numbers of floral organs, and in the constants expressing the variation of these numbers, in several plants. Just before the publication of this article, Mr G. H. Shull published in the *American Naturalist*, Vol. XXXVI. No. 422, an elaborate "Quantitative Study of Variation in the Bracts, Rays and Disk-florets "of *Aster Shortii* Hook., *A. Novae Angliae* L., *A. puniceus* L., and *A. prenanthoides* Muhl., "from Yellow Springs, Ohio." In this paper the constants expressing the variation in and correlation between bracts, rays, and disk-florets are fully determined for each species; and in addition, the author describes the results obtained by examining four sets of flowers, collected from the same series of individuals of *A. prenanthoides* on four different days. The first of these four gatherings was made on September 27, the last on October 8; a comparison between the four sets of results gives a remarkable and instructive demonstration of the change which may occur in individual plant characters within a short space of time. The following table, abridged from the fuller tables given by Mr Shull, shows some features of the change.

TABLE 1.

Variation in Number and S. D. of Floral Organs on four different Days.

	September 27 (117 Capitula)	September 30 (143 Capitula)	October 4 (139 Capitula)	October 8 (176 Capitula)
Mean No. Bracts ...	47·410 ± 0·345	44·343 ± 0·291	43·835 ± 0·302	41·920 ± 0·249
S. D. of Bracts ...	5·524 ± 0·244	5·152 ± 0·205	5·276 ± 0·213	4·890 ± 0·176
Mean No. Rays ...	30·769 ± 0·249	28·706 ± 0·201	28·252 ± 0·200	26·335 ± 0·153
S. D. of Rays ...	3·986 ± 0·176	3·569 ± 0·142	3·501 ± 0·142	3·010 ± 0·108
Mean No. Disk-florets	56·427 ± 0·249	51·713 ± 0·282	49·158 ± 0·279	45·778 ± 0·242
S. D. Disk-florets ...	3·986 ± 0·176	4·995 ± 0·199	4·885 ± 0·198	4·777 ± 0·171

The change in mean and standard deviation of all the organs studied during the short interval between September 27 and September 30 is of especial interest in connection with the asserted multimodal distributions so often described in floral organs.

Mr Shull lays little stress on the peaks of his frequency curves, but it is worth notice that in this carefully collected material there is not a single case of a many-peaked curve in which the "modes" coincide with the numbers of the Fibonacci series, and the author criticises the process by which previously recorded "modes" are brought into accord with this series. He says "The members of the series, along with Ludwig's 'Unterzahlen,' which are "made up from the Fibonacci numbers by multiplication or addition,—e.g., 10=(2×5), "29=(8+21) etc.,—include so large a proportion of all the smaller numbers that many modes "must fall on or near one of them, even if there be no fundamental relation existing between "this complex series and the number of floral parts or other organs under consideration. "To account for modes which do not fall on any of these, Ludwig creates the 'Scheingipfel,' "which is formed by the overlapping of curves having their modes on adjacent numbers of "the Fibonacci-Ludwig complex. Thus, if the maximum fall on 9, it is a 'Scheingipfel' "formed by the union of curves having maxima upon 8 and 10; if it fall upon 11, it is "made up of curves having maxima upon 10 and 13, etc. It is evident that such a scheme "will furnish an explanation of almost any condition which might arise."

It is to be hoped that Mr Shull's admirable essay will induce future investigators to consider more carefully the sources of error in their process of collection before they assert that even statistically significant "multimodality" is an indication of actual polymorphism in plants, and to realise the importance of prolonged study before the differences between local races can be usefully insisted upon.

W. F. R. W.

ON THE NUMBER AND ARRANGEMENT OF THE BONY PLATES OF THE YOUNG JOHN DORY.

By L. W. BYRNE.

The accompanying notes were made some years ago at the suggestion of my friend Mr E. W. L. Holt[*]. It had been my intention to make them more complete by the examination of adult specimens from the same locality, but for this I have had neither time nor material and they are only published now as throwing, perhaps, a little light upon one of the natural causes which may have assisted in the derivation of the asymmetrical *Pleuronectidae* from some more normally-formed member of the group *Zeorhombi*[†].

The material examined consisted of 250 specimens of *Zeus faber* from $2\frac{3}{4}$ to 5 inches in length and probably about a year old, captured by Plymouth trawlers.

For convenience the bony plates are in these notes expressed by four letters or numbers as follows:—

$$\begin{array}{c} \text{Dorsal} \\ \text{Left}\ \dfrac{9 \mid 7}{8 \mid 10}\ \text{Right} \\ \text{Ventral} \end{array}$$

the diagram $\dfrac{9 \mid 7}{8 \mid 10}$, for instance, representing a specimen with 9 left and 7 right dorsal plates, and 8 left and 10 right anal plates. In the Plate *dp*. 1 indicates the position of the anterior dorsal and *ap*. 1 of the anterior anal plate.

The most striking peculiarity of the 250 specimens taken as a whole is the very small proportion in which the plates are perfectly symmetrical in arrangement, the number of plates on the left side equalling those on the right in both the dorsal and anal series; this arrangement may be expressed by the diagram $\dfrac{n \mid n}{m \mid m}$ where n is either less than, equal to, or more than m, e.g. $\dfrac{8 \mid 8}{9 \mid 9}$, $\dfrac{9 \mid 9}{9 \mid 9}$, or $\frac{9 \mid 9}{8 \mid 8}$. This symmetry only occurs in 23·6 per cent. of the total number examined.

---

[*] See Boulenger: *Ann. Mag. Nat. Hist.*, Series 7, x. p. 303, 1902, where the circumstances which led Mr Holt to make this suggestion are explained. For the material used I am indebted to him and to the Marine Biological Association.

[†] Boulenger: *loc. cit.* p. 295.

The arrangement of the plates.

The most convenient way of dealing with the various combinations of numbers of plates found in these specimens seems to be by reference to their symmetry in one or both series.

Among the 250 specimens:—

1. Both series of plates are symmetrical in 59 or 23·6 °/o.
2. One series is asymmetrical in 112 or 44·8 °/o.
3. Both series are asymmetrical in 79 or 31·6 °/o.

1. Among the symmetrical specimens the following arrangements occur:—

$$\left[\frac{n \mid n}{m \mid m}\right].$$

$\frac{8 \mid 8}{7 \mid 7}$	$\frac{7 \mid 7}{8 \mid 8}$	$\frac{8 \mid 8}{8 \mid 8}$	$\frac{9 \mid 9}{8 \mid 8}$	$\frac{8 \mid 8}{9 \mid 9}$	$\frac{9 \mid 9}{9 \mid 9}$	$\frac{10 \mid 10}{9 \mid 9}$	$\frac{8 \mid 8}{10 \mid 10}$	$\frac{9 \mid 9}{10 \mid 10}$
2	1	11	2	25	14	1	1	2

2. Among the 112 asymmetrical in one series the following arrangements occur:—

(*a*) Of the formula $\frac{n+k \mid n}{m \mid m}$, 33 or 13·2 °/o.

$\frac{8 \mid 7}{8 \mid 8}$	$\frac{9 \mid 7}{8 \mid 8}$	$\frac{9 \mid 8}{8 \mid 8}$	$\frac{10 \mid 9}{8 \mid 8}$	$\frac{8 \mid 7}{9 \mid 9}$	$\frac{9 \mid 7}{9 \mid 9}$	$\frac{9 \mid 8}{9 \mid 9}$	$\frac{10 \mid 8}{9 \mid 9}$	$\frac{10 \mid 9}{9 \mid 9}$	$\frac{9 \mid 8}{10 \mid 10}$	$\frac{10 \mid 8}{10 \mid 10}$
2	1	4	1	3	1	14	2	1	3	1

(*b*) Of the formula $\frac{n \mid n+k}{m \mid m}$, 32 or 12·8 °/o.

$\frac{7 \mid 8}{8 \mid 8}$	$\frac{7 \mid 9}{8 \mid 8}$	$\frac{8 \mid 9}{8 \mid 8}$	$\frac{7 \mid 8}{9 \mid 9}$	$\frac{8 \mid 9}{9 \mid 9}$	$\frac{8 \mid 10}{9 \mid 9}$	$\frac{9 \mid 10}{9 \mid 9}$	$\frac{8 \mid 9}{10 \mid 10}$	$\frac{8 \mid 10}{10 \mid 10}$	$\frac{9 \mid 10}{10 \mid 10}$	$\frac{9 \mid 11}{10 \mid 10}$
1	1	1	2	17	2	4	1	1	1	1

(*c*) Of the formula $\frac{n \mid n}{m+k \mid m}$, 26 or 10·4 °/o.

$\frac{8 \mid 8}{8 \mid 7}$	$\frac{7 \mid 7}{9 \mid 8}$	$\frac{8 \mid 8}{9 \mid 8}$	$\frac{9 \mid 9}{9 \mid 8}$	$\frac{8 \mid 8}{10 \mid 8}$	$\frac{8 \mid 8}{10 \mid 9}$	$\frac{9 \mid 9}{10 \mid 9}$	$\frac{10 \mid 10}{10 \mid 9}$	$\frac{9 \mid 9}{12 \mid 9}$
3	1	4	6	1	4	4	2	1

(*d*) Of the formula $\frac{n \mid n}{m \mid m+k}$, 21 or 8·4 $^{\circ}/_{\circ}$.

$\frac{7 \mid 7}{7 \mid 8}$	$\frac{8 \mid 8}{7 \mid 9}$	$\frac{9 \mid 9}{7 \mid 9}$	$\frac{8 \mid 8}{8 \mid 9}$	$\frac{9 \mid 9}{8 \mid 9}$	$\frac{8 \mid 8}{8 \mid 10}$	$\frac{9 \mid 9}{8 \mid 10}$	$\frac{8 \mid 8}{9 \mid 10}$	$\frac{9 \mid 9}{9 \mid 10}$	$\frac{9 \mid 9}{10 \mid 11}$
1	1	1	5	6	1	1	1	3	1

3. Among the 79 asymmetrical in both series the following arrangements occur:—

(i) Where both series have more plates on the same side, 40 or 16 $^{\circ}/_{\circ}$.

(*a*) Of the formula $\frac{n+k \mid n}{m+l \mid m}$, 18 or 7·2 $^{\circ}/_{\circ}$.

$\frac{8 \mid 7}{9 \mid 7}$	$\frac{8 \mid 7}{9 \mid 8}$	$\frac{9 \mid 8}{9 \mid 8}$	$\frac{10 \mid 8}{9 \mid 8}$	$\frac{10 \mid 9}{9 \mid 8}$	$\frac{10 \mid 9}{10 \mid 7}$	$\frac{9 \mid 7}{10 \mid 8}$	$\frac{9 \mid 8}{10 \mid 8}$	$\frac{9 \mid 8}{10 \mid 9}$	$\frac{10 \mid 9}{10 \mid 9}$
1	2	2	1	1	1	1	1	7	1

(*b*) Of the formula $\frac{n \mid n+k}{m \mid m+l}$, 22 or 8·8 $^{\circ}/_{\circ}$.

$\frac{7 \mid 8}{7 \mid 9}$	$\frac{8 \mid 9}{7 \mid 9}$	$\frac{7 \mid 8}{8 \mid 9}$	$\frac{8 \mid 9}{8 \mid 9}$	$\frac{8 \mid 9}{8 \mid 10}$	$\frac{8 \mid 9}{9 \mid 10}$	$\frac{9 \mid 10}{9 \mid 10}$	$\frac{10 \mid 11}{9 \mid 10}$	$\frac{8 \mid 9}{9 \mid 11}$
1	1	5	6	1	4	2	1	1

(ii) Where the plates of one side preponderate in one series and of the other in the other.

(*a*) Where the total number of plates on the right side equals the total number on the left and the specimen attains a "secondary symmetry," 32 or 12·8 $^{\circ}/_{\circ}$.

(1) Of the formula $\frac{n+k \mid n}{m \mid m+k}$, 13 or 5·2 $^{\circ}/_{\circ}$.

$\frac{8 \mid 7}{8 \mid 9}$	$\frac{9 \mid 8}{8 \mid 9}$	$\frac{10 \mid 9}{8 \mid 9}$	$\frac{9 \mid 7}{8 \mid 10}$	$\frac{9 \mid 8}{9 \mid 10}$	$\frac{10 \mid 9}{9 \mid 10}$	$\frac{9 \mid 7}{9 \mid 11}$
1	5	1	1	3	1	1

(2) Of the formula $\frac{n \mid n+k}{m+k \mid m}$, 19 or 7·6 $^{\circ}/_{\circ}$.

$\frac{7 \mid 8}{9 \mid 8}$	$\frac{8 \mid 9}{9 \mid 8}$	$\frac{7 \mid 8}{10 \mid 9}$	$\frac{8 \mid 9}{10 \mid 9}$	$\frac{9 \mid 10}{10 \mid 9}$	$\frac{8 \mid 10}{11 \mid 9}$
1	13	1	2	1	1

(*b*) Where the plates of one side or the other preponderate there are 7 or 2·8 °/o of the following formulæ and arrangements:—

$\frac{n+k \mid n}{m \mid m+k+l}$		$\frac{n+k+l \mid n}{m \mid m+k}$	$\frac{n \mid n+k+l}{m+k \mid m}$		
$\frac{9 \mid 8}{9 \mid 10}$	$\frac{8 \mid 7}{8 \mid 10}$	$\frac{9 \mid 7}{8 \mid 9}$	$\frac{7 \mid 9}{9 \mid 8}$	$\frac{8 \mid 10}{9 \mid 8}$	$\frac{8 \mid 10}{10 \mid 11}$
1	1	2	1	1	1

The following are the particular combinations which occur in more than 5 per cent. of the 250 specimens:—

$\frac{8 \mid 8}{9 \mid 9}$ in 25 specimens or 10 per cent of the total.
$\frac{8 \mid 9}{9 \mid 9}$ in 17 ,, ,, 6·8 ,, ,,
$\frac{9 \mid 8}{9 \mid 9}$ in 14 ,, ,, 5·6 ,, ,,
$\frac{9 \mid 9}{9 \mid 9}$ in 14 ,, ,, 5·6 ,, ,,
$\frac{8 \mid 9}{9 \mid 8}$ in 13 ,, ,, 5·2 ,, ,,

The number of the plates.

Dr Günther in the first edition of the British Museum Catalogue gives the total number of plates in *Z. faber* as 9·8, or 7 occasionally 10 in each dorsal series and 9 in each anal. In the specimens under consideration the maxima and minima observed were:—

Dorsal L. Max. 10 Min. 7 R. Max. 11 Min. 7
Anal ,, ,, 12 ,, 7 ,, ,, 11 ,, 7

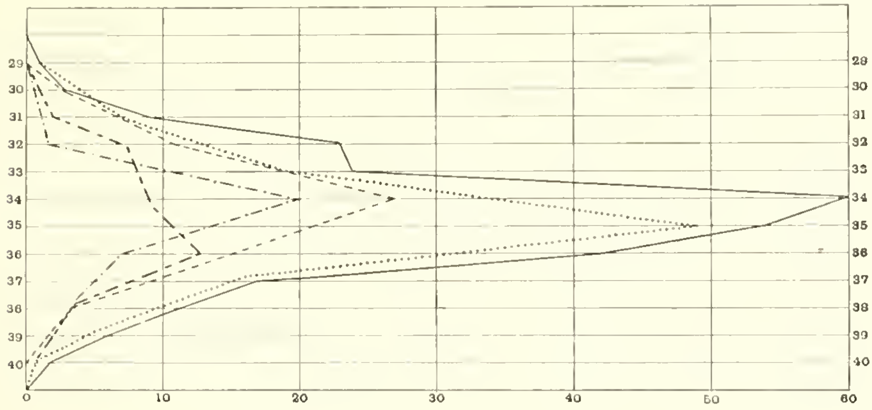
and the variation may be diagrammatically expressed $\frac{10-7 \mid 7-11}{12-7 \mid 7-11}$.

The average number of plates was 34·6, $\frac{8{\cdot}432 \mid 8{\cdot}464}{8{\cdot}864 \mid 8{\cdot}840}$, the largest in any individual 40, in two instances, $\frac{10 \mid 11}{9 \mid 10}$ and $\frac{9 \mid 11}{10 \mid 10}$, and the smallest 29, $\frac{7 \mid 7}{7 \mid 8}$.

The accompanying table and diagram show the number of plates occurring in the individuals of each formula, and the nearly regular grouping of them in each instance round a maximum of 34 or 35; (it must be borne in mind that in a specimen primarily or secondarily symmetrical there must be an even number of plates and that in a specimen asymmetrical in one series an even number is very uncommon).

Number of Plates

	29	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	Total	Per cent.
1. $\frac{n \mid n}{m \mid m}$ ...	—	3	—	11	—	27	—	15	—	3	—	—	59	23·6
2. (*a*) $\frac{n+k \mid n}{m \mid m}$	—	—	2	1	7	1	15	2	4	1	—	—	33	13·2
(*b*) $\frac{n \mid n+k}{m \mid m}$	—	—	1	1	3	—	17	2	5	1	1	1	32	12·8
(*c*) $\frac{n \mid n}{m+k \mid m}$	—	—	4	—	4	1	10	—	4	—	3	—	26	10·4
(*d*) $\frac{n \mid n}{m \mid m+k}$	1	—	—	1	5	2	7	1	3	—	1	—	21	8·4
3. (i) (*a*) $\frac{n+k \mid n}{m+l \mid m}$	—	—	1	2	—	3	2	9	—	1	—	—	18	7·2
(*b*) $\frac{n \mid n+k}{m \mid m+l}$	—	—	1	5	1	6	1	4	1	2	—	1	22	8·8
(ii) (*a*) (1) $\frac{n+k \mid n}{m \mid m+k}$	—	—	—	1	—	6	—	5	—	1	—	—	13	5·2
(2 $\frac{n \mid n+k}{m+k \mid m}$	—	—	—	1	—	14	—	2	—	2	—	—	19	7·6
(*b*)	—	—	—	—	4	—	2	—	—	—	1	—	7	2·8
Total	1	3	9	23	24	60	54	40	17	11	6	2		
Per cent. ...			3·6	9·2	9·6	24	21·6	16	6·8	4·4	2·4			



Total ———

1. $\frac{n \mid n}{m \mid m}$ - - - - -

2. Asymmetrical in one series* ....

3. (i) Asymmetrical in both† — · — · — ·

3. (ii) Secondarily symmetrical – · – · – · – · –

---

* Very few specimens have an even number of plates and these are omitted in tracing the diagram.

† Specimens with an odd number of plates omitted.

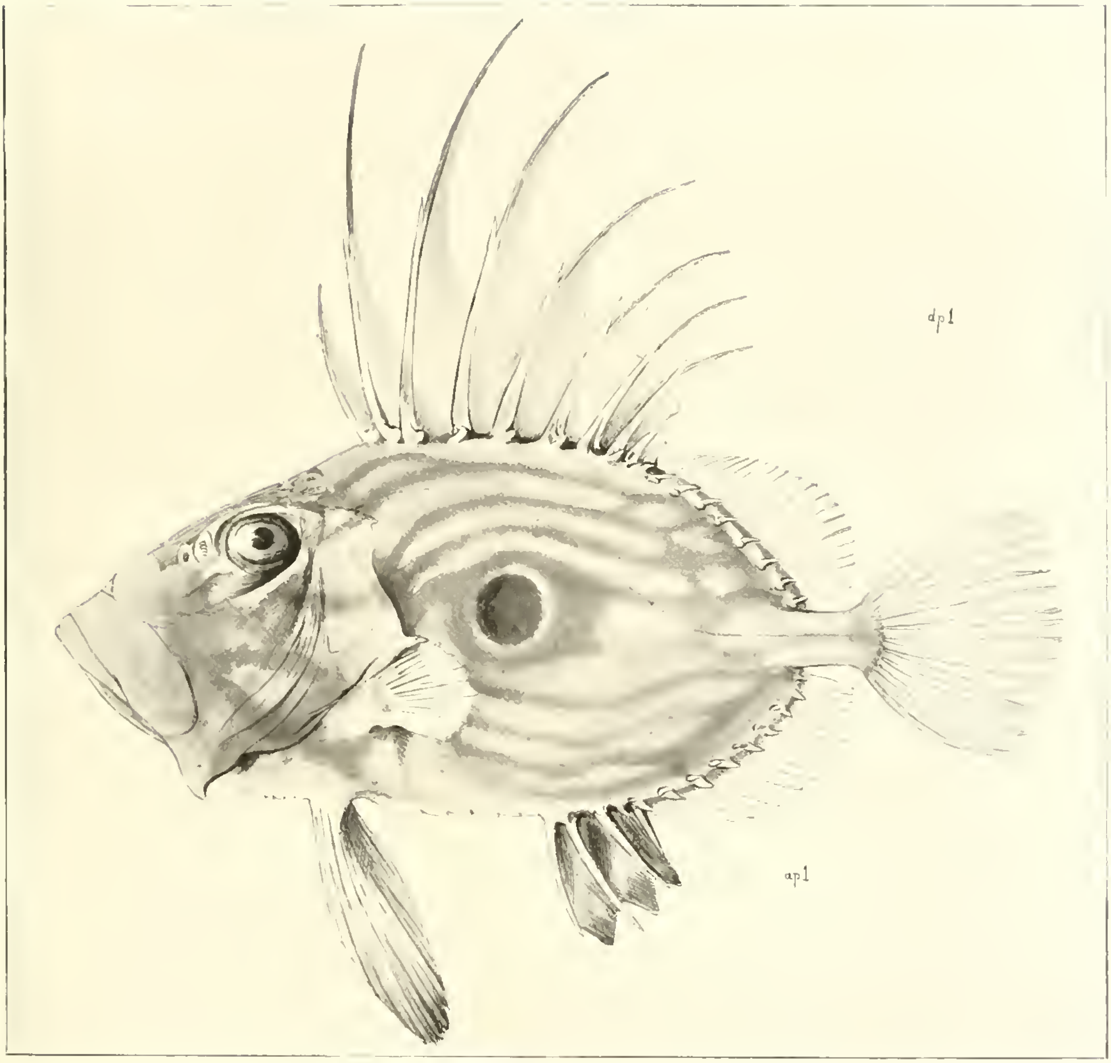
So far as can be judged from such a small number of specimens the following points seem worth noting:—

1. The plates are arranged symmetrically in only 23·6 per cent. of the whole.

2. The average number of plates is 34·6, and the actual numbers of most frequent occurrence 34, 35, and 36 (one of these three occurring in over 60 per cent. of the total).

3. The most frequent combination is $\frac{8 \mid 8}{9 \mid 9}$ which occurs in 10 per cent. of the total; the other most usual combinations are those in which one or both of the upper series are increased to 9.

4. The largest number of plates noted was 40, $\frac{10 \mid 11}{9 \mid 10}$ and $\frac{9 \mid 11}{10 \mid 10}$, and the smallest 29, $\frac{7 \mid 7}{7 \mid 8}$.



Young John Dory, showing position of anterior dorsal and anal plates.

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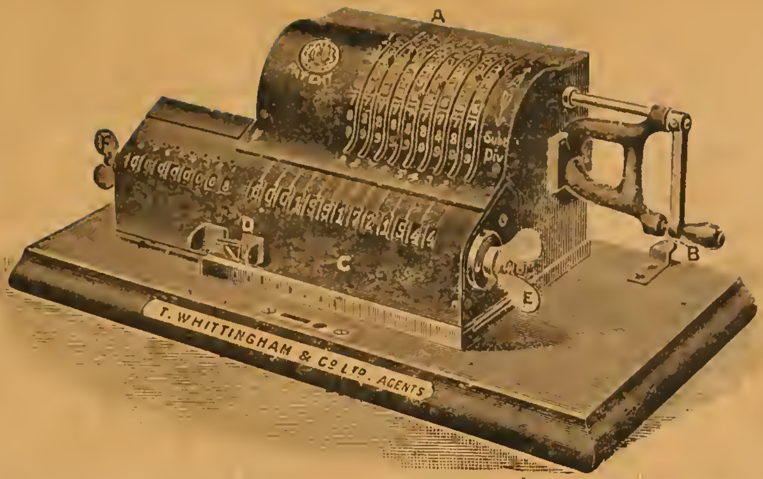
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BY

W. F. R. WELDON KARL PEARSON
AND
C. B. DAVENPORT

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VOLUME II FEBRUARY, 1903 No. 2

NOTES ON THE THEORY OF ASSOCIATION OF ATTRIBUTES IN STATISTICS.

BY G. UDNY YULE.

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THE simplest possible form of statistical classification is "division" (as the logicians term it) "by dichotomy," i.e. the sorting of the objects or individuals observed into one or other of two mutually exclusive classes according as they do or do not possess some character or *attribute*; as one may divide men into sane and insane, the members of a species of plants into hairy and glabrous, or the members of a race of animals into males and females. The mere fact that we do employ such a classification in any case must not of course be held to imply a natural and clearly defined boundary between the two classes; e.g. sanity and insanity, hairiness and glabrousness, may pass into each other by such fine gradations that judgments may differ as to the class in which a given individual should be entered. The judgment must however be finally *decisive*; intermediates not being classed as such even when observed.

The theory of statistics of this kind is of a good deal of importance, not merely because they are of a fairly common type—the statistics of hybridisation experiments given by the followers of Mendel may be cited as recent examples—but because the ideas and conceptions required in such theory form a useful introduction to the more complex and less purely logical theory of variables. The classical writings on the subject are those of De Morgan*, Boole † and Jevons ‡, the method and notation of the latter being used in the following Notes, the first three sections of which are an abstract of the two memoirs referred to below §.

---

* *Formal Logic*, chap. VIII., "On the Numerically Definite Syllogism," 1847.

† *Analysis of Logic*, 1847. *Laws of Thought*, 1854.

‡ "On a General System of Numerically Definite Reasoning," *Memoirs of Manchester Literary and Philosophical Society*, 1870. Reprinted in *Pure Logic and other Minor Works*, Macmillan, 1890.

§ "On the Association of Attributes in Statistics," *Phil. Trans.* A, Vol. 194 (1900), p. 257. "On the theory of Consistence of Logical Class Frequencies," *Phil. Trans.* A, Vol. 197 (1901), p. 91.

1. *Notation; terminology; relations between the class frequencies; tabulation.*

The notation used is as follows*:

N = total number of observations,
(A) = no. of objects or individuals possessing attribute A,
(α) = „ „ *not* possessing attribute A,
(AB) = „ „ possessing both attributes A and B,
$(A\beta)$ = „ „ „ attribute A but not B.
(αB) = „ „ „ attribute B but not A,
$(\alpha\beta)$ = „ „ not possessing either attribute A or B,

and so on for as many attributes as are specified. A class specified by n attributes in this notation may be termed a class of the nth order. The attributes denoted by English capitals may be termed *positive* attributes, and their *contraries*, denoted by the Greek letters, *negative* attributes. If two classes are such that every attribute in the one is the negative or contrary of the corresponding attribute in the other they may be termed contrary classes, and their frequencies contrary frequencies; (AB) and $(\alpha\beta)$, $(AB\gamma)$ and $(\alpha\beta C)$ are for instance pairs of contraries.

If the complete series of frequencies arrived at by noting n attributes is being tabulated, frequencies of the same order should be kept together. Those of the same order are best arranged by taking separately the set or "aggregate" of frequencies, derivable from each positive class by substituting negatives for one or more of the positive attributes. Thus the frequencies for the case of three attributes may conveniently be tabulated in the order—

$$\left.\begin{array}{ll}\text{Order 0.} & N\\ \text{Order 1.} & (A), (\alpha) : (B), (\beta) : (C), (\gamma)\\ \text{Order 2.} & (AB), (A\beta), (\alpha B), (\alpha\beta) : (AC), (A\gamma), (\alpha C), (\alpha\gamma) : (BC), (B\gamma),\\ & \quad (\beta C), (\beta\gamma)\\ \text{Order 3.} & (ABC), (\alpha BC), (A\beta C), (AB\gamma), (\alpha\beta C), (\alpha B\gamma), (A\beta\gamma), (\alpha\beta\gamma)\end{array}\right\} \quad (1).$$

But since all frequencies are used non-exclusively, (A) denoting the frequency of objects possessing the attribute A with or without others and so forth, the frequency of any class can always be expressed in terms of the frequencies of classes of higher order; that is to say we have

$$\left.\begin{aligned} N &= (A) + (\alpha) = (B) + (\beta) = \text{etc.}\\ &= (AB) + (A\beta) + (\alpha B) + (\alpha\beta) = \text{etc.}\\ (A) &= (AB) + (A\beta)\\ &= (ABC) + (AB\gamma) + (A\beta C) + (A\beta\gamma) = \text{etc.}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots(2).$$

---

* I have substituted small Greek letters for Jevons' italics. Italics are rather troublesome when speaking, as one has to spell out a group like $AbcDE$, "big A, little b, little c, big D, big E." It is simpler to say $A\beta\gamma DE$. The Greek becomes more troublesome when many letters are wanted, owing to the non-correspondence of the alphabets, but this is not often of consequence.

Hence it is quite *unnecessary* to state *all* the frequencies as under (1); if space is of importance no more need be given than the eight frequencies of the third order, the *ultimate* frequencies as they may be termed (i.e. frequencies of classes specified by the whole number of attributes noted). The number of frequencies in an aggregate of order n, is evidently 2^n, so that not more than 2^n frequencies need be stated in any case where n attributes are observed.

Any other set of 2^n independent frequencies may however be chosen instead of the 2^n ultimate frequencies, the set formed by taking N together with the frequencies of all the positive classes offering several advantages. It is not difficult to see that any frequency whatever can be expressed in terms of the number of observations and the positive-class frequencies by using the relations (2). We have for instance

$$\left.\begin{aligned}(\alpha) &= N-(A)\\ (\alpha B) &= (B)-(AB)\\ (\alpha\beta) &= (\alpha)-(\alpha B)=N-(A)-(B)+(AB)\\ (\alpha\beta\gamma) &= (\alpha\beta)-(\alpha\beta C)=(\alpha\beta)-(\alpha C)+(\alpha BC)\\ &= N-(A)-(B)-(C)+(AB)+(AC)+(BC)-(ABC)\end{aligned}\right\}\quad\ldots\ldots(3),$$

and so on.

To take a very simple example with two attributes only, consider the statement of results of one of Mr Bateson's experiments on hybridisation with poultry, Leghorn-Dorking hybrids when crossed *inter se* produced offspring of varied forms; some having the rose comb and some not, some having the extra toe that characterises the Dorking and some not. Using A to denote "rose comb," B to denote extra toe, the numerical results may be completely expressed in either of the following forms

I	II
$(AB) = 208$	$N = 336$
$(A\beta) = 63$	$(A) = 271$
$(\alpha B) = 54$	$(B) = 262$
$(\alpha\beta) = 11$	$(AB) = 208.$

The advantages of the second form of tabulation are obvious; it gives at sight the whole number of observations and the numbers of A's and B's. The first table gives neither without reckoning, yet both are equally complete.

A rather interesting case arises where the frequencies of contrary classes are equal, as may be the case if the character dealt with is really variable and the points of division between A's and α's are taken at the medians. Such a condition implies necessary relations between the class-frequencies of any odd order and the frequencies of next lower order, but for the discussion of the case I must refer to the first memoir mentioned in my note § on p. 121.

2. *Consistence and Inference.*

Although the positive-class frequencies (including N under that heading) are all independent in the sense that no single one can be expressed in terms of the others, they are nevertheless subject to certain limiting conditions if they are to be self-consistent, i.e. such as might have been observed in one and the same field of observation or "universe," to use the convenient term of the logicians. Consider the case of three attributes, for example. It is evident that we must have

$$\left.\begin{array}{lll} (AB) \nless 0 & \text{as } (AB) & \text{must not be negative} \\ \nless (A)+(B)-N & \text{as } (\alpha\beta) & \quad ,, \qquad\qquad ,, \\ \ngtr (A) & \text{as } (A\beta) & \quad ,, \qquad\qquad ,, \\ \ngtr (B) & \text{as } (\alpha B) & \quad ,, \qquad\qquad ,, \end{array}\right\} \dots\dots\dots (4);$$

and similar conditions must hold for (AC) and (BC). But these are not the only conditions that must hold. The second-order frequencies must not only be such as not to imply negative values for the frequencies of other classes of their own aggregates, but also *must not imply negative values for any of the third-order frequencies.* Expanding all the third-order frequencies in terms of the frequencies of positive classes, and putting the resulting expansion $\nless 0$, we have

$$\left.\begin{array}{lr} & \begin{array}{r}\text{or the frequency given}\\ \text{below will be negative}\end{array} \\ (ABC) \nless 0 & (ABC)\ [1] \\ \nless (AB)+(AC)-(A) & (A\beta\gamma)\ [2] \\ \nless (AB)+(BC)-(B) & (\alpha B\gamma)\ [3] \\ \nless (AC)+(BC)-(C) & (\alpha\beta C)\ [4] \\ \ngtr (AB) & (AB\gamma)\ [5] \\ \ngtr (AC) & (A\beta C)\ [6] \\ \ngtr (BC) & (\alpha BC)\ [7] \\ \ngtr (AB)+(AC)+(BC)-(A)-(B)-(C)+N & (\alpha\beta\gamma)\ [8] \end{array}\right\} \dots (5).$$

But if any one of the minor limits [1]—[4] be greater than any one of the major limits [5]—[8] these conditions are impossible of fulfilment. There are four minor limits to be compared with four major limits or sixteen comparisons in all to be made; but the majority of these, twelve in all, only lead back to conditions of the form (4). The four comparisons of expansions due to contrary frequencies alone lead to new conditions—viz.

$$\left.\begin{array}{r} (AB)+(AC)+(BC) \nless (A)+(B)+(C)-N \\ (AB)+(AC)-(BC) \ngtr (A) \\ (AB)-(AC)+(BC) \ngtr (B) \\ -(AB)+(AC)+(BC) \ngtr (C) \end{array}\right\} \dots\dots\dots\dots\dots (6).$$

These conditions give limits to any one of the three frequencies (AB), (AC) and (BC) in terms of the other two and the frequencies of the first order, i.e. enable us to infer limits to the one class-frequency in terms of the others. It will very usually happen in practical statistical cases that the limits so obtained are valueless, lying outside those given by the simpler conditions (4), but that is merely because in practice the values of the assigned frequencies, e.g. (AB) and (AC), seldom approach sufficiently closely to their limiting values to render inference possible.

3. *Association.*

Two attributes, A and B, are usually defined to be independent, within any given field of observation or "universe," when the chance of finding them together is the product of the chances of finding either of them separately. The physical meaning of the definition seems rather clearer in a different form of statement, viz. if we define A and B to be independent *when the proportion of A's amongst the B's of the given universe is the same as in that universe at large.* If for instance the question were put "What is the test for independence of small-pox attack and vaccination?", the natural reply would be "The percentage of vaccinated amongst the attacked should be the same as in the general population" or "The percentage of attacked amongst the vaccinated should be the same as in the general population." The two definitions are of course identical in effect, and permit of the same simple symbolical expression in our notation; the criterion of independence of A and B is in fact

$$(AB)=\frac{(A)(B)}{N} \quad \text{.................................(7).}$$

In this equation the attributes specifying the universe are understood, not expressed. If all objects or individuals in the universe are to possess an attribute or series of attributes K it may be written

$$(ABK)=\frac{(AK)(BK)}{(K)}.$$

An equation of such form must be recognised as the criterion of independence for A and B within the universe K. As I have shewn in the first memoir referred to in note §, p. 121, if the relation (7) hold good, the three similar relations for the remaining frequencies of the "aggregate"—i.e. the set of frequencies obtained by substituting their contraries α, β for A or B or both—must also hold, viz.

$$\left.\begin{aligned}(\alpha B) &= \frac{(\alpha)(B)}{N}\\ (A\beta) &= \frac{(A)(\beta)}{N}\\ (\alpha\beta) &= \frac{(\alpha)(\beta)}{N}\end{aligned}\right\} \quad \text{.................................(8).}$$

for we have at once

$$(\alpha B) = (B) - (AB) = \frac{(B)\{N - (A)\}}{N} = \frac{(B)(\alpha)}{N},$$

and so on. The case of two attributes is thus quite simple; the definition of independence is almost intuitive, and the criterion need only be applied to one frequency of the aggregate. The two attributes are termed positively or negatively *associated* according as (AB) is greater or less than the value it would have in the case of independence, or, to put the same thing another way, according as $(AB)/(A)$ is greater or less than $(B)/N$ or $(AB)/(B)$ greater or less than $(A)/N$.

If more than two attributes are noted in the record they must be dealt with, in the first instance, pair by pair as above, but subsequently the association between each pair should be observed in the sections of material or sub-universes defined by the other attributes. In the case of three attributes, for instance, we have not only to deal with the association between A and B in the universe at large but also in the universe of C's and in the universe of γ's—associations which will be tested by comparing $(ABC)/(AC)$ with $(BC)/(C)$ or $(ABC)/(BC)$ with $(AC)/(C)$, and $(AB\gamma)/(A\gamma)$ with $(B\gamma)/(\gamma)$ and so on. Such "partial" associations are of great practical importance, as a test for the correctness or otherwise of physical interpretations placed on any "total" associations observed. When A and B are found to be associated it is a common form of argument to say that the association is not "direct" but due to the association of A with C and B with C; the argument may be tested at once by finding whether A and B are still associated in the separated universes C and γ. If they are the argument is baseless. It has been said for instance that the association between vaccination and protection from small-pox is due to the association of both with sanitary conditions (a larger proportion of the upper classes than of the lower classes being vaccinated, according to the argument). To test the argument the "universe" or material observed should be limited *either* to those living wholly in sanitary conditions *or* to those living wholly in insanitary conditions (it does not matter which). "Partial" associations again are of importance to the biologist in the theory of heredity. If an attribute be heritable, its presence in the parent or the grandparent is associated with its presence in the offspring; but the physical interpretation to be placed on such inheritance depends very largely on whether there is also a *partial* inheritance from the grandparent, the presence of the attribute in the grandparent being associated with its presence in the offspring even when the parents either all possess or all do not possess the attribute.

It is important to notice that the test for association is necessarily based on a *comparison* of percentages or proportions, e.g. $(AB)/(A)$ and $(B)/N$. The mere fact that a certain number of A's are B gives no physical information; besides knowing how many A's are B you must know also how many non-A's are B or what proportion of A's exists in the given universe at large. In an investigation as to the causation of A it is therefore just as important to observe non-A's as A's.

This point is frequently forgotten. In an investigation as to the inheritance of deaf-mutism in America*, for instance, only the offspring of deaf-mutes were observed, and the argument consequently breaks down on page after page into conjectural statements as to points on which the editor has no information—e.g. the proportion of deaf-mutes amongst the children of normals.

The difference of $(AB)/(A)$ from $(B)/N$ and of $(AB)/(B)$ from $(A)/N$ are of course not, as a rule, the same, and it would be useful and convenient to measure the "association" by some more symmetrical method—a "coefficient of association" ranging between ± 1 like the coefficient of correlation. In the first memoir referred to in note §, p. 121, such a coefficient, of empirical form, was suggested, but that portion of the memoir should now be read in connection with a later memoir by Professor Pearson †.

4. *On the theory of complete independence of a series of Attributes.*

The tests for independence are by no means simple when the number of attributes is more than two. Under what circumstances should we say that a series of attributes $ABCD...$ were completely independent? I believe not a few statisticians would reply at once "if the chance of finding them together were equal to the product of the chances of finding them separately," yet such a reply would be in error. The mere result

$$\frac{(ABCD\ldots)}{N}=\frac{(A)}{N}\cdot\frac{(B)}{N}\cdot\frac{(C)}{N}\cdot\frac{(D)}{N}\cdots\cdots \qquad \ldots\ldots\ldots\ldots(9)$$

does not in general give any information as to the independence or otherwise of the attributes concerned. If the attributes are *known to be* completely independent then certainly the relation (9) holds good, but the converse is not true. "Equations of independence" of the form (9) must be shewn to hold for more than one class of any aggregate, of an order higher than the second, before the complete independence of the attributes can be inferred.

From the physical point of view complete independence can only be said to subsist for a series of attributes $ABCD...$ within a given universe, when every pair of such attributes exhibits independence not only within the universe at large but also in every sub-universe specified by one or more of the remaining attributes of the series, or their contraries. Thus three attributes A, B, C are completely independent within a given universe if AB, AC and BC are independent within that universe and also

AB independent within the universes C and γ,
AC „ „ „ B „ β,
BC „ „ „ A „ α.

---

* *Marriages of the Deaf in America*, ed. by E. A. Fay. Volta Bureau, Washington, 1898.

† *Phil. Trans.* Vol. 195, p. 16.

If a series of attributes are completely independent according to this definition relations of the form (9) must hold for the frequency of every class of every possible order. Take the class-frequency $(ABCD)$ of the fourth order for instance. A and B are, by the terms of the definition, independent within the universe CD. Therefore

$$(ABCD)=\frac{(ACD)(BCD)}{(CD)}.$$

But A and C, and also B and C, are independent within the universe D. Therefore the fraction on the right is equal to

$$\frac{1}{(CD)}\cdot\frac{(AD)(CD)}{(D)}\cdot\frac{(BD)(CD)}{(D)}=\frac{(AD)(BD)(CD)}{(D)^2}.$$

But again AD, BD, CD are each independent within the universe at large; therefore finally

$$(ABCD)=\frac{1}{(D)^2}\cdot\frac{(A)(D)}{N}\cdot\frac{(B)(D)}{N}\cdot\frac{(C)(D)}{N}=\frac{(A)(B)(C)(D)}{N^3}.$$

Any other frequency can be reduced step by step in precisely the same way.

Now consider the converse problem. The total frequency N is given and also the n frequencies (A), (B), (C), etc. In how many of the ultimate frequencies $(ABCD\ldots MN)$, $(\alpha BCD\ldots MN)$, etc. must "relations of independence" of the form

$$(ABCD\ldots MN)=\frac{(A)(B)(C)(D)\ldots(M)(N)}{N^{n-1}}$$

hold good, in order that complete independence of the attributes may be inferred? The answer is suggested at once by the following consideration. The number of ultimate frequencies (frequencies of order n) is 2^n; the number of frequencies given is $\overline{n+1}$. If then all but $\overline{n+1}$ of the ultimate frequencies are given in terms of the equations of independence, the remaining frequencies are determinate; either these determinate values must be those that would be given by equations of independence, *or a state of complete independence must be impossible.* Suppose all the ultimate class-frequencies to have been tested and found to be given by the equations of independence, with the exception of the negative class $(\alpha\beta\gamma\delta\ldots\mu\nu)$ and the n classes with one positive attribute $(A\beta\gamma\delta\ldots\mu\nu)$, $(\alpha B\gamma\delta\ldots\mu\nu)$, etc. Take any one of these untested class-frequencies, $(A\beta\gamma\delta\ldots\mu\nu)$, and we have for example

$$\begin{aligned}(A\beta\gamma\delta\ldots\mu\nu)=(A)&-(ABCD\ldots MN)\\&-(ABCD\ldots M\nu)\\&-\text{other terms with one negative}\\&-(ABCD\ldots\mu\nu)\\&-\text{other terms with two negatives}\\&-\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\&-(AB\gamma\delta\ldots\mu\nu)\\&-\text{other terms with } n-2 \text{ negatives.}\end{aligned}$$

But all the frequencies on the right are given by the relations of independence. Therefore

$$\begin{aligned}(A\beta\gamma\delta\ldots\mu\nu) = \frac{(A)}{N^{n-1}}\{N^{n-1} &- (B)(C)(D)\ldots(M)(N)\\ &- (B)(C)(D)\ldots(M)(\nu)\\ &- \ldots\ldots\ldots\ldots\ldots\ldots\\ &- (B)(C)(D)\ldots(\mu)(\nu)\\ &- \ldots\ldots\ldots\ldots\ldots\ldots\\ &- \ldots\ldots\ldots\ldots\ldots\ldots\\ &- (B)(\gamma)(\delta)\ldots(\mu)(\nu)\\ &- \ldots\ldots\ldots\ldots\ldots\ldots\}.\end{aligned}$$

Now consider the terms on the right in the bracket. With the exception of the one term $(B)(\gamma)(\delta)\ldots(\mu)(\nu)$, the remainder can be grouped in pairs of which the one member contains (B) and the other (β), the following frequencies in each member of the pair being the same. Carrying out this rearrangement the expression will read

$$\begin{aligned}(A\beta\gamma\delta\ldots\mu\nu) = \frac{(A)}{N^{n-1}}\{N^{n-1} &- (B)(\gamma)(\delta)\ldots(\mu)(\nu)\\ &- (B)(C)(D)\ldots(M)(N)\\ &- (\beta)(C)(D)\ldots(M)(N)\\ &- (B)(C)(D)\ldots(M)(\nu)\\ &- (\beta)(C)(D)\ldots(M)(\nu)\\ &- \ldots\ldots\ldots\ldots\ldots\ldots\\ &- (B)(C)(\delta)\ldots(\mu)(\nu)\\ &- (\beta)(C)(\delta)\ldots(\mu)(\nu)\\ &- \ldots\ldots\ldots\ldots\ldots\ldots\}.\end{aligned}$$

Replace (B) by $N-(\beta)$ throughout and rearrange the terms in similar pairs containing C and γ. (B) and (β) are then eliminated from all the terms and the expression then becomes

$$\begin{aligned}(A\beta\gamma\delta\ldots\mu\nu) = \frac{(A)}{N^{n-1}}\{N^{n-1} &+ (\beta)(\gamma)(\delta)\ldots(\mu)(\nu)\\ &- N(\gamma)(\delta)\ldots(\mu)(\nu)\\ &- N(C)(\delta)\ldots(\mu)(\nu)\\ &- \ldots\ldots\ldots\ldots\ldots\ldots\\ &- N(C)(D)\ldots(M)(N)\\ &- N(\gamma)(D)\ldots(M)(N)\\ &- \ldots\ldots\ldots\ldots\ldots\ldots\\ &- N(C)(D)\ldots(M)(\nu)\\ &- N(\gamma)(D)\ldots(M)(\nu)\\ &- \ldots\ldots\ldots\ldots\ldots\ldots\}.\end{aligned}$$

Replacing C by $N-(\gamma)$ and regrouping in similar pairs of terms containing (D) and (δ) this will become

$$\begin{aligned}(A\beta\gamma\delta \dots \mu\nu) = \frac{(A)}{N^{n-1}}\{N^{n-1} &+ (\beta)(\gamma)(\delta)\dots(\mu)(\nu)\\ &- N^2(D)(E)\dots(M)(N)\\ &- N^2(\delta)(E)\dots(M)(N)\\ &- \text{etc.}\}\end{aligned}$$

and continuing the same process until all the frequencies $(D)(E)\dots(M)(N)$ are eliminated, i.e. $\overline{n-1}$ times altogether,

$$(A\beta\gamma\delta \dots \mu\nu) = \frac{(A)(\beta)(\gamma)(\delta)\dots(\mu)(\nu)}{N^{n-1}}.$$

That is to say the theorem must be true quite generally: "A series of n attributes $ABC\dots MN$ are completely independent if the relations of independence are *proved* to hold for $(2^n - \overline{n+1})$ of the 2^n ultimate frequencies; such relations must then hold for the remaining $\overline{n+1}$ frequencies also." If the ultimate frequencies are only given by the relations of independence in n cases or less, independence may exist for certain pairs of attributes in certain universes but not in general. The mere fact of the relation holding for one class, e.g.

$$(ABCD\dots MN) = \frac{(A)(B)(C)(D)\dots(M)(N)}{N^{n-1}},$$

implies nothing—in striking contrast to the simple case of two attributes, where $2^n - \overline{n+1} = 1$ and only the one class-frequency need be tested in order to see if independence exists. In the case of three attributes the number of third-order classes is eight, of which four must be tested in order to be certain that complete independence exists. In the case of four attributes there are sixteen fourth-order classes of which eleven must be tested, and so on.

I have dealt with the problem hitherto on the assumption that only the first-order and the nth order frequencies were given, and that the frequencies of intermediate orders were unknown—or at least uncalculated, for of course the frequencies of all lower orders may be expressed in terms of those of the nth order. If however the frequencies of all orders may be supposed known, the above result may be thrown into a somewhat interesting form. It will be remembered that the frequency of any class of any order may be expressed in terms of the frequencies of the *positive* classes $[(A)(AB)(AC)(ABC)$ etc.] of its own and lower orders. Then *complete independence* exists for a series of attributes *if the criterion of independence hold for all the positive-class frequencies up to that of the nth order.* If we have for instance

$$(ABCD\dots MN) = \frac{1}{N^{n-1}}\{(A)(B)(C)(D)\dots(M)(N)\},$$

and also

$$(BCD \ldots MN) = \frac{1}{N^{n-2}} \{(B)(C)(D) \ldots (M)(N)\},$$

we must have

$$\begin{aligned}(\alpha BCD \ldots MN) &= (BCD \ldots MN) - (ABCD \ldots MN)\\ &= \frac{1}{N^{n-1}} \{(B)(C)(D) \ldots (M)(N)\} \{N - (A)\}\\ &= \frac{1}{N^{n-1}} (\alpha)(B)(C)(D) \ldots (M)(N),\end{aligned}$$

and so on. The number of class-frequencies to be tested in order to demonstrate the existence of complete independence is, of course, the same as before, viz. $2^n - \overline{n+1}$.

It should be noted as a consequence of these results that the definition of "complete independence" given on p. 127 is redundant in its terms. It is quite true that if complete independence subsist for a series of attributes every possible pair must exhibit independence in every possible sub-universe as well as in the universe at large, but it is not necessary to apply the criterion of independence to *all* these possible cases. In the case of three attributes for instance the criterion of independence need only be applied to four frequencies, as we have just seen, in order to demonstrate complete independence; it cannot then be *necessary*, as suggested by the definition, to test nine different associations, viz.

$$\begin{array}{lll} |AB| & |AB \mid C| & |AB \mid \gamma|, \\ |AC| & |AC \mid B| & |AC \mid \beta|, \\ |BC| & |BC \mid A| & |BC \mid \alpha|, \end{array}$$

in the notation of my memoir on Association (an expression like $|AB \mid C|$ specifying "the association between A and B in the universe of C's"). It is in fact only necessary to test $|AB|$, $|AC|$, $|BC|$, and $|AB \mid C|$ (or one of the other three partial associations in positive universes). If these are zero, the remaining associations must be zero also; for we are given

$$(ABC) = \frac{1}{(C)} (AC)(BC) = \frac{1}{N^2} (A)(B)(C),$$

$$\begin{aligned}\therefore (ABC) &= \frac{1}{(B)} (AB)(BC)\\ &= \frac{1}{(A)} (AB)(AC) = \text{etc.}\end{aligned}$$

i.e. $|AC \mid B|$, $|BC \mid A|$, etc. are zero. Quite generally, it is only necessary, if the testing be supposed to proceed from the second order classes upwards, to test *one* of all the possible partial associations corresponding to each positive class. If there be four attributes $ABCD$, the six total associations $|AB|$, $|AC|$, $|AD|$, $|BC|$

etc. must first be tried; if they are zero, then follow on with $|AB|C|$, $|AB|D|$, $|AC|D|$ and $|BC|D|$, but not $|AC|B|$ or $|AD|B|$ etc.; if they are zero then finally try $|AB|CD|$, if it also be zero then the attributes are completely independent. It is not necessary to try further $|AC|BD|$ or $|AD|BC|$ etc.

The inadequacy of the usual treatment of independence arises from the fact that it proceeds wholly *à priori*, and generally has reference solely to cases of artificial chance. The result is an illusory appearance of simplicity. It is pointed out that if one "event" can "succeed" in a_1 and "fail" in b_1 ways, a second succeed in a_2 and fail in b_2 ways, and so on, the combined events can take place (succeed or fail) in

$$(a_1+b_1)(a_2+b_2)\ldots(a_n+b_n)$$

ways and succeed in

$$a_1a_2\ldots a_n$$

ways. The chance of entire "success" is therefore

$$\frac{a_1a_2\ldots a_n}{(a_1+b_1)(a_2+b_2)\ldots(a_n+b_n)},$$

the chance of the first event failing and the rest succeeding is

$$\frac{b_1a_2\ldots a_n}{(a_1+b_1)(a_2+b_2)\ldots(a_n+b_n)},$$

and so on for all other possible cases. In short the chance of occurrence of the combined independent events is the product of the chances of the separate events. There the treatment stops, and all practical difficulties are avoided. In such text-book treatment it is *given* that the events are independent and *required* to deduce the consequences; in the problems that the statistician has to handle the consequences—the bare facts—are given and it is required to find whether the "events" or attributes are independent, wholly or in part.

5. *On the fallacies that may be caused by the mixing of distinct records.*

It follows from the preceding work that we cannot infer independence of a pair of attributes within a sub-universe from the fact of independence within the universe at large. From $|AB|=0$, we cannot infer $|AB|C|=0$ or $|AB|\gamma|=0$, although we can of course make the corresponding inference in the case of complete association—i.e. from $|AB|=1$ we do infer $|AB|C|=|AB|\gamma|=\text{etc.}=1$. But the converse theorem is also true; a pair of attributes does not necessarily exhibit independence within the universe at large even if it exhibit independence in *every* sub-universe; given

$$|AB|C|=0, \qquad |AB|\gamma|=0,$$

we cannot infer $|AB|=0$. The theorem is of considerable practical importance from its inverse application; i.e. even if $|AB|$ have a sensible positive or negative value we cannot be sure that nevertheless $|AB|C|$ and $|AB|\gamma|$ are not both zero. Some given attribute might, for instance, be inherited neither in the male line nor the female line; yet a mixed record might exhibit a considerable apparent inheritance. Suppose for instance that 50 % of the fathers and of the sons exhibit the attribute, but only 10 % of the mothers and daughters. Then if there be no inheritance in either line of descent the record must give (approximately)

fathers with attribute and sons with attribute 25 %
" " " " without " 25 %
" without " " with " 25 %
" " " " without " 25 %

mothers with attribute and daughters with attribute 1 %
" " " " " without " 9 %
" without " " " with " 9 %
" " " " " without " 81 %.

If these two records be mixed in equal proportions we get

parents with attribute and offspring with attribute 13 %
" " " " " without " 17 %
" without " " " with " 17 %
" " " " " without " 53 %

Here $13/30 = 43\frac{1}{3}$ % of the offspring of parents with the attribute possess the attribute themselves, but only 30 % of offspring in general, i.e. there is quite a large but illusory inheritance created simply by the mixture of the two distinct records. A similar illusory association, that is to say an association to which the most obvious physical meaning must not be assigned, may very probably occur in any other case in which different records are pooled together or in which only one record is made of a lot of heterogeneous material.

Consider the case quite generally. Given that $|AB|C|$ and $|AB|\gamma|$ are both zero, find the value of (AB). From the data we have at once

$$(AB\gamma) = \frac{(A\gamma)(B\gamma)}{(\gamma)} = \frac{[(A)-(AC)]\,[(B)-(BC)]}{[N-(C)]},$$

$$(ABC) = \frac{(AC)(BC)}{(C)}.$$

Adding

$$(AB) \quad = \frac{N(AC)(BC)-(A)(C)(BC)-(B)(C)(AC)+(A)(B)(C)}{(C)[N-(C)]}.$$

Write

$$(AB)_0 = \frac{1}{N}(A)(B), \quad (AC)_0 = \frac{1}{N}(A)(C), \quad (BC)_0 = \frac{1}{N}(B)(C),$$

subtract $(AB)_0$ from both sides of the above equation, simplify, and we have

$$(AB) - (AB)_0 = \frac{N[(AC) - (AC)_0][(BC) - (BC)_0]}{C[N - (C)]}.$$

That is to say, *there will be apparent association between A and B in the universe at large unless either A or B is independent of C.* Thus, in the imaginary case of inheritance given above, if A and B stand for the presence of the attribute in the parents and the offspring respectively, and C for the male sex, we find a positive association between A and B in the universe at large (the pooled results) because A and B are both positively associated with C, i.e. the males of both generations possess the attribute more frequently than the females. The "parents with attribute" are mostly males; as we have only noted offspring of the same sex as the parents, their offspring must be mostly males in the same proportion, and therefore more liable to the attribute than the mostly-female offspring of "parents without attribute." It follows obviously that if we had found no inheritance to exist in any one of the *four* possible lines of descent (male-male, male-female, female-male, and female-female), no fictitious inheritance could have been introduced by the pooling of the *four* records. The pooling of the two records for the crossed-sex lines would give rise to a fictitious negative inheritance—disinheritance—cancelling the positive inheritance created by the pooling of the records for the same-sex lines. I leave it to the reader to verify these statements by following out the arithmetical example just given should he so desire.

The fallacy might lead to seriously misleading results in several cases where mixtures of the two sexes occur. Suppose for instance experiments were being made with some new antitoxin on patients of both sexes. There would nearly always be a difference between the case-rates of mortality for the two. If the female cases terminated fatally with the greater frequency and the antitoxin were administered most often to the males, a *fictitious* association between "antitoxin" and "cure" would be created at once. The general expression for $(AB) - (AB)_0$ shews how it may be avoided; it is only necessary to *administer the antitoxin to the same proportion of patients of both sexes.* This should be kept constantly in mind as an essential rule in such experiments if it is desired to make the most use of the results.

The fictitious association caused by mixing records finds its counterpart in the spurious correlation to which the same process may give rise in the case of continuous variables, a case to which attention was drawn and which was fully discussed by Professor Pearson in a recent memoir*. If two separate records, for each of which the correlation is zero, be pooled together, a spurious correlation will necessarily be created unless the mean of one of the variables, at least, be the same in the two cases.

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* *Phil. Trans.* A, Vol. 192, p. 277.

A FURTHER STUDY OF STATISTICS RELATING TO VACCINATION AND SMALLPOX.

BY W. R. MACDONELL, LL.D.

(1) THE following paper is a continuation of the Note published in *Biometrika*, Vol. I. No. 3, but I have been able to extend my results to a comparison of the differential character of two epidemics in the same city, and to some preliminary consideration of the influence of occupation on the existence or non-existence of vaccination and on the severity of a smallpox attack. My statistical material is taken from: *A Summary of Statistics relating to Vaccination and Smallpox as observed in the Cases admitted to the City of Glasgow Smallpox Hospital, Belvidere, between* 10*th April,* 1900, *and* 30*th June,* 1901, by Dr R. S. Thomson and Dr Fullarton—a paper read before the Royal Philosophical Society of Glasgow on 2nd April, 1902—and from supplementary statistics relating to the same epidemic kindly sent to me by Dr Brownlee, Physician Superintendent of the Belvidere Hospital. The tables were prepared, and the coefficients of correlation with their probable errors calculated in the same manner as in the earlier paper. I may repeat that I use "mild" as equal to "discrete," and "severe" as equal to "confluent" and "haemorrhagic." The few cases (only 2½ per cent.) of "doubtful" vaccination have been excluded from my tables. The following are my tables for the Glasgow epidemic of 1900–1.

TABLE I.

	Recoveries	Deaths	Totals
Vaccinated...	1493	150	1643
Unvaccinated	59	63	122
Totals	1552	213	1765

$r = \cdot 6294 \pm \cdot 0296$.

TABLE II.

	Mild	Severe	Totals
Vaccinated...	1361	282	1643
Unvaccinated	53	69	122
Totals	1414	351	1765

$r = \cdot 5162 \pm \cdot 0322.$

TABLE III.

Scars	Mild	Severe	Totals
Foveated ...	573	82	655
Unfoveated...	788	200	988
Totals	1361	282	1643

$r = \cdot 1929 \pm \cdot 0374.$

TABLE IV.

Area of Scar	Mild	Severe	Totals
Over half square inch	805	113	918
Half square inch and under	556	169	725
Totals	1361	282	1643

$r = \cdot 2646 \pm \cdot 0301.$

TABLE V.

Number of Scars	Mild	Severe	Totals
Two and upwards	652	105	757
One	709	177	886
Totals	1361	282	1643

$r = \cdot 1511 \pm \cdot 0306.$

TABLE VI.

Vaccinated Cases.

	Recoveries	Deaths	Totals
Mild	1336	25	1361
Severe... ...	157	125	282
Totals	1493	150	1643

$r = \cdot 8603 \pm \cdot 0150.$

The statistics also enable me to find the coefficient of correlation between age and severity of attack.

TABLE VII.

Vaccinated Cases.

Years	Mild	Severe	Totals
0 to *20*... ...	244	14	258
20 and over...	1117	268	1385
Totals	1361	282	1643

$r = \cdot 3693 \pm \cdot 0386$.

If the division be made at 25 years instead of 20, $r = \cdot 3297 \pm \cdot 0314$; if at 35 years, $r = \cdot 3180 \pm \cdot 0293$. Calculating this coefficient for the Glasgow epidemic of 1892–95, I find $r = \cdot 3366 \pm \cdot 0715$, for a division at 20 years; I am unable to calculate it for a division at 25 or 35 years, as there are no divisions at these periods in the statistics of the earlier epidemic.

(2) For the sake of comparison, the coefficients for the two Glasgow epidemics are collected in the table below.

TABLE VIII.

Coefficient of correlation between	Epidemic of 1900-1	Epidemic of 1892–5
Vaccination and strength of resistance	·6294 ± ·0296	·7783 ± ·0365
Vaccination and severity of attack	·5162 ± ·0322	·9123 ± ·0181
Foveation of scar and severity of attack	·1929 ± ·0374	·3951 ± ·0594
Area of scar and severity of attack	·2646 ± ·0301	·3520 ± ·0584
Number of scars and severity of attack	·1511 ± ·0306	·2323 ± ·0616
Strength of resistance and severity of attack ...	·8603 ± ·0150	—
Age (division at 20 years) and severity of attack	·3693 ± ·0386	·3366 ± ·0715
Age (,, 25 ,,) ,, ,, ,,	·3297 ± ·0314	
Age (,, 35 ,,) ,, ,, ,,	·3180 ± ·0293	—

It will be observed that in the recent epidemic the coefficient of correlation between vaccination and strength of resistance, while less than in 1892–95, was very much the same as in previous epidemics in other towns*. On the other hand, the correlation between vaccination and degree of severity of attack is very much less than in 1892–95, which points to a marked difference in the character of the two epidemics, the earlier being the milder. Also the correlation between degree of severity and (1) foveation, (2) area, and (3) number, of scars is less even than in 1892–95; but I understand from Dr Brownlee that in concluding whether a scar is good or bad, clinically, he would take into consideration its area, the area of foveation, and the amount of depression and "puckering." The fact is also clearly brought out that adults are considerably more liable to the severe forms of the disease than the young.

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* *Biometrika*, Vol. I. No. 3, p. 380.

(3) The next part of my inquiry presents some features which, I think, are novel, and is based on figures relating to the Glasgow epidemic of 1900–1, supplied by Dr Brownlee. He grouped the male patients according to their occupations, adopting the grouping of the Registrar-General's Reports with three modifications: (1) Professions, Clerks, etc. are all grouped together, as the number of patients other than clerks was very small; e.g. only four professional men were admitted to Belvidere: (2) Railway men are shown separately from Transport Service: (3) Shopkeepers include all kinds, instead of the limited number adopted in the Registrar-General's Reports. Cases of doubtful vaccination (25) are again excluded.

The following table is prepared from Dr Brownlee's figures:

TABLE IX.

Smallpox Cases admitted into Belvidere Hospital 1900–1. (*Males.*)

Occupations	Vaccinated			Unvaccinated
	Recoveries	Deaths	Totals	
Miners...	25	3	28	2
Labourers	167	25	192	9
Metal Workers	159	14	173	0
Shopkeepers	70	10	80	5
Railway men	13	2	15	1
Transport Service ...	85	11	96	3
Other Trades	46	10	56	3
Spirit Salesmen... ...	11	3	14	0
Professions, Clerks, etc.	65	6	71	1
Building Trades ...	74	9	83	1
Textiles	22	1	23	0
Unoccupied	10	1	11	1
Totals	747	95	842	29

From the statistics of Table IX. I wanted to ascertain the correlation between the social status of the patients and (1) the presence or absence of the scar, and (2) in the case of the vaccinated their power to resist the disease. For this purpose it was necessary to divide my material into two classes having a higher and a lower status. No doubt considerable diversity of opinion may exist as to the components which should fall into these two classes. I consulted from this standpoint the death-rates of various classes as given by the Registrar-General.

My chief difficulty arose from the group of metal workers who form by far the largest single occupation group after the labourers, and yet exhibit not a single unvaccinated person. It is clear that such a class according as it is included in the higher or lower status group may completely change the correlation between status and vaccination. If the metal workers included filecutters and lead-workers, they would have a high death-rate and we ought to put them in the group of lower status. On the other hand in some districts the metal workers belong to the most trained and the best paid class of craftsmen, men who are likely to be well nourished and with fairly healthy homes. I accordingly wrote to Dr Brownlee inquiring as to the status of the Glasgow metal workers and as to the absence of unvaccinated cases among them. His reply is as follows:

"No filecutters or leadworkers were included in the metal workers. The latter "were in great majority made up of shipbuilding and forge employees, these "being the main metal industries in Glasgow. A very few brassworkers were "included. The status of these metal workers as we received them was not so high "as that of either the men who work on the Railways or in the Building Trades, "and I think that they should be included in your second class. The absence of "unvaccinated cases is to a certain extent explainable. Most of these workers were "from the neighbouring forges where on account of the contiguity of the hospital "the employers have for a long time exercised a certain amount of supervision of "the vaccination of those employed in the works. Smallpox has several times "invaded these works since 1890, so that I think unvaccinated persons must "be very few indeed."

It must be at once confessed that this more or less enforced vaccination of the large group of metal workers is a very disturbing factor in the consideration of the relation between status and vaccination. Adopting Dr Brownlee's view I first put the metal workers in the class of lower status.

As a first grouping I took for those of higher status: Professions, Clerks, etc., and Shopkeepers. I obtained:

TABLE X.

First Grouping.

Class	Vaccinated	Unvaccinated	Totals
Lower	691	20	711
Higher ...	151	9	160
Totals	842	29	871

$r = \cdot 1862 \pm \cdot 0737$.

I now included in the class of higher status, Miners, Railway men and Building Trades as well as Professions, Clerks and Shopkeepers.

TABLE XI.

Second Grouping.

Class	Vaccinated	Unvaccinated	Totals
Lower	565	16	581
Higher ...	277	13	290
Totals	842	29	871

$r = \cdot 1362 \pm \cdot 0695$.

There is thus for both cases a quite sensible if not very large correlation between status and vaccination,—*cases of unvaccinated persons occur more frequently in the classes of higher than of lower status.*

Putting on one side Dr Brownlee's views on the status of the metal workers I now formed two tables in which the metal workers were included in the group of higher status, adding them first to Professions, Clerks, etc. and Shopkeepers, as a third grouping, and finally to all these together with Miners, Railway men and Building Trades as a fourth grouping. I found:

TABLE XII.

Third Grouping.

Class	Vaccinated	Unvaccinated	Totals
Lower	518	20	538
Higher ...	324	9	333
Totals	842	29	871

$r = -\cdot 0872 \pm \cdot 0713$.

TABLE XIII.

Fourth Grouping.

Class	Vaccinated	Unvaccinated	Totals
Lower	392	16	408
Higher ...	450	13	463
Totals	842	29	871

$r = -\cdot 0939 \pm \cdot 0691$.

The correlation has clearly swung round, and there is now a very slight correlation, hardly sensible considering the probable error, between higher status and vaccination. In view, however, of Dr Brownlee's opinion as to the character of the metal workers, I do not think this classification is legitimate. Considering also that vaccination appears to be more or less compulsory among a considerable section of them I determined to omit them altogether, and then obtained the following table:

TABLE XIV.

First Grouping, Metal Workers excluded.

Class	Vaccinated	Unvaccinated	Totals
Lower	518	20	538
Higher ...	151	9	160
Totals	669	29	698

$r = \cdot 1167 \pm \cdot 0763$.

Comparing this with Tables X. and XII. we see that there is a sensible but small correlation between higher status and unvaccinated condition.

Now I lay no particular stress on these results because the material is far too sparse*, but I believe that the above statistics are the only ones hitherto dealt with with a view to determining whether the classes of higher status—presumably the better fed and healthier classes of the community—are or are not more frequently vaccinated than the lower, presumably the less nourished and less healthy classes. No dogmatic conclusion can be drawn from these data, but they exhibit no evidence at all for the unvaccinated class being of lower status than the vaccinated class; on the contrary, there is slight evidence to show that the unvaccinated in Glasgow occur rather more frequently in the classes of higher status.

(4) I turn now to the question whether among the vaccinated there exists a correlation between status and severity of the disease. I obtained the following tables:

TABLE XV.

First Grouping.

Class	Deaths	Recoveries	Totals
Lower	79	612	691
Higher ...	16	135	151
Totals	95	747	842

$r = \cdot 0249 \pm \cdot 0566$.

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* The fewness of unvaccinated cases possibly arises from the fact that during the 1892–95 epidemic in Glasgow vaccination was performed on a large scale amongst all classes, so that the epidemic of 1900–1 found the great majority of the population vaccinated.

TABLE XVI.

Second Grouping.

Class	Deaths	Recoveries	Totals
Lower ...	65	500	565
Higher	30	247	277
Totals	95	747	842

$r = {\cdot}0216 \pm {\cdot}0500.$

That is to say there is a positive but practically insensible correlation between status and power to resist the disease.

I now included the metal workers in the higher status and found with the former groupings:

TABLE XVII.

Third Grouping.

Class	Deaths	Recoveries	Totals
Lower	65	453	518
Higher ...	30	294	324
Totals	95	747	842

$r = {\cdot}1086 \pm {\cdot}0490.$

TABLE XVIII.

Fourth Grouping.

Class	Deaths	Recoveries	Totals
Lower	51	341	392
Higher ...	44	406	450
Totals	95	747	842

$r = {\cdot}1052 \pm {\cdot}0478.$

Thus the correlation between better status and recovery is slightly increased. Leaving the metal workers as before out of consideration, I find:

TABLE XIX.

First Grouping without Metal Workers.

Class	Deaths	Recoveries	Totals
Lower	65	453	518
Higher ...	16	135	151
Totals	81	588	669

$r = {\cdot}0579 \pm {\cdot}0594.$

This is probably the most satisfactory result as about the mean of the previous tables. We accordingly conclude that there is a very slight relation between status and recovery from attack.

The way in which the metal workers increase this correlation when they are placed in the better class is remarkable; they have in Glasgow a comparatively small mortality from smallpox, yet we find from the *55th Annual Report for England of the Registrar-General—Medical Supplement* that the general mortality among metal workers is higher at all ages, especially after 35, than that of occupied males. The figures are as follows, if the standard rate of mortality among occupied males at each age be taken as 100:

Ages	15—	20—	25—	35—	45—	55—	65 and up.
Metal Workers	105	106	103	111	122	129	128

It would be interesting to compare the special classes grouped as metal workers in Glasgow, with those embraced under the same heading in the Registrar-General's Reports.

(5) *Conclusions.*

(i) The statistical constants for vaccination and smallpox differ sensibly for the same place with two different epidemics, i.e. epidemics seem to be differentiated in character.

(ii) The statistics of Glasgow do not indicate that those of lower status —and therefore probably worse nourished and housed—provide the bulk of non-vaccinated cases. On the contrary there seems to be a slight tendency for the non-vaccinated to be of higher status.

(iii) There is a slight although scarcely sensible correlation between status and power to resist a smallpox attack.

The Glasgow statistics do not go very far, but as far as they go they do not justify the statement: That the apparent protection of vaccination is due to the unvaccinated belonging to classes of lower status which have a far smaller power of resistance to the disease than the better nourished classes of a higher status in which the members are more generally vaccinated.

I have not dealt with the statistics as to female patients as the great bulk of them are classified merely as "Housewives," which throws no light on their social position.

In concluding this paper I venture to express the hope that the statistics of the recent London epidemic may soon be issued and in a form which will admit of due consideration of the problems referred to in this paper. Their magnitude gives them extreme value and their publication is no doubt anxiously awaited by a wide circle of scientific inquirers at home and abroad.

NOTE.

From the "Times" of 20 November 1902, it appears that the Medical Officer of Health for Islington has prepared his final report on the 1901–2 epidemic so far as his own district was concerned, but I have been unable to obtain a copy of it. The paragraph in the "Times", however, enables me to form the following table:

TABLE XX.

Smallpox in Islington, 27 August 1901—29 *August* 1902.

	Recoveries	Deaths	Totals
Vaccinated ...	207	29	236
Unvaccinated...	35	30	65
Totals	242	59	301

$r = \cdot 5744 \pm \cdot 0560.$

This value of r is in extremely close agreement with that obtained for 1017 cases which were "completed" in London in 1901 [*], viz. $r = \cdot 5779 \pm \cdot 0311$. To show that the closeness of the agreement holds throughout the investigation, I give the equations from which r was calculated in both cases:

For the above 301 cases:

$$\cdot 070608\, r^6 + \cdot 002326\, r^5 + \cdot 151410\, r^4 + \cdot 017030\, r^3 + \cdot 336384\, r^2 + r = \cdot 707743.$$

For the 1017 cases:

$$\cdot 071607\, r^6 + \cdot 001780\, r^5 + \cdot 149636\, r^4 + \cdot 0198442\, r^3 + \cdot 326092\, r^2 + r = \cdot 710100.$$

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[*] *Biometrika*, Vol. I. No. 3, p. 379.

COOPERATIVE INVESTIGATIONS ON PLANTS.

II. VARIATION AND CORRELATION IN LESSER CELANDINE FROM DIVERS LOCALITIES.

(1) In view of the data for *F. ranunculoides* published by Dr F. Ludwig* and Prof. MacLeod† and the statistical constants determined for them‡ it seemed desirable to obtain rather more statistical material and a more comprehensive series of constants for the purposes of a comparative study. There were two points to be considered, namely: (i) the influence of locality and (ii) the influence of the time during the flowering season at which the flowers were gathered. Unfortunately we were in the present season in no case able to obtain from our collectors two series from the same locality with a month's interval between the gatherings. All the collecting except in one case had to be done during the brief Easter vacation of our workers, and this did not admit of a double gathering in the same locality at a suitable interval. The one exception is that of the Bordighera collection. Mr Francis Galton most kindly offered, as he was wintering on the Mediterranean, to provide a double series of Lesser Celandine flowers. The first series was gathered about February 19th and at once dispatched to England. On arrival it was found that practically every sepal, every petal, and nearly every stamen had fallen from the flower-heads. This lost us our earlier series, but we learnt a most valuable lesson, namely: that transit of any kind, even by hand, will cost the flower if nearly full-blown one or more sepals, petals or stamens. Our plans had therefore to be changed; each celandine flower was now gathered *as a bud* and wrapped up in a small piece of tissue paper. This involved a great increase in the labour of gathering and a much greater one in that of counting, a good deal of which had to be done under a lens, but we were thus certain of preserving all the parts of the flower intact. Mr Francis Galton suggested and carried out this arrangement in a second series gathered between March 4th and 7th which reached England safely, but three or four weeks later than this there were no flowering celandines to be obtained in Bordighera.

Mr Galton's plan was carried out in the further collections made in Guernsey, Dorset and Surrey, the collectors gathering the buds, and wrapping them up

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* *Biometrika*, Vol. i. pp. 11—20.

† *Ibid.*, pp. 125—128.

‡ *Ibid.*, pp. 316—319.

in situ in a small piece of tissue paper. The buds dried and were preserved for weeks, to be counted at leisure. Even if a petal or sepal fell off it was preserved in the paper wrapper. We believe it is largely due to this method of gathering that our results show such a totally different distribution of sepals and petals to those of other observers.

Take the case of sepals. In the Guernsey series we found only three individuals with less than three sepals. On examination under the microscope in one of these cases an abortive sepal was found, in another the sepal had clearly once been attached, but in the third case the head was unfortunately lost before microscopic examination. We think that we may safely affirm that no true case of less than three sepals was found in the Guernsey plants. In the Dorsetshire gathering no cases of less than three sepals occurred. In the Surrey gathering there were in the material six cases of less than three sepals, and in five out of these six cases the rule of gathering buds only had been disregarded by a young collector[*]; and the sixth was an aborted flower in which the stamens were not properly developed and there were no pistils at all! In the Bordighera celandines there are only two cases of less than three sepals. These have not been excluded from the calculations, because we had not when counting them learnt from the Dorset and Guernsey series to regard all cases of less than three sepals with grave suspicion and examine such cases under the microscope. But if in 624 cases only two such individuals occurred, it seems extremely probable that even these were cases of sepals knocked off or aborted. In a total of 2149 heads, there were 11 cases of less than three sepals, two of these were aborted flowers, five of them were old flowers with parts loose, one had once had a sepal which had been lost, and three were not closely examined. The experience we have had leads us to believe that each flower ought to be gathered as a bud and at once wrapped up. It seems to us that all we can admit is a *possibility* of three heads with less than three sepals in 2149 cases, while Dr Ludwig's 3000 from Greiz show no less than 60 definite cases[†]. We feel fairly confident that had the Greiz flowers been collected as buds and possibly gathered and counted by adults instead of school children they would not have differed so widely from our material in this respect. As far as our experience reaches, we question the existence of any normal flower with less than three sepals. This view may be modified when further material from central Europe gathered in bud and, if necessary, microscopically examined, is available. We hope next season that this problem of the sepal may be directly investigated.

(2) *Material of the present investigation.*

(A) 624 heads gathered in a vineyard at Bordighera, Italy, between March 4th and 7th. Due to Mr Francis Galton. The sepals, petals, stamens and pistils were counted by Mr N. Blanchard. The tables were prepared and the statistical constants calculated by Miss Alice Lee. These belonged to a variety of Lesser Celandine classed by the French botanists as *Ficaria calthaefolia* and considered by them as distinct from *F. ranunculoides*. Table I., however, exhibits less differentiation in the floral parts of *F. calthaefolia* and *F. ranunculoides* than can occur between two local races of the latter. The *Index Kewensis* gives the former name as a synonym of the latter, but there appears to be a more sensible differentiation of the leaf.

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[*] The calculator's notes are: No. 51, "only one sepal and this hanging loose"; No. 52, "old flower with petals detached"; No. 53, "Pistils loose"; No. 64, "very old flower, all parts loose"; No. 69, "very old flower." No remarks as to age of this kind occur in other cases.

[†] See *Biometrika*, Vol. I. pp. 13–15.

(B) 520 heads gathered on roadside banks at St Peter Port, Guernsey, between April 25th and 30th. Due to Miss Caroline Herford. The characters were counted, the tables formed and the statistical constants determined by Miss Mary Beeton.

(C) 500 heads gathered in two lanes and a meadow near Thursley, on the north side of Hind Head, Surrey. The material was gathered between 12th and 17th of April by a number of collectors under the supervision of K. Pearson. The characters were counted by Mr N. Blanchard and the tables and statistical constants are due to Dr Lee.

(D) 505 heads gathered in a lane at Studland, Dorsetshire, on April 7 by Mr N. L. Blanchard. The counting is due to Miss Edna Lea-Smith and to Mr Blanchard. The statistical tables and constants are again due to Dr Lee.

In reducing the material Sheppard's corrections for the moments were not used, partly because we are dealing with variation by units, and partly because it is clear in the case of the sepals, and probably true for the petals, that the frequency distribution has not high contact at the low end of the range. Otherwise the calculation of means, standard deviations, coefficients of correlation, and of variation proceeded in the usual manner.

(3) Table I. gives a summary of all the statistical constants* for the four new series and places alongside them those already found for other localities. Now this table shows at once the remarkable differences, which period in season and environment can have on mean, variability and correlation. Not one of these quantities has the least approximation to constancy for all local races of *Ficaria*, nor for the same race at two parts of the same reason. The early Belgian celandines, judging from the constants for stamens and pistils, which are all we have, are practically identical with those from Italy, while the later ones diverge widely and perhaps may be considered nearer to the Swiss Trogen series than to any other. A brief study of this table will at once convince the reader of two fundamental points:

(*a*) Local races in plants cannot be defined or distinguished by the existence of differences many times the value of their probable errors between their means, variations or correlations.

(*b*) The influences of environment and season are for plants of supreme importance and very widely or indeed entirely screen any differences due to local race.

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* These are tabulated to four places of decimals—not because such are exact or necessary for present purposes, but because for future investigations when we come to consider the evolutionary history of local races, it will be needful to have the constants to this number of places in order to calculate the partial regression coefficients true for each series to one or two places of decimals.

TABLE I. *Statistical Constants of Lesser Celandine from different localities.*

Locality		Italy	Guernsey	England		Belgium		Germany		Switzerland		Europe
		Bordighera	St Peter Port	Surrey	Dorset	Early	Late	Greiz	Gera *	Trogen	Gais	Means
Year		1902	1902	1902	1902	1899	1899	1900–1901	?	1900	1901	1899–1902
Date		March 1–7	April 25–30	April 12–17	April 7–9	Feb. 27—March 17	April 17–23	?	?	?	?	—
Number		624	520	500	505	268	373	4000	1000	285	1000	—
Means	Sepals	3·0625 ± ·0071	3·0096 ± ·0029	3·0100 ± ·0030	3·0297 ± ·0054	—	—	3·650 ± ·0171	3·309 ± ·0137	3·386 ± ·0219	4·538 ‡ ± ·0131	3·374
	Petals	8·1186 ± ·0117	8·1327 ± ·0137	8·0680 ± ·0139	8·7149 ± ·0304	—	—	8·351 ± ·0260	8·225 ± ·0237	8·144 ± ·0306	9·722 ‡ ± ·0273	8·435
	Stamens... ...	26·7179 ± ·1018	27·9558 ± ·1419	32·9380 ± ·1721	35·5881 ± ·1803	26·7313 ± ·1550	17·8633 ± ·1152	—	—	20·3682 ± ·1528	23·8250 ‖ ± ·2177	26·498
	Pistils	17·7051 ± ·1068	23·9115 ± ·1877	24·5360 ± ·1931	28·3287 ± ·2242	17·4478 ± ·1604	12·1475 ± ·1183	—	—	13·2635 ± ·1223	18·1125 ‖ ± ·3234	19·432
Standard Deviations	Sepals	·2612 ± ·0050	·0976 ± ·0020	·0995 ± ·0021	·1811 ± ·0038	—	—	·8037 ± ·0121	·6406 ± ·0097	·5487 ± ·0155	·6158 ‡ ± ·0093	·4060
	Petals	·4334 ± ·0083	·4638 ± ·0097	·4598 ± ·0098	1·0118 ± ·0215	—	—	1·2189 ± ·0184	1·1113 ± ·0168	·7663 ± ·0216	1·2819 ‡ ± ·0193	·8434
	Stamens ...	3·7720 ± ·0720	4·7978 ± ·1003	5·7039 ± ·1217	6·0058 ± ·1275	3·7609 ± ·1096	3·2984 ± ·0815	—	—	3·8234 ± ·1080	2·8872 ‖ ± ·1540	4·2562
	Pistils	3·9568 ± ·0755	6·3470 ± ·1327	6·4001 ± ·1365	7·4713 ± ·1586	3·8943 ± ·1135	3·3878 ± ·0837	—	—	3·0606 ± ·0865	4·2885 ‖ ± ·2288	4·8508
Coefficients of Variation	Sepals ...	8·53	3·24	3·31	5·98	—	—	22·02	19·36	16·20	13·57	11·53
	Petals ...	5·34	5·70	5·70	11·61	—	—	14·60	13·51	9·41	13·19	9·88
	Stamens...	14·12	17·16	17·32	16·88	14·07	18·46	—	—	18·77	12·12	16·11
	Pistils ...	22·35	26·54	26·08	26·37	22·32	27·89	—	—	23·08	23·68	24·79
Correlations	Sepals & Petals...	·1044 ± ·0267	·0993 ± ·0293	·2912 ± ·0276	·2300 ± ·0284	—	—	·2451 ± ·0200	·1928 ± ·0205	[– ·1821 ?]† ± ·0386	·2237 ‡ ± ·0203	·1981
	Sepals & Stamens	·0228 ± ·0270	·0584 ± ·0295	·0222 ± ·0300	·2225 ± ·0285	—	—	—	—	—	—	·0815
	Sepals & Pistils...	·0706 ± ·0269	·0324 ± ·0295	·1078 ± ·0297	·2490 ± ·0282	—	—	—	—	—	—	·1150
	Petals & Stamens	·2243 ± ·0256	·2939 ± ·0264	·2690 ± ·0280	·3825 ± ·0256	—	—	—	—	—	—	·2924
Petals and Pistils...		·1961 ± ·0260	·2222 ± ·0281	·3508 ± ·0265	·3527 ± ·0263	—	—	—	—	—	—	·2805
Stamens and Pistils		·4387 ± ·0218	·5206 ± ·0216	·6601 ± ·0170	·6682 ± ·0166	·5065 ± ·0306	·7489 ± ·0153	—	—	·5328 ± ·0290	·3913 ‖ ± ·0639	·5584

* A second series from Gera while less variable gives sensibly the same means and correlation. A third series, more or less intermediate in variation, gives an insensible negative correlation: see *Biometrika*, Vol. I. p. 317.

† This result is very discordant and is not included in the European mean: see *Biometrika*, Vol. I. p. 317.

‡ A second short series for Gais, with somewhat different means and variabilities, is fairly accordant in correlation.

‖ 80 heads only from 1900.

These results form a further proof, were any needful, that race cannot be defined by correlation*. Plants are, of course, more subject to environment than animal life, and the means and variabilities change with change of environment and of season very readily. Man, perhaps, of all animal life seems least affected by environment, but a table like the above showing how different the mean, variability and correlation are when there is slight change in locality or season, ought to render us cautious of cataloguing even local races of man by the means alone of small craniological series. Nurture and environment during growth may sensibly influence even human mean characters. The biometrician, far from being discouraged by the fact that the simpler biometric quantities—means, variabilities, correlations—are not constant for local races, ought rather to recognise that if natural selection be a reality these are the very quantities which he would expect to change from one local race to another, and from one environment to a second. Indeed it is exactly these differences which form the foundation upon which we hope to build up in the future the evolutionary history of local races; they are the material from which we must determine not only the characters which in each case have been selected, but also the absolute constants which define the species itself†.

(4) Comparing the first four columns—the new data—with the last four columns—the old data for central Europe—one is at once struck with the high degree of variability in sepals and petals of the latter as compared with the former. The sepals are more than four times, the petals are nearly twice, as variable in the second group, i.e. the mean values are ·6544 and 1·0946 as against ·1598 and ·5944 respectively. Now we see in the case of the Dorset group that the petal variability can rise to double the value it had in Bordighera, Guernsey or Surrey. But an examination of the tables below shows that much of the difference arises from the absence of individuals with less than 3 sepals or 7 petals in the first group, and as our data are drawn from very diverse districts we must await with interest the result of *bud* countings from Germany.

TABLE II.

Frequency of Sepals per 1000.

Number of Sepals	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*
Bordighera	—	—	3·2	932·7	62·5	1·6	—	—
St Peter Port ...	—	—	—	990·1	9·6	—	—	—
Surrey...	—	—	—	990·0	10·0	—	—	—
Dorset...	—	—	—	972·3	25·7	2·0	—	—
Greiz	4·2	1·8	9·0	546·8	251·7	174·2	11·5	0·8
Gera	3·0	0·0	21·0	733·0	152·0	86·0	5·0	—
Trogen	—	—	—	680·7	259·7	52·6	7·0	—
Gais	—	—	2·0	91·0	283·0	616·0	7·0	1·0

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* *Phil. Trans.* Vol. 187, A, pp. 266, 280.

† "Mathematical Contributions to the Theory of Evolution. XI. On the Influence of Natural Selection on Variation and Correlation." *Phil. Trans.* Vol. 200, A, pp. 18—21.

TABLE III.

Frequency of Petals per 1000.

Number of Petals	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
Bordighera ..	—	—		1·6	14·1	868·6	96·2	17·6	1·6				—
St Peter Port	—	—	—	—	13·5	869·2	96·1	13·5	7·7	—	—	—	—
Surrey ...	—	—	—	4·0	40·0	858·0	84·0	10·0	1·0	—	—	—	—
Dorset ...		—	—	—	7·9	560·4	231·7	124·7	63·4	7·9	4·0	—	—
Greiz	0·2	0·8	6·8	32·7	148·0	473·0	180·0	99·0	35·3	17·2	6·0	0·5	0·5
Gera	—	—	16·0	31·0	112·0	563·0	175·0	62·0	24·0	11·0	5·0	0·0	1·0
Trogen ...	—	—	3·5	14·0	119·3	642·4	150·9	59·7	10·5	—	—	—	—
Gais	—	—	—	2·0	34·0	139·0	265·0	296·0	175·0	69·0	18·0	2·0	—

But making all allowance for this the German and Swiss series show a marked increase in the number of both sepals and petals over our series, culminating in Gais with its modes of 5 sepals and 10 petals. Our Dorsetshire series indicates a substantial advance in the same direction as far as the petals are concerned, the Bordighera tends somewhat in the same direction for the petals. It remains as a task for next spring to determine whether the Guernsey and Gais series cannot be linked together by a continuous system of intermediate series, if only the flowers be gathered at a sufficiently wide range of places and at different seasons in those places.

(5) Turning to the correlations which involve sepals one is hardly surprised at their irregularity, for their values depend in our new data on the distribution of a very few individuals with more than four sepals. These for Bordighera are less than 7 per cent., for Dorset less than 3 per cent., and for Guernsey and Surrey not more than 1 per cent.! Hence one or two irregular or anomalous individuals cause the correlation to alter in a very remarkable manner. A much safer set of correlations are those between petals, stamens and pistils. Unfortunately the counting of stamens and pistils is a much harder task, and for a series of 500 bud-gatherings requires a week or two of very laborious work.

Looking at our new data, which are arranged according to number of stamens, we see several important and almost uniform results:

(*a*) If the different series be arranged in ascending order of stamens, this will also be an ascending order for the number of pistils.

With one exception—Surrey—this is also an ascending order for the number of petals.

(*b*) The ascending order for stamens or pistils is also an ascending order for the variability of both stamens and pistils.

With the same exception—Surrey—it is an ascending order for variability of petals.

(*c*) The ascending order for means or variabilities of stamens or pistils is an ascending order for the three correlations between stamens, pistils and petals.

The slight exception to this rule—Surrey correlation of petals and stamens—is well within the probable error of the determinations.

Summing up these results in a single statement, we conclude from the data that were gathered and counted on one uniform plan that:

The local races of the lesser celandine which have flowers with more numerous parts, have those parts more variable and more highly correlated.

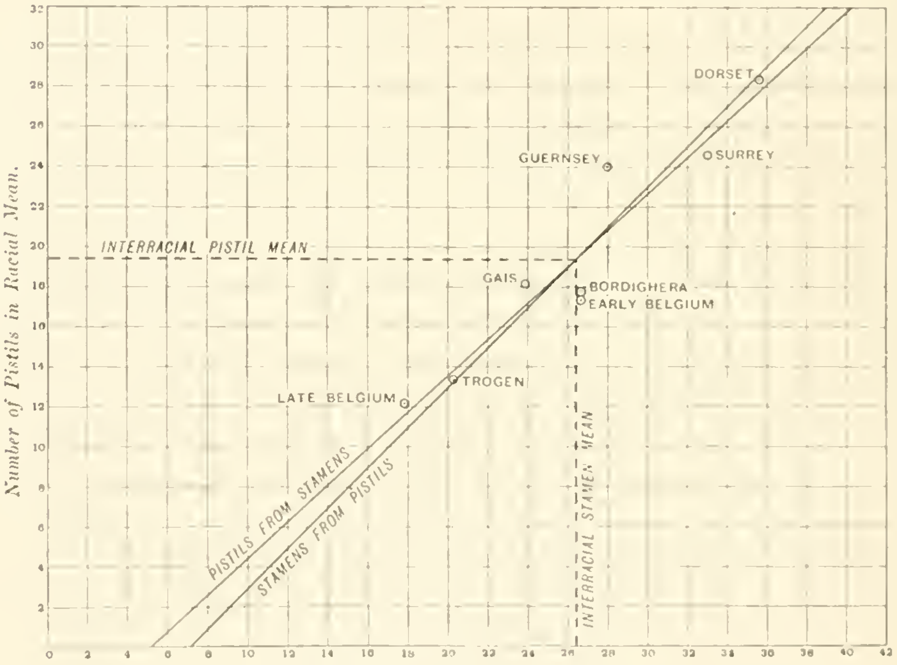
Let us compare these results with the data obtained by other observers. Germany contributes nothing, for stamens and pistils have not yet been counted. The Swiss data, if individually a little anomalous, form, if clubbed together, a striking confirmation of our argument,—they have fewer pistils and stamens than our four series, the variations are less, and the correlations for stamens and pistils are low—Gais has the lowest result of all, and Trogen only about the Guernsey value. Unfortunately we can only compare one correlation and this has been worked out for only small numbers, 285 in one case and 80 in the other. With regard to Belgium the comparative results are not so satisfactory. The early Belgium flowers have about the same number of stamens as the Italian and fewer pistils, the variability of both stamens and pistils is less. The late Belgium flowers have the least number of stamens and pistils of any series, their variability in both stamens and pistils is less than all four of our series, and less than the mean of the two Swiss series. Here again we have unfortunately only one correlation available for comparison, that of stamens and pistils. It is for the early flowers somewhat greater than that for the Italian series, when it ought to be about equal to it, while for the late flowers, it is the highest of all when it should be the least from the above rule. But just in this material a new factor comes into play, which, if not the source of the result, deserves to be taken into account. The late flowers are said to be from the *same* plants as the early flowers. But pistils and stamens cannot be counted unless the flower be gathered. Hence the plants from which the later flowers are gathered may be heterogeneous in character, namely plants naturally flowering late, and plants which have been more or less injured by one or more flowers being removed and which have reflowered in a more or less exhausted condition. Such heterogeneity would tend to increase correlation. Clearly in this as in so many cases the conditions are very complex, and we cannot without further investigation club together flowers from plants (*a*) which have had earlier flowers gathered, (*b*) which naturally flower late, with the late flowers of plants which have been flowering throughout the season undisturbed, and term the group 'late flowers' without danger of introducing heterogeneity. Till therefore the whole subject of the correlation of late and early flowers has been gone into carefully with ample data, we may take, we hold, the above general principle—that increase in the number of parts denotes increase in the variability and correlation of those parts—as a working hypothesis to be demonstrated or modified by further researches. We, of course, do not extend the principle beyond the species for which the data are deduced, even as a "working hypothesis," and we state it only for petals, stamens and pistils.

(6) It may be useful to examine the interracial result from the correlation standpoint, although at present we have rather meagre data. We have eight local series; considering them of equal weight, although the Belgian and Swiss series are very inferior in number to ours, we find:

Mean of local means for stamens ... ... = 26·498 ± 1·321
Mean of local means for pistils ... ... = 19·432 ± 1·265
Variability of local means for stamens ... = 5·5409 ± ·9343
Variability of local means for pistils ... = 5·3037 ± ·8943
Correlation of local means for stamens and pistils = ·9514 ± ·0235.

The probable errors are very high because we have only eight pairs. It is therefore difficult, perhaps, to make a definite statement that the interracial is greater than the average intraracial variability, but it appears like it; even the relative order of variability of pistils and stamens may be changed when we pass from intraracial to interracial constants. On the other hand the correlation is so high* that we can make a very definite statement about it, i.e.:

Lesser Celandine. Interracial Regression Lines. Stamens and Pistils.



The interracial correlation of the mean numbers of stamens and pistils is very much greater than the mean intraracial correlation between stamens and pistils, being to the latter nearly in the ratio of 12 *to* 7.

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* And therefore its probable error even with only eight pairs, so small.

These conclusions show us how extremely cautious we must be in extending conclusions deduced by intraracial calculations to interracial relations. Intraracial and interracial variabilities and correlations may be of a totally different order. By the usual theory of regression we have the following formulæ to find the probable number of stamens in a race given the mean number of pistils, and the probable number of pistils given the mean number of stamens:

$$\left.\begin{array}{l}\text{Probable mean number of stamens in a}\\ \quad\text{local race}\end{array}\right\} = \cdot 9939 \times (\text{observed mean number of pistils}) + 7\cdot 185.$$

$$\left.\begin{array}{l}\text{Probable mean number of pistils in a}\\ \quad\text{local race}\end{array}\right\} = \cdot 9106 \times (\text{observed mean number of stamens}) - 4\cdot 697.$$

Comparing with actual values we have:

	Stamens from Pistils		Pistils from Stamens	
	Observed	*Calculated*	*Observed*	*Calculated*
Bordighera	26·7	24·9	17·7	19·6
St Peter Port	28·0	31·0	23·9	20·8
Surrey	32·9	31·7	24·5	25·3
Dorset	35·6	35·3	28·3	27·7
Early Belgian	26·7	24·5	17·4	19·6
Late Belgian	17·9	19·3	12·1	11·6
Trogen	20·4	20·4	13·3	13·8
Gais	23·8	25·2	18·1	17·0

The maximum error in the number of stamens is 3·0 and the mean error 1·3; for the number of pistils it is 3·1 and the mean error again 1·3. Thus our formulæ will give the number of stamens from pistils, or pistils from stamens in any race with an average error of about one pistil or stamen, and a maximum of about three. They are thus close enough for most practical purposes. The diagram gives the lines to read stamens from pistils and pistils from stamens.

(7) There are a number of other points which are suggested by our table, but it seems better to defer their discussion until more ample data are forthcoming. There is hardly any district from which at present a series of 500—600 celandine heads would not be of value. But these series ought to be gathered on a uniform plan, i.e. gathered as buds, each bud being wrapped in a separate soft paper cover. Further since the labour of counting is very great, those who will count as well as gather are the more valuable helpers. In counting great care must be taken to separate the individual pistils, and all heads with less than three sepals ought to be carefully examined with a microscope or powerful lens. If possible two series should be taken from each locality, one early and one late in the season, but the flowers should not be taken off the same plants, but off plants having substantially the same environment. A record of date, environment and

locality should accompany each series. The most important localities at present seem to be, Mid-France, South and Mid-Italy, Mid and North England, Switzerland and South Germany in as many districts as possible, and any northern European stations, where workers may be available. It would be of great value to have one species of plant thoroughly worked out from the biometric standpoint over a wide area. This can only be done by cooperative labour.

CORRELATION TABLES. LESSER CELANDINE.

A. SEPALS AND PETALS.

I. *Bordighera.*

Sepals.

Petals.	*2*	*3*	*4*	*5*	Totals
6	—	1	–	—	1
7	—	3	6	—	9
8	2	521	19	—	542
9	–	46	14	—	60
10	—	10	—	1	11
11	—	1	—	—	1
Totals	2	582	39	1	624

II. *Guernsey.*

Sepals.

Petals.	*3*	*4*	Totals
7	6	1	7
8	450	2	452
9	49	1	50
10	7	—	7
11	3	1	4
Totals	515	5	520

III. *Surrey.*

Sepals.

Petals.	*3*	*4*	Totals
6	2	—	2
7	20	—	20
8	429	—	429
9	38	4	42
10	5	—	5
11	1	1	2
Totals	495	5	500

IV. *Dorset.*

Sepals.

Petals.	*3*	*4*	*5*	Totals
7	4	—	—	4
8	279	4	—	283
9	115	2	—	117
10	59	4	—	63
11	31	1	—	32
12	3	1	–	4
13	—	1	1	2
Totals	491	13	1	505

B. SEPALS AND STAMENS.

V. *Bordighera.*

Stamens.

Sepals.	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	*41*	*42*	*43*	*44*	*45*	*46*	*47*	*48*	Totals
2	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
3	1	1	6	4	9	17	20	46	38	85	65	61	77	45	27	25	23	8	7	5	4	4	1	1	—	—	—	1	—	—	—	1	—	582
4	—	—	2	—	—	1	1	3	4	5	—	6	9	5	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	39
5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	1	1	8	4	9	18	21	49	42	90	65	67	86	51	28	28	23	8	7	5	4	4	1	1	—	—	—	1	—	—	—	1	1	624

VI. *Guernsey.*

Stamens.

Sepals.	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	*41*	*42*	*43*	Totals
3	1	—	—	2	3	1	11	15	29	39	29	38	53	44	43	25	30	31	24	23	22	18	10	6	6	8	3	—	—	1	515
4	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	1	—	—	1	—	—	—	—	—	—	—	1	—	—	5
Totals	1	—	—	2	3	1	11	15	29	40	29	38	53	45	43	25	31	31	24	24	22	18	10	6	6	8	3	1	—	1	520

VII. *Surrey.*

Stamens.

Sepals.	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	*41*	*42*	*43*	*44*	*45*	*46*	*47*	*48*	*49*	*50*	*51*	*52*	Totals
3	—	—	—	1	—	—	6	11	2	14	29	20	34	29	27	42	31	30	41	33	24	23	22	20	13	8	10	1	5	3	2	5	4	2	2	—	1	495
4	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	1	—	—	—	—	—	—	—	5
Totals	—	—	1	1	—	—	6	11	2	14	29	20	35	29	27	42	31	30	41	33	24	23	23	20	13	8	11	1	5	4	2	5	4	2	2	—	1	500

VIII. *Dorset.*

Stamens.

Sepals.	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	*41*	*42*	*43*	*44*	*45*	*46*	*47*	*48*	*49*	*50*	*51*	*52*	*53*	*54*	*55*	*56*	*57*	*58*	*59*	*60*	*61*	Totals
3	1	—	—	—	—	—	—	2	1	1	5	5	7	8	8	14	17	23	20	35	47	36	34	25	29	37	22	29	17	16	18	6	6	3	1	9	3	3	2	1	—	—	—	—	—	—	—	—	—	491
4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2	1	—	3	—	—	1	—	—	2	—	1	1	—	—	—	—	—	—	—	—	—	—	—	1	—	—	13
5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	1	—	—	—	—	—	—	2	1	1	5	5	7	8	8	14	17	23	20	35	48	36	36	26	29	40	22	29	18	16	18	8	6	4	2	9	3	3	2	1	—	—	—	—	—	—	1	—	1	505

C. SEPALS AND PISTILS.

IX. *Bordighera.*

Pistils.

Sepals.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	*41*	Totals
2	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
3	1	—	—	—	1	3	9	24	63	71	75	82	71	50	31	31	24	14	6	7	1	5	2	1	1	—	2	1	—	1	2	1	—	1	—	1	582
4	—	—	—	—	—	—	—	—	5	4	7	3	7	5	2	3	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	39
5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
Totals	1	—	—	—	1	3	9	24	68	75	82	85	79	56	33	34	25	15	6	7	1	5	2	1	1	—	2	1	—	1	2	3	—	1	—	1	624

X. *Guernsey.*

Pistils.

Sepals.	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	*41*	*42*	*43*	*44*	*45*	*46*	*47*	*48*	*49*	*50*	Totals
3	5	4	10	17	15	28	32	28	28	33	35	30	30	40	23	28	20	16	13	11	19	15	5	5	5	6	4	2	—	2	—	3	—	2	—	—	—	—	1	515
4	—	—	—	—	1	—	—	—	—	1	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	5
Totals	5	4	10	17	16	28	32	28	28	34	35	30	31	40	24	28	20	16	13	11	19	15	5	5	5	6	4	2	—	2	—	4	—	2	—	—	—	—	1	520

XI. *Surrey.*

Pistils.

Sepals.	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	*41*	*42*	*43*	*44*	*45*	*46*	Totals
3	2	4	3	5	10	15	21	26	25	23	34	34	41	31	31	28	31	21	17	12	15	8	13	5	11	4	6	3	3	4	4	—	1	—	2	1	1	495
4	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	5
Totals	2	4	3	5	10	15	22	26	25	23	34	34	41	31	31	28	31	21	17	13	15	9	13	5	11	4	6	4	3	5	4	—	1	—	2	1	1	500

XII. *Dorset.*

Pistils.

Sepals.	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	*41*	*42*	*43*	*44*	*45*	*46*	*47*	*48*	*49*	*50*	*51*	*52*	*53*	*54–56*	*57*	*58–75*	*76*	Totals
3	1	—	1	1	2	1	2	5	8	7	17	15	18	28	28	29	28	26	36	28	19	33	26	15	17	13	14	13	8	9	8	4	7	8	4	2	2	4	1	1	—	—	1	—	1	—	—	—	—	491
4	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	1	1	—	—	—	1	—	1	—	—	—	1	—	—	1	—	1	—	1	—	1	—	—	—	—	—	—	—	—	—	1	—	—	13
5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	1	—	1	1	2	1	2	5	8	7	17	15	18	29	29	30	29	27	36	28	19	34	26	16	17	13	14	14	8	9	9	4	8	8	5	2	3	4	1	1	—	—	1	—	1	—	1	—	1	505

D. PETALS AND STAMENS.

XIII. *Bordighera.*

Stamens.

Petals.	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	*41*	*42*	*43*	*44*	*45*	*46*	*47*	*48*	Totals
6	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
7	—	—	—	—	—	1	1	—	—	3	—	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9
8	1	1	7	3	7	16	18	45	37	79	63	60	74	40	24	25	22	8	7	3	—	2	—	—	—	—	—	—	—	—	—	—	—	542
9	—	—	—	1	2	1	2	3	3	8	1	4	10	11	4	2	1	—	—	1	3	1	1	1	—	—	—	—	—	—	—	—	—	60
10	—	—	1	—	—	—	—	1	2	—	1	—	—	—	—	1	—	—	—	—	1	1	—	—	—	—	—	1	—	—	—	1	1	11
11	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—		1
Totals	1	1	8	4	9	18	21	49	42	90	65	67	86	51	28	28	23	8	7	5	4	4	1	1	—	—	—	1	—	—		1	1	624

XIV. *Guernsey.*

Stamens.

Petals.	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	Totals
7	—	—	—	—	—	—	1	—	—	1	—	1	—	1	—	—	—	—	1	1	—	—	—	1	—	—	—	—	—	—	7
8	1	—	—	2	3	1	10	15	28	38	28	36	51	38	39	23	28	23	18	19	17	15	4	3	5	5	1	—	—	1	452
9	—	—	—	—	—	—	—	—	1	—	1	1	2	5	4	2	3	8	4	3	4	2	3	2	1	3	1	—	—	—	50
10	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1	1	1	1	2	—	—	—	—	—	—	—	7
11	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1	—	—	—	1	1	—	—	4
Totals	1	—	—	2	3	1	11	15	29	10	29	38	53	45	43	25	31	31	24	24	22	18	10	6	6	8	3	1	—	1	520

XV. *Surrey.*

Stamens.

Petals.	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	Totals
6	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
7	—	1	—	—	—	1	—	1	1	—	3	2	1	4	2	—	—	—	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	20
8	—	—	—	—	6	10	2	13	28	19	29	23	25	35	26	28	38	29	22	17	19	16	11	5	8	1	4	2	2	4	3	2	2	—	—	429
9	1	—	—	—	—	—	—	—	—	1	2	3	1	3	2	1	2	1	—	4	3	4	1	3	3	—	1	2	—	—	1	—	—	—	—	42
10	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	1	5
11	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	2
Totals	1	1	—	—	6	11	2	14	29	20	35	29	27	42	31	30	41	33	24	23	23	20	13	8	11	1	5	4	2	5	4	2	2	—	1	500

XVI. *Dorset.*

Stamens.

Petals.	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	Totals
7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
8	—	—	—	—	—	—	—	2	1	1	5	5	5	6	6	12	11	17	13	23	35	22	20	14	17	19	10	13	8	4	8	1	2	—	—	3	—	—	—	—	—	—	—	—	—	—	—	—	—	283
9	—	—	—	—	—	—	—	—	—	—	—	—	2	1	1	1	4	5	5	7	5	5	7	6	8	14	9	10	3	7	4	2	3	1	1	2	3	—	—	1	—	—	—	—	—	—	—	—	—	117
10	1	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	1	1	—	2	5	7	6	4	3	3	3	4	4	3	4	2	1	2	—	1	—	1	2	—	—	—	—	—	—	—	—	—	—	63
11	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2	2	3	2	1	3	—	2	2	2	2	2	—	1	1	3	—	2	—	—	—	—	—	—	—	—	1	—	—	32
12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
13	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2
Totals	1	—	—	—	—	—	—	2	1	1	5	5	7	8	8	14	17	23	20	35	48	36	36	26	29	40	22	29	18	16	18	8	6	4	2	9	3	3	2	1	—	—	—	—	—	—	1	—	1	505

E. PETALS AND PISTILS.

XVII. *Bordighera.*

Pistils.

Petals.	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	*41*	Totals
6	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
7	—	—	—	—	—	—	—	—	1	1	2	—	3	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	9
8	1	—	—	—	1	2	9	23	63	67	73	77	66	45	29	25	25	13	6	6	1	3	1	—	—	—	1	—	—	1	2	1	—	1	—	—	542
9	—	—	—	—	—	1	—	1	2	7	6	7	9	8	4	7	—	1	—	1	—	—	1	1	1	—	—	1	—	—	—	1	—	—	—	1	60
10	—	—	—	—	—	—	—	—	2	—	1	1	—	—	—	2	—	1	—	—	—	2	—	—	—	—	1	—	—	—	—	1	—	—	—	—	11
11	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	1	—	—	—	1	3	9	24	68	75	82	85	79	56	33	34	25	15	6	7	1	5	2	1	1	—	2	1	—	1	2	3	—	1	—	1	624

XVIII. *Guernsey.*

Pistils.

Petals.	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	*41*	*42*	*43*	*44*	*45*	*46*	*47*	*48*	*49*	*50*	Totals
7	—	—	—	—	—	—	—	1	1	—	—	1	1	—	1	—	—	1	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	7
8	5	4	10	16	16	27	30	25	24	32	30	29	28	35	18	26	16	13	9	11	14	11	4	3	3	4	3	2	—	1	—	—	—	2	—	—	—	—	1	452
9	—	—	—	—	—	1	1	2	2	2	4	—	2	5	5	2	2	2	4	—	5	4	1	1	2	1	—	—	—	—	—	2	—	—	—	—	—	—	—	50
10	—	—	—	1	—	—	1	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	1	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	7
11	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	—	—	4
Totals	5	4	10	17	16	28	32	28	28	34	35	30	31	40	24	28	20	16	13	11	19	15	5	5	5	6	4	2	—	2	—	4	—	2	—	—	—	—	1	520

XIX. *Surrey.*

Pistils.

Petals.	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	Totals
6	—	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
7	—	1	—	—	—	—	3	3	2	1	1	—	1	1	1	3	—	—	—	—	1	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	—	20
8	2	3	3	5	9	15	17	23	22	20	29	31	37	27	27	23	28	18	12	12	12	7	12	4	10	4	2	3	2	3	3	—	1	—	2	—	1	429
9	—	—	—	—	1	—	2	—	1	2	3	3	2	3	3	2	3	2	4	—	1	2	—	—	1	—	3	1	1	1	1	—	—	—	—	—	—	42
10	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1	—	5
11	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	2
Totals	2	4	3	5	10	15	22	26	25	23	34	34	41	31	31	28	31	21	17	13	15	9	13	5	11	4	6	4	3	5	4	—	1	—	2	1	1	500

XX. *Dorset.*

Pistils.

Petals.	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54–56	57	58–75	76	Totals
7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
8	1	—	1	—	2	1	2	4	7	6	15	10	15	20	18	15	17	19	20	15	12	14	13	8	10	7	5	3	3	3	4	2	2	3	2	1	1	1	1	—	—	—	—	—	—	—	—	—	—	283
9	—	—	—	1	—	—	—	—	—	—	1	4	2	7	7	8	7	4	8	10	4	9	9	4	4	1	3	7	3	2	4	1	2	1	1	1	—	—	—	1	—	—	1	—	—	—	—	—	—	117
10	—	—	—	—	—	—	—	1	1	1	1	—	1	2	1	4	5	3	5	2	2	5	3	4	1	5	2	4	—	—	—	1	2	4	—	—	2	1	—	—	—	—	—	—	—	—	—	—	—	63
11	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	2	—	—	3	1	—	5	—	—	2	—	4	—	2	4	1	—	1	—	1	—	—	2	—	—	—	—	—	—	1	—	1	—	—	32
12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
13	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2
Totals	1	—	1	1	2	1	2	5	8	7	17	15	18	29	29	30	29	27	36	28	19	34	26	16	17	13	14	14	8	9	9	4	8	8	5	2	3	4	1	1	—	—	1	—	1	—	1	—	1	505

F. STAMENS AND PISTILS.

XXI. *Bordighera.*

Pistils.

Stamens.	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	Totals
16	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
17	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
18	—	—	—	—	1	—	—	1	1	1	1	1	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
19	—	—	—	—	—	1	—	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
20	—	—	—	—	—	—	1	—	1	1	2	1	—	1	1	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	9
21	—	—	—	—	—	—	1	2	3	4	3	1	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	18
22	—	—	—	—	—	—	—	2	6	1	7	1	2	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	21
23	—	—	—	—	—	1	4	2	7	9	8	3	5	5	2	—	—	—	—	—	—	1	1	—	—	—	—	—	—	1	—	—	—	—	—	—	49
24	—	—	—	—	—	—	—	4	7	10	4	7	5	3	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	42
25	—	—	—	—	—	1	—	7	11	14	16	13	9	5	6	4	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	90
26	—	—	—	—	—	—	1	3	11	13	6	11	5	1	5	3	3	1	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	65
27	—	—	—	—	—	—	—	1	9	9	10	9	11	5	3	4	2	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	67
28	—	—	—	—	—	—	1	2	9	6	11	12	24	9	4	3	1	2	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	86
29	1	—	—	—	—	—	—	—	1	3	6	14	4	11	2	5	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	51
30	—	—	—	—	—	—	—	—	—	2	4	6	6	5	—	2	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	28
31	—	—	—	—	—	—	—	—	—	1	2	4	2	4	5	3	6	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	28
32	—	—	—	—	—	—	—	—	—	—	1	2	3	5	1	3	4	1	—	1	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	23
33	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	2	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	8
34	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1	1	—	1	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	7
35	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	2	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
36	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
37	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	1	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	4
38	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
39	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
40	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
41	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
42	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
43	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
44	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
45	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
46	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
47	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
48	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
Totals	1	—	—	—	1	3	9	24	68	75	82	85	79	56	33	34	25	15	6	7	1	5	2	1	1	—	2	1	—	1	2	3	—	1	—	1	624

XXII. *Guernsey.*

Pistils.

Stamens.	*12*	*13*	*14*	*15*	*16*	*17*	*18*	*19*	*20*	*21*	*22*	*23*	*24*	*25*	*26*	*27*	*28*	*29*	*30*	*31*	*32*	*33*	*34*	*35*	*36*	*37*	*38*	*39*	*40*	*41*	*42*	*43*	*44*	*45*	*46*	*47*	*48*	*49*	*50*	Totals
14	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
15	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
16	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
17	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
18	—	—	—	—	—	—	—	1	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
19	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
20	2	—	—	—	—	1	—	4	—	2	—	—	—	1	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	11
21	—	—	1	1	—	3	2	2	3	1	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	15
22	—	1	2	3	2	3	1	4	2	2	1	2	1	2	1	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	29
23	1	1	—	4	3	3	6	2	2	4	1	1	3	1	—	2	—	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	40
24	—	—	1	1	1	4	1	3	3	1	4	5	—	4	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	29
25	—	1	2	2	4	1	3	2	2	3	2	2	6	3	—	—	1	—	1	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	38
26	—	—	2	1	3	1	3	—	4	5	5	5	3	5	5	3	—	1	—	2	2	—	—	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	53
27	—	—	1	2	—	1	5	2	6	4	2	1	3	4	1	2	4	3	1	1	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	45
28	—	—	—	—	1	4	4	—	2	4	3	5	1	2	3	1	4	—	2	1	2	3	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	43
29	—	—	1	1	—	—	3	2	1	—	1	—	5	4	2	1	—	1	—	2	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	25
30	—	1	—	—	1	—	3	2	—	2	1	2	1	4	3	1	1	1	1	—	2	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	31
31	—	—	—	—	—	3	—	2	1	1	3	1	3	1	—	6	1	3	1	—	—	—	—	—	1	2	1	—	—	—	—	1	—	—	—	—	—	—	—	31
32	—	—	—	—	—	2	1	2	2	2	3	1	—	—	—	2	1	2	2	1	2	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	24
33	1	—	—	—	—	—	—	—	—	2	1	2	2	2	2	1	1	1	1	1	2	1	1	—	—	—	1	1	—	1	—	—	—	—	—	—	—	—	—	24
34	—	—	—	—	—	—	—	—	—	1	1	2	—	2	1	1	4	—	2	—	—	3	1	1	—	1	—	—	—	—	—	—	—	2	—	—	—	—	—	22
35	—	—	—	1	—	1	—	—	—	—	—	1	1	1	1	1	—	—	1	—	4	1	1	1	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	18
36	—	—	—	—	—	—	—	—	—	—	1	—	1	—	1	2	1	—	—	—	1	1	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	10
37	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1	2	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	6
38	—	—	—	—	—	—	—	—	—	—	—	—	1	2	1	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	6
39	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	2	1	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	8
40	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3
41	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
42	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
43	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	5	4	10	17	16	28	32	28	28	31	35	30	31	40	24	28	20	16	13	11	19	15	5	5	5	6	4	2	—	2	—	4	—	2	—	—	—	—	1	520

XXIII. *Surrey.*

Pistils.

Stamens.	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	Totals
18	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
19	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
20	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
21	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
22	1	1	—	—	—	1	1	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6
23	1	1	1	—	—	—	2	1	1	1	—	—	—	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	11
24	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
25	—	—	—	—	1	2	1	3	1	2	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	14
26	—	1	1	3	1	3	—	3	1	4	2	3	1	2	1	—	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	29
27	—	1	1	—	—	2	1	1	3	1	1	1	2	1	2	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	20
28	—	—	—	1	3	1	5	3	1	3	—	3	5	2	1	2	—	—	2	1	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	35
29	—	—	—	—	2	2	2	2	1	—	8	3	3	2	1	—	1	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	29
30	—	—	—	—	1	1	2	3	2	—	3	—	2	4	1	2	3	—	—	1	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	27
31	—	—	—	—	—	2	3	3	4	—	3	6	5	2	4	4	2	3	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	42
32	—	—	—	1	—	—	2	2	1	—	2	3	1	4	3	2	5	—	1	—	2	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	31
33	—	—	—	—	1	1	2	1	1	3	3	4	—	3	—	2	3	4	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	30
34	—	—	—	—	—	—	—	1	6	2	5	3	5	3	4	5	1	1	1	1	—	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	41
35	—	—	—	—	—	—	—	1	—	3	3	3	5	2	1	1	2	2	2	1	1	—	2	—	1	—	1	—	1	—	1	—	—	—	—	—	—	33
36	—	—	—	—	—	—	—	1	1	1	—	—	2	1	3	1	3	1	1	3	2	—	1	—	1	2	—	—	—	—	—	—	—	—	—	—	—	24
37	—	—	—	—	—	—	—	—	—	1	1	1	4	1	4	—	2	1	1	1	1	2	—	—	2	—	—	—	—	1	—	—	—	—	—	—	—	23
38	—	—	—	—	—	—	—	—	—	—	—	1	3	—	1	1	2	2	1	3	1	1	3	—	1	—	1	1	—	1	—	—	—	—	—	—	—	23
39	—	—	—	—	—	—	—	—	—	1	1	1	—	1	1	3	2	3	2	1	1	1	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	20
40	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2	2	1	1	1	1	—	—	1	1	—	—	—	1	—	1	—	—	—	—	—	—	—	13
41	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	3	1	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	8
42	—	—	—	—	1	—	—	—	—	—	—	—	—	1	1	1	—	—	—	—	—	—	2	—	—	—	—	—	2	1	2	—	—	—	—	—	—	11
43	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
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45	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	1	—	—	1	—	—	—	—	—	—	—	4
46	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	2
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52	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
Totals	2	4	3	5	10	15	22	26	25	23	34	34	41	31	31	28	31	21	17	13	15	9	13	5	11	4	6	4	3	5	4	—	1	—	2	1	1	500

XXIV. *Dorset.*

Pistils.

Stamens.	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	No reading 58–75	76	Totals
13	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1
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22	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1
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25	–	–	–	1	–	–	–	1	–	–	3	–	–	1	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	7
26	–	–	–	–	–	–	–	1	–	1	1	1	–	2	1	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	8
27	–	–	–	–	–	–	–	1	–	–	1	–	–	1	–	1	1	–	2	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	8
28	–	–	–	–	–	–	–	–	–	1	2	1	1	3	1	–	–	–	2	1	–	1	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	14
29	–	–	–	–	1	–	–	1	1	–	–	1	2	1	3	2	1	–	2	1	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	17
30	–	–	–	–	–	–	–	1	2	1	1	1	–	2	3	3	1	3	–	–	–	1	2	–	–	–	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	23
31	–	–	–	–	–	–	–	–	1	–	–	4	1	2	5	1	2	–	–	–	–	3	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	20
32	–	–	–	–	–	–	–	–	2	1	1	–	4	1	6	1	2	4	1	3	3	2	2	–	1	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	35
33	–	–	–	–	–	–	–	–	1	1	–	3	4	5	2	2	6	3	5	1	3	5	2	2	–	2	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	48
34	–	–	–	–	1	–	–	–	–	–	1	1	2	2	1	5	2	1	4	3	2	3	2	2	–	2	1	1	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	37
35	–	–	–	–	–	–	1	–	–	–	1	1	1	2	1	4	2	3	6	3	–	2	2	1	2	1	–	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	35
36	–	–	–	–	–	–	–	–	–	2	1	–	–	2	1	2	1	2	2	2	2	1	5	1	–	1	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	26
37	–	–	–	–	–	–	–	–	–	–	–	–	1	3	1	2	2	1	4	1	4	1	1	2	1	1	–	1	–	–	1	–	–	–	–	–	1	–	1	–	–	–	–	–	–	–	–	–	–	–	–	29
38	–	–	–	–	–	–	–	–	–	–	1	2	–	–	1	2	1	6	1	3	1	2	1	4	2	3	1	4	–	2	–	–	2	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	40
39	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2	–	1	–	4	1	2	4	1	2	1	1	–	1	–	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	22
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41	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	2	2	1	3	–	–	1	–	1	2	–	2	1	1	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	18
42	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	1	–	–	–	1	2	–	3	2	1	–	2	–	–	1	1	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	16
43	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	1	–	3	–	–	2	–	–	1	–	–	1	1	–	–	1	1	–	3	1	–	1	–	1	–	–	–	–	–	–	–	–	–	–	–	18
44	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	1	–	1	–	–	1	1	–	1	–	1	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	8
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Totals	1	–	1	1	2	1	2	5	8	7	17	15	18	29	29	30	29	27	36	28	19	34	26	16	17	13	14	14	8	9	9	4	8	8	5	2	3	4	1	1	–	–	1	–	1	–	–	–	1	–	1	505

SECOND REPORT ON THE RESULT OF CROSSING JAPANESE WALTZING MICE WITH EUROPEAN ALBINO RACES.

By A. D. DARBISHIRE, Balliol College, Oxford.

The present paper is an account of the continuation of work, the first part of which was published in the last number of this Journal. It contains, first, an additional record of the results of crossing Japanese waltzing mice with albino mice, embodying the previously recorded nine families and adding eleven new ones; and secondly the result of pairing hybrids resulting from such crosses, and of crossing these hybrids with albinos.

Crosses between Japanese Waltzing and Albino Mice.

The number of young obtained from such crosses has increased since the last paper from 48 to 154; but the uniformity of the result (the almost universal presence of patches of colour like that of the house-mouse) has not been maintained: there have appeared besides more yellow mice, black mice, and black and white.

The classification of colours in the hybrid has therefore had to be re-organized. The hybrids nearly always shew a considerable amount of white: they are now classified according to the amount of whiteness into five groups: each group is then subdivided into five classes according to the colour itself.

Group 1 (Fig. 1) has more white and less extent of coloured patches than the normal waltzing mouse. The distribution of colour on a waltzer is shewn in Fig. 6.

Group 2 (Fig. 2) has about as much white as a normal waltzing mouse.

Group 3 (Fig. 3) has less white (i.e. greater extent of coloured patches) than a waltzer.

Group 4 (Fig. 4) has still less white and leads on to

Group 5 (Fig. 5) which has no pure white: but the belly is whitish not gray.

Group 6 contains mice whose bellies are nearly the same colour as their backs. A gray mouse of group 6 (6*c* or 6*d*) is therefore indistinguishable from a house-mouse.

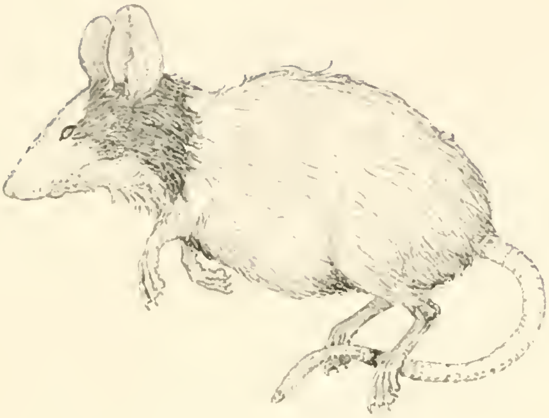

Fig. 1.

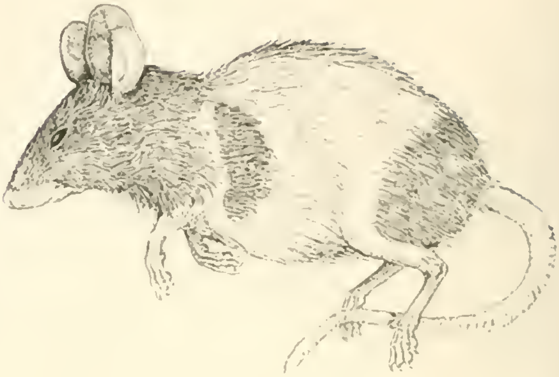

Fig. 2.

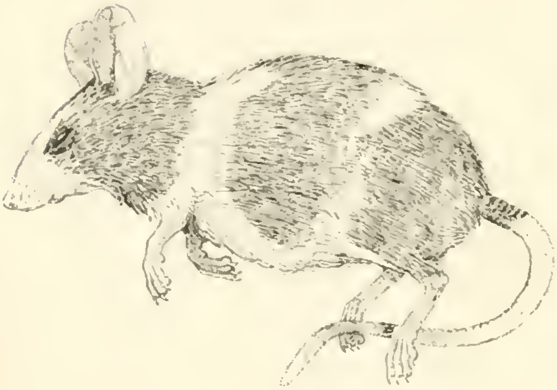

Fig. 3.

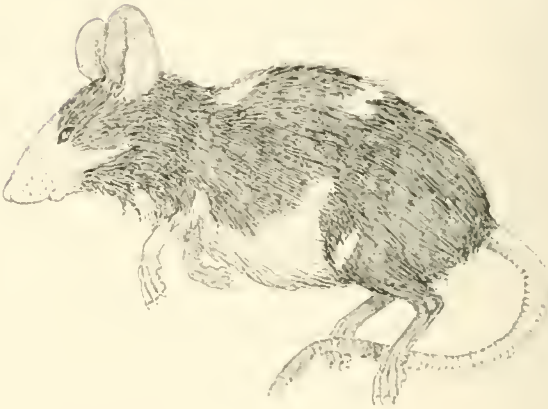

Fig. 4.

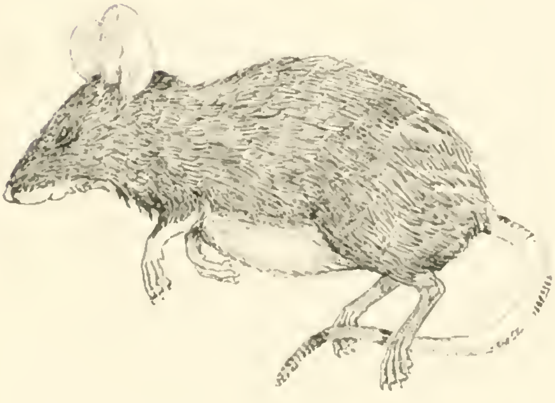

Fig. 5.

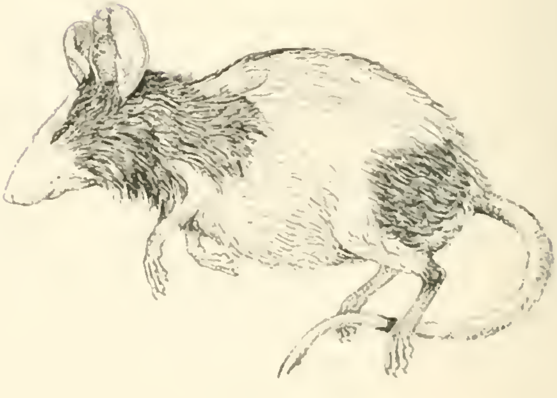

Fig. 6.

The individuals of each group are then classified according to the colour of the coloured patches.

Class a = yellow.
Class b = fawn yellow.
Class c = pale wild colour } (i.e. that of the house-mouse).
Class d = dark wild colour }
Class e = black,
and *Class* f = "lilac" = pale blue gray; at present only exhibited by the offspring of hybrids.

It is not suggested that these form one colour series. Table I. shews the distribution of colour and the colour itself in all the individuals of twenty families produced by crossing waltzing with albino mice. All the mice in the Table have black eyes.

TABLE I.

Number of Cross	Group I.					Group II.					Group III.					Group IV.					Group V.					Group VI.					Totals
	a	*b*	*c*	*d*	*e*	*a*	*b*	*c*	*d*	*e*	*a*	*b*	*c*	*d*	*e*	*a*	*b*	*c*	*d*	*e*	*a*	*b*	*c*	*d*	*e*	*a*	*b*	*c*	*d*	*e*	
i	–	–	2	–	–	–	–	1	1	–	–	–	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	6
ii	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1
vii	–	–	–	–	–	–	–	2	1	–	–	–	1	1	–	–	–	–	–	–	–	–	2	–	–	–	–	–	–	–	7
viii	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	6	–	–	–	–	–	–	–	6
ix	–	–	–	–	–	–	–	5	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	6
xii	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2	–	–	3	–	–	–	–	–	–	5
xiii	–	–	–	–	–	–	–	–	–	–	–	–	2	–	–	–	–	–	–	–	–	–	1	1	–	–	–	–	–	–	4
xvi	–	–	–	–	–	–	–	–	–	–	–	–	2	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	3
xx	–	–	–	–	–	–	–	1	2	–	–	–	–	1	–	–	–	–	–	–	–	–	–	2	–	–	–	–	–	–	6
xxvii*	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1
xxxiii*	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	1
xxxv	–	–	–	–	–	–	–	1	1	–	–	–	–	2	–	–	–	–	2	–	–	–	–	–	–	–	–	–	–	–	6
xlv	–	–	–	–	–	–	–	4	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	4
lix*	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	–	–	1	–	2
lxi	–	–	–	–	–	–	–	1	–	–	–	–	1	–	–	–	–	1	–	–	–	–	4	–	–	–	–	–	–	–	7
lxii	–	–	–	–	–	–	–	1	–	–	–	–	2	–	–	–	–	–	–	–	–	–	3	–	–	–	–	–	–	–	6
lxxiii	–	–	–	–	–	–	–	–	–	–	–	–	2	–	–	–	–	–	–	–	–	–	4	–	–	–	–	–	–	–	6
lxxiv	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	6	–	–	–	–	–	–	–	–	–	–	–	–	7
lxxv	–	–	1	–	–	–	–	2	–	–	–	–	1	–	–	–	–	1	–	–	–	–	–	–	–	–	–	–	–	–	5
lxxvii	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2	–	1	–	–	4	–	1	–	–	–	–	–	8
lxxviii	–	–	–	–	–	–	–	–	–	1	–	–	–	–	1	–	–	–	–	–	–	–	–	1	1	–	–	–	–	–	4
lxxxii*	–	–	–	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	–	–	1	–	–	–	–	–	–	–	2
lxxxiv	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	5	–	4	–	–	–	–	–	–	–	9
lxxxv	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2	–	–	–	1	–	–	–	–	3	1	–	–	–	–	–	7
lxxxvi	–	–	–	–	–	–	–	–	–	–	–	–	2	–	–	–	–	–	–	–	–	–	5	–	–	–	–	–	–	–	7
xciv*	–	–	–	–	–	–	–	–	–	–	–	–	2	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	2
c	–	–	–	–	–	–	–	1	–	–	–	–	3	–	–	–	–	–	–	–	–	–	5	–	–	–	–	–	–	–	9
ci	–	–	–	–	–	–	–	1	–	–	–	–	2	–	–	–	–	1	–	–	–	–	5	–	–	–	–	–	–	–	9
ciii	–	–	–	–	–	–	–	1	–	–	–	–	3	–	–	–	–	–	–	–	–	–	4	–	–	–	–	–	–	–	8
Totals	–	–	3	–	–	–	–	21	7	2	–	–	27	5	3	–	–	12	3	1	7	–	49	10	3	–	–	–	1	–	154

* In these crosses the parents were, ♀ waltzing mouse × ♂ albino; in all the rest the parents were ♀ albino × ♂ waltzer.

From the following re-classification of the above Table I. the frequency of degrees of whiteness among 154 hybrids will be at once evident.

Group 1	3
Group 2	30
Group 3	35
Group 4	16
Group 5	69
Group 6	1
	154

That is to say the majority of hybrids have coloured patches in, roughly, the same place as the waltzers have; *or* are wild-coloured mice except for the belly.

The following arrangement shews the frequency of colours among 154 hybrids.

Class a	7
Class b	0
Class c	112
Class d	26
Class e	9
	154

That is to say by far the greater number of hybrids exhibit wild colour.

Offspring of Hybrids.

Hybrids have been paired *inter se* and crossed with albinos.

A. *Offspring of pairs of Hybrids.*

The number of mice which waltz in this generation is eight. At present nine albinos have appeared: the actual arrangement of the colours and the colours themselves are here shewn: the letter *w* indicates that the mouse exhibits waltzing movements, and *p* that it has pink eyes. It is unnecessary to say that albinos have pink eyes.

H_7. ♀ 2*d* × ♂ 2*c*.
3 *Young*. 2*f* (*wp*), 2*a* (*p*), 2*b*.

H_8. ♀ 1*c* × ♂ 2*c*.
4 *Young*. 3*b* (*p*), 3*c*, 4*a* (*p*), 4*a* (*p*).

H_9. ♀ 2*d* × ♂ 3*c*.
5 *Young*. 3*a*, 3*b*, 3*b*, 3*e* (*w*), 3*f*.

H_{18}. ♀ 2*c* × ♂ 2*c*.
7 *Young*. 4 *albinos*, 3*b* (*wp*), 3*b* (*p*), 3*c*.

H_{19}. ♀ 2*c* × ♂ 3*c*.
7 *Young*. 2 *albinos*, 2*c*, 2*c*, 2*c*, 2*c* (*w*), 4*e* (*w*).

H_{20}. ♀ 2*c* × ♂ 2*c*.
6 *Young*. *Albino*. *Albino* (*w*), 2*c* (*w*), 3*c* (*w*), 2*e*, 2*f*.

H_{29}. ♀ 2*c* × ♂ 3*c*.
5 *Young*. *Albino*, 2*c*, 2*c*, 4*e*, 4*d*.

That is to say there are 37 young from 14 hybrid parents of degrees of whiteness included in Groups 2 and 3. The young may be classified according to degrees of whiteness and according to their colour.

Degrees of Whiteness	
Albinos	9
Group 1	0
Group 2	12
Group 3	11
Group 4	5
	37

Nature of Colour	
Albinos	9
Class a	4
Class b	6
Class c	9
Class d	1
Class e	5
Class f	3
	37

B. *Offspring of Hybrids and Albinos.*

The following is a record of young produced by crossing a hybrid with an albino.

H_1. ♀ albino × ♂ 3*c*.
4 *Young*. 5*c*, 5*c*, 5*c*, 5*c*.

H_2. ♀ albino × ♂ 3*c*.
9 *Young*. 5 *albinos* and 4*e*, 4*d*, 4*d*, 5*d*.

H_4. ♀ albino × ♂ 5*c*.
5 *Young*. 2 *albinos* and 5*c*, 5*d*, 6*e*.

H_5. ♀ albino × ♂ 5*c*.
5 *Young*. 1 *albino* and 6*d*, 6*d*, 6*d*, 6*c*.

H_6. ♀ albino × ♂ 5*c*.
4 *Young*. 2 *albinos* and 6*d*, 6*d*.

H_{11}. ♀ albino × ♂ 3*d*.
6 *Young*. 5 *albinos* and 6*e*.

H_{12}. ♀ 2*c* × ♂ albino.
5 *Young*. 2 *albinos* and 3*c*, 6*e*, 6*e*.

H_{13}. ♀ 5*c* × ♂ albino.
3 *Young*. 1 *albino* and 5*d*, 6*e*.

H_{14}. ♀ 5*c* × ♂ albino.
6 *Young.* 4 *albinos* and 2*c*, 3*e*.

H_{15}. ♀ albino × ♂ 5*c*.
8 *Young.* 4 *albinos*, 3*a*, 3*a*, 6*a*, 3*e*.

H_{17}. ♀ albino × ♂ 5*c*.
5 *Young.* 2 *albinos*, 3*e*, 3*d*, 6*d*.

H_{24}. ♀ albino × ♂ 5*c*.
6 *Young.* 3 *albinos*, 4*d*, 4*d*, 6*d*.

H_{26}. ♀ albino × ♂ 3*c*.
4 *Young.* 1 *albino*, 6*e*, 5*d*, 5*d*.

H_{28}. ♀ 5*c* × ♂ albino.
2 *Young.* 5*d*, 5*c*.

H_{31}. ♀ 5*d* × ♂ albino.
1 *Young.* 1 *albino*.

H_{32}. ♀ 5*c* × ♂ albino.
6 *Young.* 2 *albinos* and 6*c*, 6*c*, 4*c*, 6*e*.

H_{33}. ♀ albino × ♂ 5*c*.
9 *Young.* 4 *albinos* and 6*e*, 6*e*, 6*e*, 6*e*, 4*c*.

That is to say there is a fairly sharp segregation into albino and wild-coloured mice. Out of 88 young there are 39 albino, 31 wild-coloured, 15 black and 3 yellow mice.

Discussion of Results.

1. *The First Generation.*

As Table I. shews, the first generation has not a uniform colour: any modification of Mendel's hypothesis involves the uniformity of the first generation. For it is one of the Mendelian principles that segregation of characters does not occur until the generation produced by pairing individuals of the first (hybrid) generation. The generation, thus produced, essentially consists of 25 °/₀ organisms with the recessive character and 75 °/₀ with the dominant; and this not only in the case of a simple character, but in that of a complex one; for example the hybrids produced by Mendel's crossing a white flowered with a purple flowered (a complex colour according to Mendel) bean were all purple. But the offspring produced by the pairing of these purple hybrids exhibited the most marked heterogeneity. The essential point is that segregation of however complex characters never occurs before the second generation[*].

Now in these mice this segregation seems to take place in the first generation, for there appear in it besides the wild colour, yellow and black, as can be seen from Table I.

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[*] G. Mendel: "Versuche mit Pflanzen-Hybriden," *Verhandl. des Naturforsch. Ver. Brünn*, IV. Band. 1865 (Abhandlungen), pp. 34, 35.

The view that albinism is "recessive" in Mendel's sense implies that albinos of any ancestry, provided they themselves are really albino, will behave in the same way when crossed. Now, the young of all the albinos are not the same and at the end of my last paper it was pointed out that a difference in the kind of the litter corresponded to a difference in the ancestry of the albino mother: that is to say, that wild-coloured mice only appeared in the litters of pure-bred albinos: and the generalization that the more in-bred a mouse was, the less power it had of transmitting its whiteness, was suggested. The evidence brought forward for this were the results of von Guaita who in four crosses between waltzing mice and albinos which were in-bred for 29 generations always got absolutely wild-coloured mice; and my own results consisting of nine such crosses. This generalization is borne out by the additional eleven litters which have appeared since my last paper was written; as a classification of litters into (*a*) those from cross-bred albinos and (*b*) those from pure-bred albinos will readily shew (Table II.).

TABLE II.

	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6
(*a*) Young from cross-bred albinos	2	11	8	4	4	—
(*b*) Young from pure-bred albinos	—	13	23	—	60*	1

In my previous paper I think I did not sufficiently emphasize the difference which seems to me to exist between in-bred and pure-bred mice: the above table is certainly of pure-bred mice; but what is not certain is whether the *relative* inability to transmit whiteness among pure-bred mice is due to the fact of their being pure-bred, or in-bred. Pure-bred mice usually are in-bred and von Guaita's unquestionably were; but this does not help us to decide whether the relative inability to transmit whiteness is due to in-breeding or pure-breeding; it only shews that it is not impossible that it may be due to in-breeding. The meanings of these terms should be clearly understood: there are two pairs of categories, one pair in which cross-bred is antithetic to pure-bred, and another pair in which out-bred is antithetic to in-bred, the former pair referring to the presence or absence of any other colour than white in the ancestry of the mouse, the latter pair to the distance or nearness of relation of the parents, grandparents, etc. of the mouse under consideration. And before the cause of the great preponderance of the number of wild-coloured mice in the litters of pure-bred over the number of those in the litters of cross-bred albinos can be ascertained waltzing mice must be crossed with in-bred and out-bred pure-bred albinos and in-bred and out-bred cross-bred albinos. (Such kinds of albinos are being reared as quickly as possible.) Whatever the explanation be, the fact remains that the ancestry of a white mouse does make a difference in the character of its offspring when crossed with a waltzing mouse.

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* The 60 in the table includes the 7 yellow mice 5*a* in which there is no whiteness.

2. *The Second Generation.*

It will be seen by inspection of the tables recording the offspring of hybrids, that when a hybrid is paired with a hybrid the offspring form a much more heterogeneous collection than does the offspring from a hybrid and a white in which the young are more sharply segregated into white and wild-coloured. And this difference is coincident with, if not causally connected with a difference in the ancestry of the two sets of young. For the heterogeneous offspring of two hybrids have a more complex ancestry than do the less variable offspring of a hybrid and an albino; and it may be a fact of similar meaning that the hybrids produced by crossing a waltzing mouse with a *cross-bred* albino are more heterogeneous than those produced by a similar cross in which the albino, however, was *pure-bred* (see Table II.)—in both cases the more heterogeneous collection of offspring comes from parents of which the ancestry is more complex; that is to say the young of the cross-bred albinos are more heterogeneous than those of the pure-bred; and the young produced by pairing two hybrids are more heterogeneous than those produced by crossing hybrids with albinos.

No theory of compound allelomorphs such as that put forward by Mendel will account for this striking difference between the variability of the two groups of offspring.

Remembering that 6*c* or 6*d* indicates a mouse indistinguishable from the common *Mus musculus*, the curious fact will be noticed that only *one* such appeared among 154 hybrids of the first generation, whereas out of the 88 offspring of a cross between a hybrid and an albino there were 10. This result may be parallel with that obtained by Darwin* when he crossed a white fantail with a black barb and also a barb with a spot and then crossed the mongrel bard fantail with the mongrel barb spot and got "a bird of as beautiful a blue colour, with the white loins, double black wing-bar, and barred and white-edged tail-feathers, as any wild rock-pigeon!"; i.e. reversion, in both cases, did not occur until the third generation.

The facts so far observed which are in possible accordance with some form of Mendelian hypothesis are (1) the apparently regular appearance of albinos when hybrids are crossed with albinos, although the evidence at present available does not suffice to shew whether these occur in Mendelian proportions (50 °/₀ albino and 50 °/₀ hybrid) or not, (2) the well-known fact that albinos of any ancestry when paired together produce albino young, exceptions to this rule being at least very rare, and (3) the appearance of waltzing and albino mice in the second hybrid generation.

The first of these results although not inconsistent with the truth of Mendel's hypothesis cannot be taken as proof that this hypothesis applies: for a similar result is observed in such cases as that of human eye-colour where Mendel's Laws have been shewn not to hold†.

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* *The Origin of Species*, pp. 17, 18.

† Karl Pearson: see below, pp. 213 *et seq.*

Postscript, added Feb. 12, 1903.

The mice, resulting from a first cross between albinos and pink-eyed waltzing mice, are now 203, all the individuals having dark eyes, while none have wholly white fur, and none waltz. In the second generation, there are 66 mice produced by pairing hybrids; of these 13 are pink-eyed albinos, and 17 have pink eyes and coats more or less coloured. In the same generation there are 205 individuals produced by crossing albinos with hybrids; of these 111 are albinos, the remainder having dark eyes and some colour in the coat.

The proportions of albinos and of individuals not albino are not in disagreement with Mendel's results, but the exhibition of these results in Mendelian form depends on the adoption of a quite artificial category of coat-colours; for albinism includes only pure whiteness of coat, while coloured coats include the whole range of conditions from white with small patches of pale yellow to dark "wild colour" or black.

The inheritance of eye-colour is not in accordance with Mendel's results. For since pink eyes occur in particoloured mice, the possession of pink eyes must on Mendel's view depend on a separate embryonic element from that which determines coat-colour. Pink eyes are however not "dominant," since the two pink-eyed parents of the first generation always produce dark-eyed young. For the same reason pink eyes are not "recessive." Yet although pink eyes disappear in the first generation (the result of crossing two pink-eyed parents) they reappear in the second; but a correlation is then established between coat-colour and eye-colour which is strong in the offspring of hybrids paired together, and at present perfect in the offspring of hybrids and albinos. The behaviour of eye-colour is thus in every respect discordant with Mendel's results.

NEW TABLES OF THE PROBABILITY INTEGRAL[*].

By W. F. SHEPPARD, M.A., LL.M.

Description of the Tables.

1. The "probability integral" expresses the area of the normal curve, or curve of error, whose equation is

$$z = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \text{ .... } (1).$$

The abscissa x is measured from the central ordinate, about which the curve is symmetrical, and its unit of measurement is the standard deviation (square root of mean square of deviation). The whole area of the curve is unity. If this whole area be divided by the ordinate z, at distance x from the central ordinate, into portions $\frac{1}{2}(1+\alpha)$ and $\frac{1}{2}(1-\alpha)$, then

$$\tfrac{1}{2}(1+\alpha) = \int_{-\infty}^{x} z\,dx, \quad \tfrac{1}{2}(1-\alpha) = \int_{x}^{\infty} z\,dx \text{ .... } (2),$$

$$\alpha = 2\int_0^x z\,dx \text{ ....} (2\text{A}),$$

where z has the value given by (1).

Tables I. and II. give the values of z and of $\frac{1}{2}(1+\alpha)$ for any value of x from ·00 to 6·00; and therefore, the curve being symmetrical about the central ordinate, they enable us to determine the values of $\frac{1}{2}(1+\alpha)$, $\frac{1}{2}(1-\alpha)$, and z, for any value of x between $-6·00$ and $+6·00$. Tables III. and IV. give the values of x and of z for any value of α from ·00 to ·80; and therefore for any value of α between $-·80$ and $+·80$.

If X denotes the measurement of an organ common to a large number N of individuals, and if the different values of X are distributed according to the normal law about a mean value m with mean square of deviation σ^2, then the numbers of individuals whose measurements are respectively less and greater than any particular value X are N_1 and N_2, where

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[*] [In the prospectus of *Biometrika*, the Editors promised to provide "numerical tables tending to reduce the labour of statistical arithmetic." The first instalment of such tables by Mr Palin Elderton was given in Vol. I. This second instalment provides for the widely felt want of probability integral and normal curve data calculated on the basis of the *standard deviation* and not on that either of the modulus or of the probable error. Editors.]

$$N_1 = \tfrac{1}{2}N(1+\alpha), \quad N_2 = \tfrac{1}{2}N(1-\alpha),$$

and the ordinate of the figure of frequency corresponding to the measurement X is

$$Nz/\sigma,$$

where $\frac{1}{2}(1+\alpha)$ and z are the values of the area and of the ordinate given by Tables I. and II. as corresponding to

$$x = (X-m)/\sigma.$$

Method of Using.

2. *Interpolation.* If a quantity u be tabulated in terms of x, and u_0 be the value of u for $x = x_0$, then its value for $x = x_0 + \theta h$, where h is the common difference of x, and θ is less than 1, is usually found from u_0 and its advancing differences by the formula

$$u = u_0 + \theta\Delta u_0 - \frac{\theta(1-\theta)}{2\,!}\Delta^2 u_0 + \frac{\theta(1-\theta)(2-\theta)}{3\,!}\Delta^3 u_0 - \dots \dots\dots \quad (3).$$

An alternative formula, in which central differences are used, is that recently given* by Professor Everett. When we do not require to go beyond third differences, this formula is most conveniently written

$$u = u_0 + \theta\Delta u_0 - \frac{\theta(1-\theta^2)}{3\,!}\delta^2 u_1 - \frac{\phi(1-\phi^2)}{3\,!}\delta^2 u_0 \quad \dots\dots\dots\dots\dots(4),$$

where $\delta^2 u_1$ and $\delta^2 u_0$ are the second central differences of u_1 and u_0 (i.e. $\delta^2 u_1 = \Delta^2 u_0$, $\delta^2 u_0 = \Delta^2 u_{-1}$), and $\phi = 1-\theta$.

In practice, when second and higher differences have to be considered, and a mechanical method of performing multiplications is available (e.g. by means of a Brunsviga), it is more convenient to use differential coefficients. Let the means of the pairs of odd differences above and below the line of u_0 be denoted by $\mu\delta u_0$, $\mu\delta^3 u_0$, ... (i.e. $\mu\delta u_0 = \frac{1}{2}(\Delta u_0 + \Delta u_{-1})$, $\mu\delta^3 u_0 = \frac{1}{2}(\Delta^3 u_{-1} + \Delta^3 u_{-2})$, ...), and let Du_0, $D^2 u_0$, $D^3 u_0$, ... be the values for $x = x_0$ of the successive differential coefficients of u with regard to x; then†, δ^2 having the same meaning as in (4),

$$\left.\begin{aligned} hDu_0 &= \mu\delta u_0 - \tfrac{1}{6}\mu\delta^3 u_0 + \tfrac{1}{30}\mu\delta^5 u_0 - \dots \\ h^2D^2u_0 &= \delta^2 u_0 - \tfrac{1}{12}\delta^4 u_0 + \dots \\ h^3D^3u_0 &= \mu\delta^3 u_0 - \tfrac{1}{4}\mu\delta^5 u_0 + \dots \\ h^4D^4u_0 &= \delta^4 u_0 - \dots \\ &\text{etc.} \end{aligned}\right\} \quad \dots\dots\dots\dots\dots\dots(5),$$

and, for $x = x_0 + \theta h$,

$$u = u_0 + \theta\left[hDu_0 + \tfrac{1}{2}\theta\left\{h^2D^2u_0 + \tfrac{1}{3}\theta\left(h^3D^3u_0 + \dots\right)\right\}\right] \dots\dots\dots\dots\dots(6).$$

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* *Journal of the Institute of Actuaries*, Vol. xxxv. p. 452.

† *Proc. of the Lond. Math. Soc.* Vol. xxxi. p. 465.

We need only use inside the square brackets the numerical values of θ and of hDu_0, $h^2D^2u_0$, ..., if we put the sign of $\theta . hDu_0$ outside, and follow the rule of signs inside. The rule is that a *minus* sign must be inserted, if θ be positive, whenever there is a change of sign in the series hDu_0, $h^2D^2u_0$, ..., or, if θ be negative, whenever there is no change of sign in this series.

Thus, in Table I., taking u to be $\frac{1}{2}(1+\alpha)$, we have for $x_0 = \cdot 40$ (omitting decimal point)

$$\begin{array}{ccc} \mu\delta u_0 & \delta^2 u_0 & \mu\delta^3 u_0 \\ 36826\frac{1}{2} & -147 & -3 \\ hDu_0 & h^2D^2u_0 & h^3D^3u_0 \\ 36827 & -147 & -3 \end{array}$$

and therefore, for $x = \cdot 40 + \cdot 01\theta$,

$$\tfrac{1}{2}(1+\alpha) = 10^{-7}(6554217 + \theta\,[36827 - \tfrac{1}{2}\theta\,\{147 + \tfrac{1}{3}\theta \,.\, 3\}]).$$

For $x = \cdot 40 - \cdot 01\theta$ we should replace $+\theta$, $-\frac{1}{2}\theta$, $+\frac{1}{3}\theta$, by $-\theta$, $+\frac{1}{2}\theta$, $-\frac{1}{3}\theta$. The formulæ given by (3) and (4) would be, for $x = \cdot 40 + \cdot 01\theta$,

$$\tfrac{1}{2}(1+\alpha) = 10^{-7}\left(6554217 + 36753\theta + 150\,\frac{\theta(1-\theta)}{2} - 3\,\frac{\theta(1-\theta)(2-\theta)}{6}\right),$$

$$\tfrac{1}{2}(1+\alpha) = 10^{-7}\left(6554217 + 36753\theta + 150\,\frac{\theta(1-\theta^2)}{6} + 147\,\frac{\phi(1-\phi^2)}{6}\right).$$

There is no difficulty about the divisions represented by the coefficients $\frac{1}{2}$, $\frac{1}{3}$, ... of θ in (6); but, if we wish to avoid them, we may calculate $h^2D^2u_0/2\,!$, $h^3D^3u_0/3\,!$, ... and write the formula

$$u = h_0 + \theta\,[hDu_0 + \theta\,\{\tfrac{1}{2}h^2D^2u_0 + \theta\,(\tfrac{1}{6}h^3D^3u_0 + \ldots)\}] \quad \ldots\ldots\ldots\ldots\ldots(7).$$

3. *Inverse Interpolation.* In most cases N_1 and $N_2\,[=\frac{1}{2}N(1+\alpha)$ and $\frac{1}{2}N(1-\alpha)]$ are known, and we require x. If $\frac{1}{2}(1+\alpha)$ and $\frac{1}{2}(1-\alpha)$ are both less than $\cdot 90$, we can use Table III., which gives x in terms of $\alpha = (N_1 - N_2)/N$. But, if either $\frac{1}{2}(1+\alpha)$ or $\frac{1}{2}(1-\alpha)$ be greater than $\cdot 90$, we must use Table I., by inverse interpolation. By (6) we have, if $x = x_0 + \theta h$,

$$\theta = (u - u_0) \div [hDu_0 + \tfrac{1}{2}\theta\,\{h^2D^2u_0 + \tfrac{1}{3}\theta\,(h^3D^3u_0 + \ldots)\}] \quad \ldots\ldots\ldots\ldots\ldots(8),$$

or, if $x = x_0 - \theta h$,

$$\theta = (u_0 - u) \div [hDu_0 - \tfrac{1}{2}\theta\,\{h^2D^2u_0 - \tfrac{1}{3}\theta\,(h^3D^3u_0 - \ldots)\}] \quad \ldots\ldots\ldots\ldots(8\text{A}).$$

The value of θ, and thence that of x, is obtained by successive approximations.

Suppose, for instance, that

$$\tfrac{1}{2}(1+\alpha) = \cdot 654.$$

If $x = \cdot 40 - \cdot 01\theta$, we have from Table I.

$$\theta = 14217 \div [36827 + \tfrac{1}{2}\theta\,\{147 - \theta\}].$$

A first approximation gives

$$\theta = 14217 \div 36827 = \cdot 38605\,;$$

and, with this value, the corrected divisor becomes

$$36855 \cdot 2,$$

which gives for a second approximation

$$\theta = \cdot 38575,$$

and therefore

$$x = \cdot 3961425.$$

The correct value, as given by Table III., is

$$x = \cdot 3961424.$$

The degree of accuracy with which x can be obtained by this method depends on the relation of magnitude between the differences of x and of u. In the above example, to a difference of ·01 in x there corresponds a difference of very little more than one-third of ·01 in $\frac{1}{2}(1+\alpha)$; and therefore, if x is calculated from Table I. to seven places of decimals, it will only be accurate within about 2 in the last figure. The possible inaccuracy of x increases as $\frac{1}{2}(1+\alpha)$ increases. But this is not important, as the "probable error" of x, for any given number of observations, also increases.

4. *Smoothing.* In arranging a table, with differences, for the calculation of any quantity v, it is usual to enter in the difference-columns the actual (or "tabular") differences of the values of u as tabulated. In the present tables I have adopted a different method, and have given the differences as near as possible to the differences of the true values of u. The object of this is to enable greater accuracy to be obtained when required. If we only want u to five or six places of decimals, it is immaterial whether we use the tabular or the corrected differences. But, if we wish to have it as accurate as possible, we can alter the tabulated values by inspection.

Looking, for instance, at the commencement of Table I., it is clear that the tabulated values of $\frac{1}{2}(1+a)$ are too great for $x = \cdot 01$ and $x = \cdot 03$, while they are too small for $x = \cdot 02$ and $x = \cdot 04$. Taking $u = \frac{1}{2}(1+a) \times 10^7$, so as to omit the decimal point, the table may be written

x	u	Δ $+$	Δ^2 $-$
$\cdot 00$	5000000		0
		$39894 - \theta$	
$\cdot 01$	$5039894 - \theta$		$4 + (1 - 2\theta - \phi)$
		$39890 - (1 - \theta - \phi)$	
$\cdot 02$	$5079783 + \phi$		$8 - (1 - \theta - 2\phi - \chi)$
		$39882 - (\phi + \chi)$	
$\cdot 03$	$5119665 - \chi$		$12 + (1 - \phi - 2\chi - \psi)$
		$39870 - (1 - \chi - \psi)$	
$\cdot 04$	$5159534 + \psi$	$\vdots$	$\vdots$
$\vdots$	$\vdots$		

By tabulating u by differences of ·05 or ·10 in x, it will be found* that the third difference in Table I., for these values of x, is almost exactly 4. We see therefore that $\theta + \phi$ and $\chi + \psi$ are both greater than $\frac{1}{2}$, and $\phi + \chi$ is less than $\frac{1}{2}$; while θ, $1 - \theta - \phi$, $\phi + \chi$, and $1 - \chi - \psi$ are all very nearly equal. The values $\theta = \cdot 4$, $\phi = \cdot 2$, $\chi = \cdot 2$, $\psi = \cdot 4$ satisfy these conditions; and, as a matter of fact, they give for $\frac{1}{2}(1+a)$ values which are correct within 1×10^{-8}.

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* For the relations between differences of u for large and small differences in x, see *Proc. Lond. Math. Soc.* Vol. XXXI. pp. 468—471.

Examples of Application.

5. *Expression of data in terms of x.* When a distribution is nearly normal, we may state the data by expressing X (see § 1) in terms of x. For an example, take the head-breadths of 3000 criminals, given on p. 214 of *Biometrika*, Vol. I. The interval in X is ·1 of a centimetre: but, for brevity, we shall take intervals of ·3 of a centimetre.

We should first note the probable errors. If, of the N values, N_1 lie below X and N_2 above it, the probable error in N_1 or N_2 is $\pm Q\sqrt{N_1N_2/N}$, where $Q = {\cdot}67449$; i.e. it is an even chance that N times the true proportion of values below X lies between $N_1 - Q\sqrt{N_1N_2/N}$ and $N_1 + Q\sqrt{N_1N_2/N}$. Thus, for $X = 14{\cdot}55$ cm., the P. E. is $\pm Q\sqrt{479 \times 2521 \div 3000} = 13{\cdot}5$. Calculating the probable errors, the data may be expressed thus:—

X	N_1	N_2	P. E.	X	N_1	N_2	P. E.
13·35	0	3000	—	*15·45*	2364	636	±15·1
13·65	8	2992	± 1·9	*15·75*	2763	237	±10·0
13·95	38	2962	± 4·1	*16·05*	2925	75	± 5·8
14·25	157	2843	± 8·2	*16·35*	2978	22	± 3·2
14·55	479	2521	±13·5	*16·65*	2997	3	± 1·2
14·85	1077	1923	±17·7	*16·95*	3000	0	—
15·15	1762	1238	±18·2				

Now calculate the values of x and of z corresponding to $N_1 = \frac{1}{2}N(1 + \alpha)$. An error of θ in x is equivalent to an error of $z\theta$ in $\frac{1}{2}(1 + \alpha)$, so that the above values of the P. E. have to be divided by $3000z$ to give the P. E. in x. We thus get the data in the form:—

X	x	P. E.	X	x	P. E.
13·35	$-\infty$	—	*15·45*	+ ·800	±·017
13·65	−2·786	±·077	*15·75*	+1·412	±·023
13·95	−2·236	±·042	*16·05*	+1·960	±·033
14·25	−1·625	±·020	*16·35*	+2·441	±·052
14·55	− ·996	±·019	*16·65*	+3·090	±·135
14·85	− ·361	±·016	*16·95*	$+\infty$	—
15·15	+ ·221	±·016			

This, it should be observed, is merely a statement of facts, and does not involve any assumption as to the distribution being really normal.

6. *Interpolation**. By means of these values of x, we can interpolate for values of X lying towards the extremities of the range, where the differences

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* For a fuller discussion of the methods employed in this and the next two sections, see *Journal of the Royal Statistical Society*, Vol. LXIII, pp. 433–451.

of X_1 or X_2 are usually irregular. Thus, calculating the values of x for $X = 14·05$ and 14·15 by means of the first difference alone, and thence calculating $\frac{1}{2}(1+\alpha)$ from Table I. (remembering that x and α are negative), we get the following results, as compared with the actual observations:—

X	x	$\frac{1}{2}(1-\alpha)$	$\frac{1}{2}(1+\alpha)$	$X_1 = \frac{1}{2}X(1+\alpha)$	
				Calculated value	Actual value
14·05	−2·032	·9789	·0211	63	61
14·15	−1·829	·9663	·0337	101	97

It is quite possible that the discrepancy between the calculated and the actual values is mainly due to the errors of random selection of the 118 individuals lying between $X = 13·95$ and $X = 14·25$.

7. *Certain Special Cases.* The method is especially useful (a) where the differences in X are irregular or are large in comparison with the standard deviation, and (b) in dealing with the "arrays" in cases of normal or nearly normal correlation. As an example of the former, Prof. Pearson has provided me with the following results obtained by Miss C. D. Fawcett.

Mottling of Mimulus Luteus.

X = Number of splotches	Less than 50	50 to 61	62 to 71	More than 71	Total
Number of individuals	18	43	87	56	204

This gives three values of x in terms of X, viz:—

X	$49\frac{1}{2}$	$61\frac{1}{2}$	$71\frac{1}{2}$
x	$-1·352 \pm ·084$	$-·527 \pm ·062$	$+·599 \pm ·063$

The differences in x are ·825 and 1·126, whereas the differences in X are in the ratio of 6 : 5. Having regard to the probable errors, it is very doubtful whether the distribution can be treated as normal. If it can be, the true values of x may be somewhat as follows:—

X	$49\frac{1}{2}$	$61\frac{1}{2}$	$71\frac{1}{2}$
x (corrected)	−1·503	−·417	+·488
Ratio of correction to P. E.	−1·8	+1·8	−1·8

These would give a mean of 66·61, and a standard deviation of 11·05, the probable errors being respectively ± ·52 and ± ·37.

This, of course, is guesswork. If there are more than three or four values of x to be dealt with, we can do our guessing by graphic methods. The values of x should be plotted as ordinates, with a mark on each side to shew the probable error: and we have then to draw a straight line which shall make the error as small as possible, allowance being made for the different values of the probable error in the different ordinates so plotted. It must be remembered that the errors are not independent, but correlated: the correlation between the errors in any pair of values of x being positive.

If we want to proceed more rigidly, we must use the actual values of x, and take account of differences. This is practically equivalent to regarding X as an unknown function of some other quantity Y, whose values are normally distributed; the relation between X and Y being such that dX/dY is always positive.

8. *Calculation of ordinates.* The ordinate Z of the curve of frequency is dN_1/dX. In most cases this can be calculated directly from the data, by the first formula of (5) (taking $u=N_1$) or a similar formula. Where the differences in u are too irregular for this, but the differences in x are comparatively regular, we can use the data in the form shewn in § 5: we have then

$$Z=N\,.\,d\tfrac{1}{2}(1+a)/dx\,.\,dx/dX=Nz\,.\,dx/dX.$$

The values of dx/dX are given by the data, and the values of z are found from Table II.

9. *Testing for normal distribution.* This is the purpose for which tables such as Table I. are most frequently employed. It is not necessary to give any examples here.

10. *Calculation of correlation-volumes.* The formula, given by Professor Pearson*, for calculating the double-integral expressing normal correlation, involves a factor

$$\frac{1}{2\pi}e^{-\frac{1}{2}(x^2+y^2)},$$

which is the product of the values of z as given by Table II. for x and for y respectively, and therefore is easily found from that Table.

Construction of the Tables.

11. The tabulated values are all given to seven decimal places (and, in the latter part of Tables I. and II., to ten decimal places). They were originally calculated to two or three more places, the final figures being then corrected. Where the final figure was doubtful, the value was calculated specially. The differences are taken from the larger table, the last figure being corrected: but doubtful values were not specially calculated.

For constructing Tables I. and II. up to $x=2{\cdot}50$, the values of z were found for the intermediate values ·005, ·015, ·025... of x by successive multiplication; each tenth value being checked by Newman's table† of $e^{-\theta}$. The differences being then taken, the values of $\frac{1}{2}(1+a)$ for

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* *Phil. Trans.* series A, Vol. 195, pp. 1—47.

† *Camb. Phil. Soc. Trans.* Vol. XIII. Pt. 3.

$x = \cdot 01, \cdot 02 \ldots$ were obtained by quadrature*, and those of z by interpolation. For the remainder of the two tables, the values of z were found from those of $\log_{10} z$, which are easily calculated, and thence the values of $\frac{1}{2}(1+\alpha)$ were obtained by quadrature. For checking the table of $\frac{1}{2}(1+\alpha)$, values were directly calculated at intervals†: and both tables were further checked by the calculations required where a final figure was doubtful.

For constructing Tables III. and IV., the values were first obtained approximately to seven places: and the tables were then extended, by a method explained elsewhere‡. The extension gave z to nine and x to eleven places. The table was checked by direct calculation for $\alpha = \cdot 1, \cdot 2, \cdot 3 \ldots$.

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* See *Proc. of Lond. Math. Soc.* Vol. XXXI. pp. 179—182.

† Some use was also made of Burgess's tables (*Trans. Roy. Soc. Edin.*, Vol. XXXIX., Pt. 2, No. 9), in which α is given (to a large number of figures) in terms of $t \equiv x/\sqrt{2}$. But they were only used incidentally and the two sets of tables may be regarded as independently calculated.

‡ *Proc. of Lond. Math. Soc.* Vol. XXXI. pp. 423, 439.

TABLES I. AND II. *Area and Ordinate in terms of Abscissa.*

Note.

For values of the abscissa x from $\cdot 00$ to $4 \cdot 50$, the values of the area $\frac{1}{2}(1+\alpha)$ and of the ordinate z are given to 7 decimal places (pp. 182–7). For values of x from $4 \cdot 50$ to $6 \cdot 00$, the values of $\frac{1}{2}(1+\alpha)$ and of z are given to ten decimal places (p. 188), but the initial figures are omitted. Hence, in using this latter portion of the tables the figures in the column for $\frac{1}{2}(1+\alpha)$ must have $\cdot 99999$ prefixed, and those in the column for z must have sufficient zeros prefixed to bring up the total of decimal figures to ten. For example, against $x = 5 \cdot 75$ we have 99955 and 264, but we must read $\frac{1}{2}(1+\alpha) = \cdot 9999999955$ and $z = \cdot 0000000264$.

TABLES I. AND II. *Area and Ordinate in terms of Abscissa.*

x	$\frac{1}{2}(1+\alpha)$	Δ +	Δ^2 −	z	Δ −	Δ^2 −	x	$\frac{1}{2}(1+\alpha)$	Δ +	Δ^2 −
·00	·5000000	39894	0	·3989423	199	399	·50	·6914625	35118	176
·01	·5039894	39890	4	·3989223	598	399	·51	·6949743	34939	179
·02	·5079783	39882	8	·3988625	997	399	·52	·6984682	34758	181
·03	·5119665	39870	12	·3987628	1395	398	·53	·7019440	34574	184
·04	·5159534	39854	16	·3986233	1793	398	·54	·7054015	34388	186
·05	·5199388	39834	20	·3984439	2191	397	·55	·7088403	34200	189
·06	·5239222	39810	24	·3982248	2588	397	·56	·7122603	34009	191
·07	·5279032	39782	28	·3979661	2984	396	·57	·7156612	33815	193
·08	·5318814	39750	32	·3976677	3379	395	·58	·7190427	33620	196
·09	·5358564	39714	36	·3973298	3773	394	·59	·7224047	33422	198
·10	·5398278	39675	40	·3969525	4166	393	·60	·7257469	33222	200
·11	·5437953	39631	44	·3965360	4558	392	·61	·7290691	33020	202
·12	·5477584	39584	48	·3960802	4948	390	·62	·7323711	32816	204
·13	·5517168	39532	51	·3955854	5337	389	·63	·7356527	32610	206
·14	·5556700	39477	55	·3950517	5724	387	·64	·7389137	32402	208
·15	·5596177	39418	59	·3944793	6110	386	·65	·7421539	32192	210
·16	·5635595	39355	63	·3938684	6493	384	·66	·7453731	31980	212
·17	·5674949	39288	67	·3932190	6875	382	·67	·7485711	31767	214
·18	·5714237	39217	71	·3925315	7255	380	·68	·7517478	31551	215
·19	·5753454	39143	74	·3918060	7633	378	·69	·7549029	31334	217
·20	·5792597	39065	78	·3910427	8008	375	·70	·7580363	31116	219
·21	·5831662	38983	82	·3902419	8381	373	·71	·7611479	30896	220
·22	·5870644	38897	86	·3894038	8752	371	·72	·7642375	30674	222
·23	·5909541	38808	89	·3885286	9120	368	·73	·7673049	30451	223
·24	·5948349	38715	93	·3876166	9485	365	·74	·7703500	30226	225
·25	·5987063	38618	97	·3866681	9847	362	·75	·7733726	30001	226
·26	·6025681	38518	100	·3856834	10207	360	·76	·7763727	29773	227
·27	·6064199	38414	104	·3846627	10564	357	·77	·7793501	29545	228
·28	·6102612	38306	107	·3836063	10917	354	·78	·7823046	29316	230
·29	·6140919	38195	111	·3825146	11268	350	·79	·7852361	29085	231
·30	·6179114	38081	114	·3813878	11615	347	·80	·7881446	28853	232
·31	·6217195	37963	118	·3802264	11958	344	·81	·7910299	28620	233
·32	·6255158	37842	121	·3790305	12298	340	·82	·7938919	28387	234
·33	·6293000	37717	125	·3778007	12635	337	·83	·7967306	28152	235
·34	·6330717	37589	128	·3765372	12968	333	·84	·7995458	27917	235
·35	·6368307	37458	131	·3752403	13297	329	·85	·8023375	27680	236
·36	·6405764	37323	135	·3739106	13623	325	·86	·8051055	27443	237
·37	·6443088	37185	138	·3725483	13944	322	·87	·8078498	27205	238
·38	·6480273	37044	141	·3711539	14262	318	·88	·8105703	26967	238
·39	·6517317	36900	144	·3697277	14575	313	·89	·8132671	26728	239
·40	·6554217	36753	147	·3682701	14885	309	·90	·8159399	26489	239
·41	·6590970	36602	150	·3667817	15190	305	·91	·8185887	26249	240
·42	·6627573	36449	153	·3652627	15491	301	·92	·8212136	26008	240
·43	·6664022	36293	156	·3637136	15787	296	·93	·8238145	25768	241
·44	·6700314	36133	159	·3621349	16079	292	·94	·8263912	25527	241
·45	·6736448	35971	162	·3605270	16367	288	·95	·8289439	25285	241
·46	·6772419	35806	165	·3588903	16650	283	·96	·8314724	25044	242
·47	·6808225	35638	168	·3572253	16928	278	·97	·8339768	24802	242
·48	·6843863	35467	171	·3555325	17202	274	·98	·8364569	24560	242
·49	·6879331	35294	173	·3538124	17470	269	·99	·8389129	24318	242
·50	·6914625		176	·3520653		264	1·00	·8413447		242

TABLES I. AND II.—(*continued*).

z	Δ −	Δ² −	x	½(1+α)	Δ +	Δ² −	z	Δ −	Δ² +
·3520653	17734	264	*1·00*	·8413447	24076	242	·2419707	24196	0
·3502919	17994	259	*1·01*	·8437524	23834	242	·2395511	24191	5
·3484925	18248	254	*1·02*	·8461358	23592	242	·2371320	24182	10
·3466677	18497	249	*1·03*	·8484950	23351	242	·2347138	24168	14
·3448180	18741	244	*1·04*	·8508300	23109	242	·2322970	24149	19
·3429439	18981	239	*1·05*	·8531409	22868	241	·2298821	24125	24
·3410458	19215	234	*1·06*	·8554277	22626	241	·2274696	24097	28
·3391243	19444	229	*1·07*	·8576903	22386	241	·2250599	24064	33
·3371799	19667	224	*1·08*	·8599289	22145	240	·2226535	24027	37
·3352132	19886	219	*1·09*	·8621434	21905	240	·2202508	23986	41
·3332246	20099	213	*1·10*	·8643339	21665	240	·2178522	23940	46
·3312147	20307	208	*1·11*	·8665005	21426	239	·2154582	23890	50
·3291840	20510	203	*1·12*	·8686431	21188	239	·2130691	23836	54
·3271330	20707	197	*1·13*	·8707619	20950	238	·2106856	23778	58
·3250623	20899	192	*1·14*	·8728568	20712	237	·2083078	23715	62
·3229724	21086	187	*1·15*	·8749281	20475	237	·2059363	23649	66
·3208638	21267	181	*1·16*	·8769756	20239	236	·2035714	23578	70
·3187371	21442	176	*1·17*	·8789995	20004	235	·2012135	23504	74
·3165929	21613	170	*1·18*	·8809999	19769	235	·1988631	23426	78
·3144317	21777	165	*1·19*	·8829768	19535	234	·1965205	23344	82
·3122539	21936	159	*1·20*	·8849303	19302	233	·1941861	23259	85
·3100603	22090	154	*1·21*	·8868606	19070	232	·1918602	23170	89
·3078513	22239	148	*1·22*	·8887676	18839	231	·1895432	23077	93
·3056274	22381	143	*1·23*	·8906514	18609	230	·1872354	22981	96
·3033893	22519	137	*1·24*	·8925123	18379	229	·1849373	22882	99
·3011374	22650	132	*1·25*	·8943502	18151	228	·1826491	22779	103
·2988724	22777	126	*1·26*	·8961653	17924	227	·1803712	22673	106
·2965948	22897	121	*1·27*	·8979577	17697	226	·1781038	22564	109
·2943050	23013	115	*1·28*	·8997274	17472	225	·1758471	22452	112
·2920038	23122	110	*1·29*	·9014747	17248	224	·1736022	22337	115
·2896916	23227	104	*1·30*	·9031995	17026	223	·1713686	22218	118
·2873689	23325	99	*1·31*	·9049021	16804	222	·1691468	22097	121
·2850364	23419	93	*1·32*	·9065825	16584	220	·1669370	21973	124
·2826945	23507	88	*1·33*	·9082409	16365	219	·1647397	21847	127
·2803438	23589	83	*1·34*	·9098773	16147	218	·1625551	21717	129
·2779849	23666	77	*1·35*	·9114920	15930	217	·1603833	21585	132
·2756182	23738	72	*1·36*	·9130850	15715	215	·1582248	21451	134
·2732444	23805	66	*1·37*	·9146565	15501	214	·1560797	21314	137
·2708640	23866	61	*1·38*	·9162067	15289	212	·1539483	21175	139
·2684774	23922	56	*1·39*	·9177356	15078	211	·1518308	21033	142
·2660852	23972	51	*1·40*	·9192433	14868	210	·1497275	20890	144
·2636880	24017	45	*1·41*	·9207302	14660	208	·1476385	20744	146
·2612863	24058	40	*1·42*	·9221962	14453	207	·1455641	20596	148
·2588805	24093	35	*1·43*	·9236415	14248	205	·1435046	20446	150
·2564713	24122	30	*1·44*	·9250663	14044	204	·1414600	20294	152
·2540591	24147	25	*1·45*	·9264707	13842	202	·1394306	20140	154
·2516443	24167	20	*1·46*	·9278550	13642	201	·1374165	19985	155
·2492277	24182	15	*1·47*	·9292191	13443	199	·1354181	19828	157
·2468095	24194	10	*1·48*	·9305634	13245	197	·1334353	19669	159
·2443904	24196	5	*1·49*	·9318879	13049	196	·1314684	19508	160
·2419707		0	*1·50*	·9331928		194	·1295176		162

TABLES I. AND II.—(*continued*).

x	$\frac{1}{2}(1+\alpha)$	Δ +	Δ^2 −	z	Δ −	Δ^2 +	x	$\frac{1}{2}(1+\alpha)$	Δ +	Δ^2 −
1·50	·9331928	12855	194	·1295176	19346	162	*2·00*	·9772499	5345	108
1·51	·9344783	12662	193	·1275830	19183	163	*2·01*	·9777844	5239	106
1·52	·9357445	12471	191	·1256646	19018	165	*2·02*	·9783083	5134	105
1·53	·9369916	12282	189	·1237628	18853	166	*2·03*	·9788217	5031	103
1·54	·9382198	12094	188	·1218775	18685	167	*2·04*	·9793248	4929	102
1·55	·9394292	11908	186	·1200090	18517	168	*2·05*	·9798178	4829	100
1·56	·9406201	11724	184	·1181573	18348	169	*2·06*	·9803007	4731	98
1·57	·9417924	11541	183	·1163225	18177	170	*2·07*	·9807738	4634	97
1·58	·9429466	11360	181	·1145048	18006	171	*2·08*	·9812372	4539	95
1·59	·9440826	11181	179	·1127042	17834	172	*2·09*	·9816911	4445	94
1·60	·9452007	11004	177	·1109208	17661	173	*2·10*	·9821356	4352	92
1·61	·9463011	10828	176	·1091548	17487	174	*2·11*	·9825708	4262	91
1·62	·9473839	10654	174	·1074061	17312	174	*2·12*	·9829970	4172	89
1·63	·9484493	10482	172	·1056748	17137	175	*2·13*	·9834142	4084	88
1·64	·9494974	10311	170	·1039611	16962	176	*2·14*	·9838226	3998	86
1·65	·9505285	10142	169	·1022649	16786	176	*2·15*	·9842224	3913	85
1·66	·9515428	9975	167	·1005864	16609	177	*2·16*	·9846137	3829	84
1·67	·9525403	9810	165	·0989255	16432	177	*2·17*	·9849966	3747	82
1·68	·9535213	9647	163	·0972823	16255	177	*2·18*	·9853713	3666	81
1·69	·9544860	9485	162	·0956568	16077	178	*2·19*	·9857379	3587	79
1·70	·9554345	9325	160	·0940491	15899	178	*2·20*	·9860966	3509	78
1·71	·9563671	9167	158	·0924591	15722	178	*2·21*	·9864474	3432	77
1·72	·9572838	9011	156	·0908870	15544	178	*2·22*	·9867906	3357	75
1·73	·9581849	8856	155	·0893326	15366	178	*2·23*	·9871263	3283	74
1·74	·9590705	8704	153	·0877961	15188	178	*2·24*	·9874545	3210	73
1·75	·9599408	8553	151	·0862773	15010	178	*2·25*	·9877755	3138	71
1·76	·9607961	8403	149	·0847764	14832	178	*2·26*	·9880894	3068	70
1·77	·9616364	8256	147	·0832932	14654	178	*2·27*	·9883962	2999	69
1·78	·9624620	8110	146	·0818278	14477	177	*2·28*	·9886962	2932	68
1·79	·9632730	7966	144	·0803801	14300	177	*2·29*	·9889893	2865	66
1·80	·9640697	7824	142	·0789502	14123	177	*2·30*	·9892759	2800	65
1·81	·9648521	7684	140	·0775379	13946	176	*2·31*	·9895559	2736	64
1·82	·9656205	7545	139	·0761433	13770	176	*2·32*	·9898296	2674	63
1·83	·9663750	7409	137	·0747663	13594	176	*2·33*	·9900969	2612	62
1·84	·9671159	7273	135	·0734068	13419	175	*2·34*	·9903581	2552	60
1·85	·9678432	7140	133	·0720649	13245	175	*2·35*	·9906133	2492	59
1·86	·9685572	7009	132	·0707404	13071	174	*2·36*	·9908625	2434	58
1·87	·9692581	6879	130	·0694333	12897	173	*2·37*	·9911060	2377	57
1·88	·9699460	6751	128	·0681436	12725	173	*2·38*	·9913437	2321	56
1·89	·9706210	6624	126	·0668711	12553	172	*2·39*	·9915758	2267	55
1·90	·9712834	6500	125	·0656158	12382	171	*2·40*	·9918025	2213	54
1·91	·9719334	6377	123	·0643777	12211	170	*2·41*	·9920237	2160	53
1·92	·9725711	6255	121	·0631566	12041	170	*2·42*	·9922397	2108	52
1·93	·9731966	6136	120	·0619524	11873	169	*2·43*	·9924506	2058	51
1·94	·9738102	6018	118	·0607652	11705	168	*2·44*	·9926564	2008	50
1·95	·9744119	5902	116	·0595947	11538	167	*2·45*	·9928572	1960	49
1·96	·9750021	5787	115	·0584409	11372	166	*2·46*	·9930531	1912	48
1·97	·9755808	5674	113	·0573038	11206	165	*2·47*	·9932443	1865	47
1·98	·9761482	5563	111	·0561831	11042	164	*2·48*	·9934309	1820	46
1·99	·9767045	5453	110	·0550789	10879	163	*2·49*	·9936128	1775	45
2·00	·9772499		108	·0539910		162	*2·50*	·9937903		44

TABLES I. AND II.—(*continued*).

z	Δ	Δ^2 +	x	$\frac{1}{2}(1+\alpha)$	Δ +	Δ^2 −	z	Δ −	Δ^2 +
·0539910	10717	162	2·50	·9937903	1731	44	·0175283	4336	92
·0529192	10557	161	2·51	·9939634	1688	43	·0170947	4246	91
·0518636	10397	160	2·52	·9941323	1646	42	·0166701	4157	89
·0508239	10238	159	2·53	·9942969	1605	41	·0162545	4069	88
·0498001	10081	157	2·54	·9944571	1565	40	·0158476	3982	86
·0487920	9924	156	2·55	·9946139	1525	39	·0154493	3897	85
·0477996	9769	155	2·56	·9947664	1487	39	·0150596	3814	84
·0468226	9616	154	2·57	·9949151	1449	38	·0146782	3731	82
·0458611	9463	153	2·58	·9950600	1412	37	·0143051	3650	81
·0449148	9312	151	2·59	·9952012	1376	36	·0139401	3571	80
·0439836	9162	150	2·60	·9953388	1341	35	·0135830	3493	78
·0430674	9013	149	2·61	·9954729	1306	35	·0132337	3416	77
·0421661	8866	147	2·62	·9956035	1272	34	·0128921	3340	76
·0412795	8720	146	2·63	·9957308	1239	33	·0125581	3266	74
·0404076	8575	145	2·64	·9958547	1207	32	·0122315	3193	73
·0395500	8432	143	2·65	·9959754	1176	32	·0119122	3121	72
·0387069	8290	142	2·66	·9960930	1145	31	·0116001	3051	70
·0378779	8149	140	2·67	·9962074	1115	30	·0112951	2981	69
·0370629	8010	139	2·68	·9963189	1085	29	·0109969	2913	68
·0362619	7873	138	2·69	·9964274	1056	29	·0107056	2847	67
·0354746	7737	136	2·70	·9965330	1028	28	·0104209	2781	66
·0347009	7602	135	2·71	·9966358	1001	27	·0101428	2717	64
·0339408	7468	133	2·72	·9967359	974	27	·0098712	2654	63
·0331939	7337	132	2·73	·9968333	948	26	·0096058	2592	62
·0324603	7206	130	2·74	·9969280	922	26	·0093466	2531	61
·0317397	7077	129	2·75	·9970202	897	25	·0090936	2471	60
·0310319	6950	127	2·76	·9971099	873	24	·0088465	2413	59
·0303370	6824	126	2·77	·9971972	849	24	·0086052	2355	57
·0296546	6699	125	2·78	·9972821	825	23	·0083697	2299	56
·0289847	6576	123	2·79	·9973646	803	23	·0081398	2244	55
·0283270	6455	122	2·80	·9974449	781	22	·0079155	2189	54
·0276816	6335	120	2·81	·9975229	759	22	·0076965	2136	53
·0270481	6216	119	2·82	·9975988	738	21	·0074829	2084	52
·0264265	6099	117	2·83	·9976726	717	21	·0072744	2033	51
·0258166	5984	116	2·84	·9977443	697	20	·0070711	1983	50
·0252182	5870	114	2·85	·9978140	678	20	·0068728	1934	49
·0246313	5757	113	2·86	·9978818	658	19	·0066793	1886	48
·0240556	5646	111	2·87	·9979476	640	19	·0064907	1839	47
·0234910	5536	110	2·88	·9980116	622	18	·0063067	1793	46
·0229374	5428	108	2·89	·9980738	604	18	·0061274	1748	45
·0223945	5322	107	2·90	·9981342	587	17	·0059525	1704	44
·0218624	5217	105	2·91	·9981929	570	17	·0057821	1661	43
·0213407	5113	104	2·92	·9982498	553	16	·0056160	1619	42
·0208294	5011	102	2·93	·9983052	537	16	·0054541	1578	41
·0203284	4910	101	2·94	·9983589	522	16	·0052963	1537	40
·0198374	4811	99	2·95	·9984111	507	15	·0051426	1497	40
·0193563	4713	98	2·96	·9984618	492	15	·0049929	1459	39
·0188850	4617	96	2·97	·9985110	478	14	·0048470	1421	38
·0184233	4522	95	2·98	·9985588	464	14	·0047050	1384	37
·0179711	4428	93	2·99	·9986051	450	14	·0045666	1347	36
·0175283		92	3·00	·9986501		13	·0044318		35

TABLES I. AND II.—(*continued*).

x	$\frac{1}{2}(1+a)$	Δ +	Δ² −	*z*	Δ −	Δ² +
3·00	·9986501	437	13	·0044318	1312	35
3·01	·9986938	424	13	·0043007	1277	35
3·02	·9987361	411	13	·0041729	1243	34
3·03	·9987772	399	12	·0040486	1210	33
3·04	·9988171	387	12	·0039276	1178	32
3·05	·9988558	375	12	·0038098	1146	32
3·06	·9988933	364	11	·0036951	1115	31
3·07	·9989297	353	11	·0035836	1085	30
3·08	·9989650	342	11	·0034751	1056	29
3·09	·9989992	332	10	·0033695	1027	29
3·10	·9990324	322	10	·0032668	999	28
3·11	·9990646	312	10	·0031669	971	27
3·12	·9990957	302	10	·0030698	944	27
3·13	·9991260	293	9	·0029754	918	26
3·14	·9991553	284	9	·0028835	893	26
3·15	·9991836	275	9	·0027943	868	25
3·16	·9992112	267	9	·0027075	843	24
3·17	·9992378	258	8	·0026231	820	24
3·18	·9992636	250	8	·0025412	797	23
3·19	·9992886	242	8	·0024615	774	23
3·20	·9993129	235	8	·0023841	752	22
3·21	·9993363	227	7	·0023089	731	21
3·22	·9993590	220	7	·0022358	710	21
3·23	·9993810	213	7	·0021649	689	20
3·24	·9994024	206	7	·0020960	669	20
3·25	·9994230	200	7	·0020290	650	19
3·26	·9994429	193	6	·0019641	631	19
3·27	·9994623	187	6	·0019010	612	18
3·28	·9994810	181	6	·0018397	595	18
3·29	·9994991	175	6	·0017803	577	17
3·30	·9995166	169	6	·0017226	560	17
2·31	·9995335	164	6	·0016666	543	17
3·32	·9995499	159	5	·0016122	527	16
3·33	·9995658	153	5	·0015595	512	16
3·34	·9995811	148	5	·0015084	496	15
3·35	·9995959	143	5	·0014587	481	15
3·36	·9996103	139	5	·0014106	467	15
3·37	·9996242	134	5	·0013639	453	14
3·38	·9996376	130	4	·0013187	439	14
3·39	·9996505	125	4	·0012748	426	13
3·40	·9996631	121	4	·0012322	413	13
3·41	·9996752	117	4	·0011910	400	13
3·42	·9996869	113	4	·0011510	388	12
3·43	·9996982	109	4	·0011122	376	12
3·44	·9997091	106	4	·0010747	364	12
3·45	·9997197	102	4	·0010383	353	11
3·46	·9997299	99	3	·0010030	342	11
3·47	·9997398	95	3	·0009689	331	11
3·48	·9997493	92	3	·0009358	320	10
3·49	·9997585	89	3	·0009037	310	10
3·50	·9997674		3	·0008727		10

x	$\frac{1}{2}(1+a)$	Δ +	Δ² −
3·50	·9997674	86	3
3·51	·9997759	83	3
3·52	·9997842	80	3
3·53	·9997922	77	3
3·54	·9997999	74	3
3·55	·9998074	72	3
3·56	·9998146	69	3
3·57	·9998215	67	2
3·58	·9998282	65	2
3·59	·9998347	62	2
3·60	9998409	60	2
3·61	9998469	58	2
3·62	·9998527	56	2
3·63	·9998583	54	2
3·64	·9998637	52	2
3·65	·9998689	50	2
3·66	·9998739	48	2
3·67	·9998787	47	2
3·68	·9998834	45	2
3·69	·9998879	43	2
3·70	·9998922	42	2
3·71	·9998964	40	2
3·72	·9999004	39	1
3·73	·9999043	37	1
3·74	·9999080	36	1
3·75	·9999116	35	1
3·76	·9999150	33	1
3·77	·9999184	32	1
3·78	·9999216	31	1
3·79	·9999247	30	1
3·80	·9999277	29	1
3·81	·9999305	28	1
3·82	·9999333	27	1
3·83	·9999359	26	1
3·84	·9999385	25	1
3·85	·9999409	24	1
3·86	·9999433	23	1
3·87	·9999456	22	1
3·88	·9999478	21	1
3·89	·9999499	20	1
3·90	·9999519	19	1
3·91	·9999539	19	1
3·92	·9999557	18	1
3·93	·9999575	17	1
3·94	·9999593	17	1
3·95	·9999609	16	1
3·96	·9999625	15	1
3·97	·9999641	15	1
3·98	·9999655	14	1
3·99	·9999670	14	1
4·00	·9999683		1

TABLES I. AND II.—(*continued*).

z	Δ $-$	Δ^2 $+$	x	$\frac{1}{2}(1+a)$	Δ $+$	Δ^2 $-$	z	Δ $-$	Δ^2 $+$
·0008727	301	10	*4·00*	·9999683	13	1	·0001338	53	2
·0008426	291	10	*4·01*	·9999696	13	1	·0001286	51	2
·0008135	282	9	*4·02*	·9999709	12	0	·0001235	49	2
·0007853	273	9	*4·03*	·9999721	12		·0001186	47	2
·0007581	264	9	*4·04*	·9999733	11		·0001140	45	2
·0007317	256	8	*4·05*	·9999744	11		·0001094	43	2
·0007061	247	8	*4·06*	·9999755	10		·0001051	42	2
·0006814	239	8	*4·07*	·9999765	10		·0001009	40	2
·0006575	232	8	*4·08*	·9999775	9		·0000969	39	2
·0006343	224	8	*4·09*	·9999784	9		·0000930	37	1
·0006119	217	7	*4·10*	·9999793	9		·0000893	36	1
·0005902	210	7	*4·11*	·9999802	8		·0000857	35	1
·0005693	203	7	*4·12*	·9999811	8		·0000822	33	1
·0005490	196	7	*4·13*	·9999819	8		·0000789	32	1
·0005294	189	6	*4·14*	·9999826	7		·0000757	31	1
·0005105	183	6	*4·15*	·9999834	7		·0000726	30	1
·0004921	177	6	*4·16*	·9999841	7		·0000697	28	1
·0004744	171	6	*4·17*	·9999848	7		·0000668	27	1
·0004573	165	6	*4·18*	·9999854	6		·0000641	26	1
·0004408	160	6	*4·19*	·9999861	6		·0000615	25	1
·0004248	155	5	*4·20*	·9999867	6		·0000589	24	1
·0004093	149	5	*4·21*	·9999872	6		·0000565	23	1
·0003944	144	5	*4·22*	·9999878	5		·0000542	22	1
·0003800	139	5	*4·23*	·9999883	5		·0000519	22	1
·0003661	135	5	*4·24*	·9999888	5		·0000498	21	1
·0003526	130	5	*4·25*	·9999893	5		·0000477	20	1
·0003396	125	4	*4·26*	·9999898	4		·0000457	19	1
·0003271	121	4	*4·27*	·9999902	4		·0000438	18	1
·0003149	117	4	*4·28*	·9999907	4		·0000420	18	1
·0003032	113	4	*4·29*	·9999911	4		·0000402	17	1
·0002919	109	4	*4·30*	·9999915	4		·0000385	16	1
·0002810	105	4	*4·31*	·9999918	4		·0000369	16	1
·0002705	102	4	*4·32*	·9999922	3		·0000354	15	1
·0002604	98	4	*4·33*	·9999925	3		·0000339	14	1
·0002506	95	3	*4·34*	·9999929	3		·0000324	14	1
·0002411	91	3	*4·35*	·9999932	3		·0000310	13	1
·0002320	88	3	*4·36*	·9999935	3		·0000297	13	1
·0002232	85	3	*4·37*	·9999938	3		·0000284	12	1
·0002147	82	3	*4·38*	·9999941	3		·0000272	12	0
·0002065	79	3	*4·39*	·9999943	3		·0000261	11	
·0001987	76	3	*4·40*	·9999946	2		·0000249	11	
·0001910	73	3	*4·41*	·9999948	2		·0000239	10	
·0001837	71	3	*4·42*	·9999951	2		·0000228	10	
·0001766	68	3	*4·43*	·9999953	2		·0000218	9	
·0001698	66	2	*4·44*	·9999955	2		·0000209	9	
·0001633	63	2	*4·45*	·9999957	2		·0000200	9	
·0001569	61	2	*4·46*	·9999959	2		·0000191	8	
·0001508	59	2	*4·47*	·9999961	2		·0000183	8	
·0001449	57	2	*4·48*	·9999963	2		·0000175	8	
·0001393	55	2	*4·49*	·9999964	2		·0000167	7	
·0001338		2	*4·50*	·9999966			·0000160		

TABLES I. AND II.—(*continued*). See Note, p. 181.

x	$\frac{1}{2}(1+\alpha)$	z	x	$\frac{1}{2}(1+\alpha)$	z	x	$\frac{1}{2}(1+\alpha)$	z
4·50	66023	159837	5·00	97133	14867	5·50	99810	1077
4·51	67586	152797	5·01	97278	14141	5·51	99821	1019
4·52	69080	146051	5·02	97416	13450	5·52	99831	965
4·53	70508	139590	5·03	97548	12791	5·53	99840	913
4·54	71873	133401	5·04	97672	12162	5·54	99849	864
4·55	73177	127473	5·05	97791	11564	5·55	99857	817
4·56	74423	121797	5·06	97904	10994	5·56	99865	773
4·57	75614	116362	5·07	98011	10451	5·57	99873	731
4·58	76751	111159	5·08	98113	9934	5·58	99880	691
4·59	77838	106177	5·09	98210	9441	5·59	99886	654
4·60	78875	101409	5·10	98302	8972	5·60	99893	618
4·61	79867	96845	5·11	98389	8526	5·61	99899	585
4·62	80813	92477	5·12	98472	8101	5·62	99905	553
4·63	81717	88297	5·13	98551	7696	5·63	99910	522
4·64	82580	84298	5·14	98626	7311	5·64	99915	494
4·65	83403	80472	5·15	98698	6944	5·65	99920	467
4·66	84190	76812	5·16	98765	6595	5·66	99924	441
4·67	84940	73311	5·17	98830	6263	5·67	99929	417
4·68	85656	69962	5·18	98891	5947	5·68	99933	394
4·69	86340	66760	5·19	98949	5647	5·69	99936	372
4·70	86992	63698	5·20	99004	5361	5·70	99940	351
4·71	87614	60771	5·21	99056	5089	5·71	99944	332
4·72	88208	57972	5·22	99105	4831	5·72	99947	313
4·73	88774	55296	5·23	99152	4585	5·73	99950	296
7·74	89314	52739	5·24	99197	4351	5·74	99953	280
4·75	89829	50295	5·25	99240	4128	5·75	99955	264
4·76	90320	47960	5·26	99280	3917	5·76	99958	249
4·77	90789	45728	5·27	99318	3716	5·77	99960	235
4·78	91235	43596	5·28	99354	3525	5·78	99963	222
4·79	91661	41559	5·29	99388	3344	5·79	99965	210
4·80	92067	39613	5·30	99421	3171	5·80	99967	198
4·81	92453	37755	5·31	99452	3007	5·81	99969	187
4·82	92822	35980	5·32	99481	2852	5·82	99971	176
4·83	93173	34285	5·33	99509	2704	5·83	99972	166
4·84	93508	32667	5·34	99535	2563	5·84	99974	157
4·85	93827	31122	5·35	99560	2430	5·85	99975	148
4·86	94131	29647	5·36	99584	2303	5·86	99977	139
4·87	94420	28239	5·37	99606	2183	5·87	99978	131
4·88	94696	26895	5·38	99628	2069	5·88	99979	124
4·89	94958	25613	5·39	99648	1960	5·89	99981	117
4·90	95208	24390	5·40	99667	1857	5·90	99982	110
4·91	95446	23222	5·41	99685	1760	5·91	99983	104
4·92	95673	22108	5·42	99702	1667	5·92	99984	98
4·93	95889	21046	5·43	99718	1579	5·93	99985	92
4·94	96094	20033	5·44	99734	1495	5·94	99986	87
4·95	96289	19066	5·45	99748	1416	5·95	99987	82
4·96	96475	18144	5·46	99762	1341	5·96	99987	77
4·97	96652	17265	5·47	99775	1270	5·97	99988	73
4·98	96821	16428	5·48	99787	1202	5·98	99989	68
4·99	96981	15629	5·49	99799	1138	5·99	99990	65
						6·00	99990	61

TABLES III. and IV. *Abscissa and Ordinate in terms of difference of Areas.*

a	*x*	Δ +	Δ² +	Δ³ +	*z*	Δ −	Δ² −	Δ³ −
·00	·0000000		0		·3989423		627	
		125335		20		313		0
·01	·0125335		20		·3989109		627	
		125354		20		940		0
·02	·0250689		39		·3988169		627	
		125394		20		1567		0
·03	·0376083		59		·3986603		627	
		125453		20		2194		0
·04	·0501536		79		·3984408		627	
		125532		20		2821		0
·05	·0627068		99		·3981587		628	
		125631		20		3449		1
·06	·0752699		119		·3978138		628	
		125750		20		4078		1
·07	·0878418		139		·3974060		629	
		125889		20		4707		1
·08	·1004337		159		·3969353		630	
		126048		20		5337		1
·09	·1130385		180		·3964016		631	
		126228		21		5967		1
·10	·1256613		201		·3958049		632	
		126429		21		6599		1
·11	·1383042		221		·3951450		633	
		126650		21		7232		1
·12	·1509692		243		·3944218		634	
		126893		21		7866		1
·13	·1636585		264		·3936352		635	
		127157		22		8501		1
·14	·1763742		286		·3927852		636	
		127443		22		9137		1
·15	·1891184		308		·3918715		638	
		127751		22		9775		2
·16	·2018935		330		·3908939		640	
		128081		23		10415		2
·17	·2147016		353		·3898525		641	
		128434		23		11056		2
·18	·2275450		376		·3887469		643	
		128811		24		11699		2
·19	·2404260		400		·3875769		645	
		129211		24		12344		2
·20	·2533471		424		·3863425		647	
		129635		25		12991		2
·21	·2663106		449		·3850434		649	
		130084		25		13641		2
·22	·2793190		474		·3836794		652	
		130559		26		14292		2
·23	·2923749		500		·3822501		654	
		131059		27		14946		3
·24	·3054808		527		·3807555		657	
		131586		27		15603		3
·25	·3186394		554		·3791952		659	
		132140		28		16262		3
·26	·3318533		582		·3775690		662	
		132722		29		16924		3
·27	·3451255		611		·3758766		665	
		133333		30		17589		3
·28	·3584588		640		·3741177		668	
		133973		30		18258		3
·29	·3718561		671		·3722919		672	
		134644		31		18929		3
·30	·3853205		702		·3703990		675	
		135346		32		19604		4
·31	·3988551		735		·3684386		679	
		136081		34		20283		4
·32	·4124631		768		·3664103		682	
		136849		35		20965		4
·33	·4261480		803		·3643138		686	
		137652		36		21651		4
·34	·4399132		839		·3621487		690	
		138490		37		22342		4
·35	·4537622		876		·3599146		695	
		139366		39		23036		4
·36	·4676988		914		·3576109		699	
		140281		40		23735		5
·37	·4817268		954		·3552374		704	
		141235		42		24439		5
·38	·4958503		996		·3527935		709	
		142231		43		25148		5
·39	·5100735		1039		·3502788		714	
		143271		45		25861		5
·40	·5244005		1085		·3476926		719	
		144355		47		26580		6
·41	·5388360		1132		·3450346		725	
		145487		49		27305		6
·42	·5533847		1181		·3423041		730	
		146668		51		28035		6
·43	·5680515		1232		·3395005		736	
		147900		54		28772		6
·44	·5828415		1286		·3366233		743	
		149186		56		29514		7
·45	·5977601		1342		·3336719		749	
		150529		59		30264		7
·46	·6128130		1402		·3306455		756	
		151930		62		31020		7
·47	·6280060		1464		·3275435		763	
		153394		65		31783		7
·48	·6433454		1529		·3243652		771	
		154923		69		32554		8
·49	·6588377		1598		·3211098		779	
		156521		72		33333		8
·50	·6744898		1670		·3177766		787	

TABLES III. AND IV.—(*continued*).

a	x	Δ +	Δ^2 +	Δ^3 +	z	Δ −	Δ^2 −	Δ^3 −
·50	·6744898	158191	1670	76	·3177766	34119	787	9
·51	·6903088	159937	1747	81	·3143646	34915	795	9
·52	·7063026	161765	1828	86	·3108732	35719	804	9
·53	·7224791	163678	1913	91	·3073013	36532	814	10
·54	·7388468	165682	2004	96	·3036481	37356	823	10
·55	·7554150	167782	2100	102	·2999125	38189	834	11
·56	·7721932	169984	2203	109	·2960936	39034	844	11
·57	·7891917	172296	2312	116	·2921902	39889	856	12
·58	·8064212	174724	2428	124	·2882013	40757	867	12
·59	·8238936	177276	2552	133	·2841256	41637	880	13
·60	·8416212	179961	2685	143	·2799619	42530	893	14
·61	·8596174	182789	2828	153	·2757089	43437	907	15
·62	·8778963	185771	2981	165	·2713653	44358	921	15
·63	·8964731	188917	3147	178	·2669295	45295	937	16
·64	·9153651	192242	3325	193	·2624000	46247	953	17
·65	·9345893	195760	3518	209	·2577753	47217	970	18
·66	·9541653	199486	3727	227	·2530535	48205	988	19
·67	·9741139	203440	3951	248	·2482330	49213	1007	20
·68	·9944579	207641	4201	271	·2433117	50240	1028	22
·69	1·0152220	212114	4472	297	·2382877	51289	1049	23
·70	1·0364334	216882	4769	326	·2331588	52362	1072	25
·71	1·0581216	221977	5095	360	·2279226	53459	1097	26
·72	1·0803193	227432	5455	399	·2225767	54582	1123	28
·73	1·1030626	233286	5854	443	·2171185	55734	1152	30
·74	1·1263911	239583	6297	495	·2115451	56916	1182	33
·75	1·1503494	246374	6792	555	·2058535	58130	1215	35
·76	1·1749868	253721	7347	625	·2000405	59380	1250	38
·77	1·2003589	261693	7972	709	·1941024	60669	1288	42
·78	1·2265281	270373	8681	808	·1880356	61999	1330	45
·79	1·2535654	279861	9488	926	·1818357	63374	1375	50
·80	1·2815516		10414		·1754983		1425	

VARIATION IN "EUPAGURUS PRIDEAUXI" (HELLER).

By E. H. J. SCHUSTER, B.A.

I.

Introduction. The work on which the following paper was based was done in the winter of 1901—1902 during my occupancy of the Oxford University table at the Naples Biological Station; and I take this opportunity of expressing my appreciation of the kindness and courtesy of those members of the staff of that institution with whom I came into contact.

The subsequent calculations were done under the direction of Professor Weldon, and I herewith tender him my sincere thanks for devoting so much time and trouble to the purpose. I have also to thank Professor Pearson for his numerous and valuable suggestions.

The work itself is an attempt to determine whether members of the species *Eupagurus prideauxi* caught in shallow water, differ with regard to certain characters from those caught in comparatively deep water.

For this purpose the following three measurements were taken:

1. From the upper articulation of the propodite with the carpopodite of the right chela to the upper and outer articulation of the propodite with the dactylopodite, called Measurement No. 1. Fig. 1, *AB*.

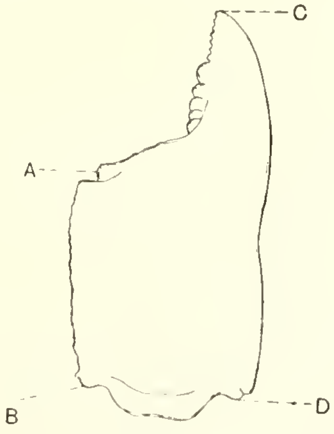


Fig. 1.

2. The length of the right chela measured from its lower articulation with the carpopodite, to the furthest point of the fixed blade of the scissors. Fig. 1, *CD*.

3. The length of the carapace along the median line.

These measurements were taken with a pair of dividers provided with a screw fine adjustment and an ivory scale divided into $\frac{1}{2}$ millimetres, they are believed to be accurate to $\frac{1}{10}$ of a millimetre.

From Measurements No. 1 and No. 2 an index called the chela index was deduced, which is intended to represent the proportion which the claw bears to the whole length of the chela.

$$\text{The chela index} = \frac{\text{Measurement No. 2} - \text{Measurement No. 1}}{\text{Measurement No. 2}}.$$

About two thousand individuals were measured, which were separated into two main groups:

I. *Shallow water forms*, from a depth of 35 metres or under;

II. *Deep water forms*, from a depth of over 35 metres.

Owing to the great difference in size between the sexes each of these groups was again subdivided into male and female. Thus finally we have to deal with four sets with about five hundred individuals in each.

II.

Comparison between deep water forms and shallow water forms as regards each measurement and the chela index taken separately. The deep water males are in each case compared with the shallow water males and the deep water females with the shallow water females.

TABLE I.

Measurement No. 1. ♂.

	Mean	Probable error of mean	Standard Deviation
Deep water forms ...	9·7078 mm.	·0847 mm.	2·7588 mm.
Shallow water forms	10·2718 mm.	·0745 mm.	2·5901 mm.
Difference	0·5640 mm.	Probable error of difference = ·1128 mm.	

This measurement is greater in the shallow water forms by an amount exactly five times as great as its probable error, and therefore almost certainly significant.

TABLE II.

Measurement No. 1. ♀.

	Mean	Probable error of mean	Standard Deviation
Deep water forms ...	7·4000 mm.	·0330 mm.	1·0607 mm.
Shallow water forms	7·4850 mm.	·0293 mm.	1·0234 mm.
Difference	0·0850 mm.	Probable error of difference = ·0441 mm.	

This measurement is greater in the shallow water forms by an amount about twice as great as its probable error.

TABLE III.

Right Chela Length. ♂.

	Mean	Probable error of mean	Standard Deviation
Deep water forms ...	17·9676 mm.	·1451 mm.	4·7279 mm.
Shallow water forms	18·6773 mm.	·1259 mm.	4·3769 mm.
Difference	·7097 mm.	Probable error of difference = ·1922 mm.	

This measurement is greater in the shallow water forms by an amount between three and four times as great as its probable error, and therefore is probably significant.

TABLE IV.

Right Chela Length. ♀.

	Mean	Probable error of mean	Standard Deviation
Deep water forms ...	14·1415 mm.	·0612 mm.	1·9679 mm.
Shallow water forms	13·9735 mm.	·0522 mm.	1·8215 mm.
Difference	·1680 mm.	Probable error of difference = ·0805 mm.	

This measurement is greater in the deep water forms by an amount twice as great as its probable error.

To sum up with regard to the absolute measurements of the right chela; in the male they are both significantly greater in the shallow water than in the deep water forms; while in the female Measurement No. 1 is greater in the shallow water forms, Measurement No. 2 in the deep water forms, and in neither case is the difference, though well marked, sufficiently great to be called significant.

TABLE V.

Chela Index. ♂.

	Mean	Probable error of mean	Standard Deviation
Deep water forms ...	·461348	·000538	·017537
Shallow water forms	·451045	·000472	·016417
Difference	·010303	Probable error of difference = ·000715	

The chela index is greater in the deep water forms by an amount more than 13 times as big as its probable error, and therefore certainly significant.

TABLE VI.

Chela Index. ♀.

	Mean	Probable error of mean	Standard Deviation
Deep water forms ...	·475051	·000458	·014721
Shallow water forms	·471312	·000385	·013429
Difference	·003739	Probable error of difference = ·000598	

The chela index is greater in the deep water forms by an amount more than six times as big as its probable error, and therefore almost certainly significant.

Thus Tables V. and VI. show that in the forms taken from deep water, both male and female, a greater proportion of the whole length of the chela is taken up by the blade of the scissors than in forms taken from shallow water.

TABLE VII.

Carapace Length. ♂.

	Mean	Probable error of mean	Standard Deviation
Deep water forms ...	8·5854 mm.	·0512 mm.	1·6696 mm.
Shallow water forms	8·4063 mm.	·0425 mm.	1·4918 mm.
Difference	·1791 mm.	Probable error of difference = ·0665 mm.	

The mean carapace length in the deep water forms is greater than that of the shallow water forms by an amount more than twice as great as its probable error.

TABLE VIII.

Carapace Length. ♀.

	Mean	Probable error of mean	Standard Deviation
Deep water forms ...	7·5405 mm.	·0293 mm.	·9417 mm.
Shallow water forms	7·1222 mm.	·0247 mm.	·8631 mm.
Difference	·4183 mm.	Probable error of difference = ·0383 mm.	

The mean carapace length in the deep water forms is greater than that of the shallow water forms by an amount about ten times as great as its probable error.

Thus in both male and female the mean carapace length is greater in the deep water forms, in the female by a certainly significant amount, in the male by a quantity which taken alone could hardly be taken as significant, but viewed in the light of the result obtained by the female it may possibly be considered to be so.

III.

On the Comparative Variability of Deep and Shallow Water Forms.

TABLE VIII *bis*.

Table	Deep Water		Shallow Water		Difference of	
Character, Sex	S. D.	C. of V.	S. D.	C. of V.	S. D.'s	C.'s of V.
Measurement No. 1 ♂	2·7588 ± ·0599	28·418 ± ·665	2·5901 ± ·0527	25·216 ± ·544	·1687 ± ·0798	3·202 ± ·865
" " ♀	1·0607 ± ·0233	14·334 ± ·322	1·0234 ± ·0207	13·673 ± ·278	·0373 ± ·0312	·661 ± ·425
Right Chela Length ♂	4·7279 ± ·1026	26·313 ± ·609	4·3769 ± ·0890	23·434 ± ·502	·3510 ± ·1359	2·879 ± ·789
" " ♀	1·9679 ± ·0433	13·916 ± ·312	1·8215 ± ·0369	13·036 ± ·269	·1464 ± ·0569	·880 ± ·412
Carapace Length ♂ ...	1·6696 ± ·0362	19·446 ± ·438	1·4918 ± ·0303	17·746 ± ·372	·1778 ± ·0470	1·700 ± ·575
" " ♀ ...	·9417 ± ·0207	12·489 ± ·279	·8631 ± ·0175	12·118 ± ·249	·0786 ± ·0271	·371 ± ·374
Chela Index ♂	·01754 ± ·00038	38·013 ± ·936	·01642 ± ·00033	36·404 ± ·833	·00112 ± ·00051	1·608 ± 1·253
" " ♀	·01472 ± ·00032	30·988 ± ·744	·01343 ± ·00027	28·493 ± ·622	·00129 ± ·00042	2·495 ± ·970

The table above gives the standard deviations and the coefficients of variation* with their probable errors, and we see:

(i) That in both forms the male, whether we judge by standard deviation or coefficient of variation, is, for the characters considered, much more variable than the female.

(ii) That there are significant differences in the variability of the deep and shallow water forms. The deep water forms are in every single case the more variable, however variability be estimated. For males the difference is always greater and often much greater than its probable error; for females the difference is less marked, but none the less quite obvious.

We can therefore conclude that the conditions of life are probably far more stringent for the shallow than the deep sea forms, and for the females than for the males. It would, perhaps, be rash to assert that the shore crabs are a selection from the deep sea form, but the facts as to variability are not only compatible with but indeed suggestive of such an hypothesis.

IV.

Comparison of the correlation between the length of carapace and each of the other measurements for deep and shallow water forms.

To commence with the consideration of the males, Table IX. shows the correlation between the length of the carapace and Measurement No. 1 for deep, Table X. for shallow water forms.

For the former $r = \cdot 9458 \pm \cdot 0032$,
For the latter $r = \cdot 9337 \pm \cdot 0037$,
The difference $= \cdot 0120 \pm \cdot 0049$.

---

* Percentage variation on the mean, i.e. 100 × standard deviation and divided by the mean.

TABLE IX.

Deep Water Forms. ♂. Correlation between Length of Carapace and Measurement No. 1.

Measurement No. 1 (millimetres).

Length of Carapace (millimetres).	*4·1–4·5*	*4·6–5·0*	*5·1–5·5*	*5·6–6·0*	*6·1–6·5*	*6·6–7·0*	*7·1–7·5*	*7·6–8·0*	*8·1–8·5*	*8·6–9·0*	*9·1–9·5*	*9·6–10·0*	*10·1–10·5*	*10·6–11·0*	*11·1–11·5*	*11·6–12·0*	*12·1–12·5*	*12·6–13·0*	*13·1–13·5*	*13·6–14·0*	*14·1–14·5*	*14·6–15·0*	*15·1–15·5*	*15·6–16·0*	*16·1–16·5*	*16·6–17·0*	*17·1–17·5*	Totals
12·1—12·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	2
11·6—12·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	4	3	3	3	1	—	15
11·1—11·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	3	6	5	2	—	—	—	1	18
10·6—11·0	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	1	2	11	6	6	4	—	—	1	—	—	33
10·1—10·5	—	—	—	—	—	—	—	—	—	1	—	—	1	—	1	7	7	3	3	2	—	1	—	1	—	—	—	27
9·6—10·0	—	—	—	—	—	—	—	—	—	—	—	1	5	11	18	8	4	2	3	—	—	—	—	—	—	—	—	52
9·1— 9·5	—	—	—	—	—	1	—	2	—	—	2	2	17	15	5	1	3	3	—	—	—	—	—	—	—	—	—	51
8·6— 9·0	—	—	—	—	—	—	—	—	2	2	11	17	15	1	1	—	—	—	—	—	—	—	—	—	—	—	—	49
8·1— 8·5	—	—	—	1	—	1	1	1	7	18	15	6	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	51
7·6— 8·0	—	—	2	—	—	—	1	6	21	4	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	37
7·1— 7·5	—	—	—	2	—	2	14	20	10	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	49
6·6— 7·0	—	—	—	1	3	10	23	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	40
6·1— 6·5	—	—	1	3	11	7	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	23
5·6— 6·0	1	—	2	12	8	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	23
5·1— 5·5	—	2	5	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
4·6— 5·0	1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
Totals	2	3	11	22	22	21	40	32	40	27	31	26	39	27	25	17	15	11	17	12	12	14	5	5	4	1	2	483

TABLE X.

Shallow Water Forms. ♂. *Correlation between Length of Carapace and Measurement No.* 1.

Measurement No. 1 (millimetres).

Length of Carapace (millimetres).

	4·1–4·5	4·6–5·0	5·1–5·5	5·6–6·0	6·1–6·5	6·6–7·0	7·1–7·5	7·6–8·0	8·1–8·5	8·6–9·0	9·1–9·5	9·6–10·0	10·1–10·5	10·6–11·0	11·1–11·5	11·6–12·0	12·1–12·5	12·6–13·0	13·1–13·5	13·6–14·0	14·1–14·5	14·6–15·0	15·1–15·5	15·6–16·0	16·1–16·5	16·6–17·0	17·1–17·5	Totals
11·6—12·0	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	3
11·1—11·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	4	3	1	—	9
10·6—11·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2	3	3	3	8	3	2	2	1	—	—	28
10·1—10·5	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	2	2	8	13	7	4	5	—	—	—	—	43
9·6—10·0	—	—	—	—	—	—	—	—	—	1	—	—	—	1	3	7	11	6	9	13	1	—	—	—	—	—	—	52
9·1— 9·5	—	—	—	—	—	—	—	—	—	—	1	2	3	6	14	12	13	5	2	—	—	—	—	—	—	—	—	58
8·6— 9·0	—	—	—	—	—	—	1	—	1	—	2	5	11	16	15	7	4	1	—	—	—	—	—	—	—	—	—	63
8·1— 8·5	—	—	—	—	1	1	—	—	2	4	18	17	24	9	—	—	—	1	—	—	—	—	—	—	—	—	—	77
7·6— 8·0	—	—	—	—	—	—	1	—	7	16	22	10	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	57
7·1— 7·5	—	—	—	—	—	—	4	12	24	12	7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	59
6·6— 7·0	—	—	—	—	1	3	9	13	11	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	39
6·1— 6·5	—	—	—	—	6	8	9	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	24
5·6— 6·0	1	—	—	8	7	4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	20
5·1— 5·5	—	—	—	10	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	12
4·6— 5·0	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
4·1— 4·5	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
Totals	2	4	1	18	17	16	24	26	45	37	50	31	38	33	33	27	32	18	22	29	16	7	8	6	5	2	—	550

TABLE XI.

Deep Water Forms. ♂. *Correlation between Length of Right Chela and Length of Carapace.*

Length of Right Chela (millimetres).

Length of Carapace (millimetres).	8·0–8·9	9·0–9·9	10·0–10·9	11·0–11·9	12·0–12·9	13·0–13·9	14·0–14·9	15·0–15·9	16·0–16·9	17·0–17·9	18·0–18·9	19·0–19·9	20·0–20·9	21·0–21·9	22·0–22·9	23·0–23·9	24·0–24·9	25·0–25·9	26·0–26·9	27·0–27·9	28·0–28·9	29·0–29·9	30·0–30·9	Totals
12·1—12·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	2
11·6—12·0	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	3	5	5	—	—	15
11·1—11·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	3	5	6	2	—	—	1	18
10·6—11·0	—	—	—	—	—	—	—	—	—	1	—	—	—	2	1	3	15	4	4	3	—	—	—	33
10·1—10·5	—	—	—	—	—	—	—	1	—	—	—	1	1	6	6	5	3	3	1	—	—	—	—	27
9·6—10·0	—	—	—	—	—	—	—	—	—	—	1	5	18	16	9	2	1	—	—	—	—	—	—	52
9·1— 9·5	—	—	—	—	—	1	2	—	—	2	6	20	11	3	6	—	—	—	—	—	—	—	—	51
8·6— 9·0	—	—	—	—	—	—	—	1	1	16	24	5	1	1	—	—	—	—	—	—	—	—	—	49
8·1— 8·5	—	—	—	1	—	2	—	4	20	20	3	1	—	—	—	—	—	—	—	—	—	—	—	51
7·6— 8·0	—	1	1	—	—	1	5	17	12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	37
7·1— 7·5	—	—	—	2	—	8	23	14	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	49
6·6— 7·0	—	—	—	—	8	25	7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	40
6·1— 6·5	—	1	1	9	9	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	23
5·6— 6·0	1	—	7	13	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	23
5·1— 5·5	—	4	6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	10
4·6— 5·0	1	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
Totals	2	8	15	25	19	40	37	37	35	39	34	32	31	28	22	11	22	14	14	11	5	—	2	483

TABLE XII.

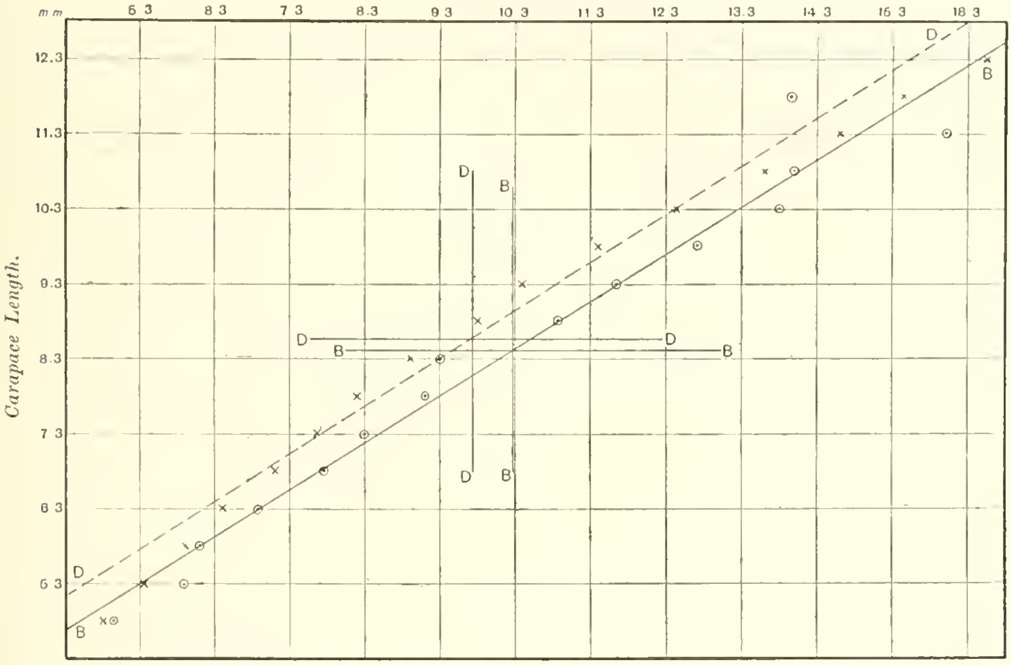
Shallow Water Forms. ♂. *Correlation between Length of Right Chela and Length of Carapace.*

Length of Right Chela (millimetres).

Length of Carapace (millimetres).	*8·0–8·9*	*9·0–9·9*	*10·0–10·9*	*11·0–11·9*	*12·0–12·9*	*13·0–13·9*	*14·0–14·9*	*15·0–15·9*	*16·0–16·9*	*17·0–17·9*	*18·0–18·9*	*19·0–19·9*	*20·0–20·9*	*21·0–21·9*	*22·0–22·9*	*23·0–23·9*	*24·0–24·9*	*25·0–25·9*	*26·0–26·9*	*27·0–27·9*	*28·0–28·9*	*29·0–29·9*	Totals
11·6—12·0	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—	1	3
11·1—11·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	4	3	—	9
10·6—11·0	—	—	—	—	—	—	—	—	—	—	—	—	1	—	2	2	4	7	10	1	1	—	28
10·1—10·5	—	—	—	—	—	—	—	—	1	—	—	—	—	1	3	7	12	11	7	1	—	—	43
9·6—10·0	—	—	—	—	—	—	—	—	—	1	—	—	2	8	19	7	13	2	—	—	—	—	52
9·1—9·5	—	—	—	—	—	—	—	—	—	2	2	3	15	20	13	3	—	—	—	—	—	—	58
8·6—9·0	—	—	—	—	—	—	1	—	2	2	11	21	17	6	3	—	—	—	—	—	—	—	63
8·1—8·5	—	—	—	—	2	—	—	1	7	21	31	11	—	1	—	—	—	—	—	—	—	—	77
7·6—8·0	—	—	—	—	—	1	—	7	27	17	4	1	—	—	—	—	—	—	—	—	—	—	57
7·1—7·5	—	—	—	—	—	1	14	30	11	3	—	—	—	—	—	—	—	—	—	—	—	—	59
6·6—7·0	—	—	—	1	2	9	20	7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	39
6·1—6·5	—	—	—	1	11	12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	24
5·6—6·0	1	—	2	15	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	20
5·1—5·5	—	—	5	7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	12
4·6—5·0	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
4·1—4·5	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
Totals	3	3	8	24	17	23	35	45	48	47	51	36	35	36	40	19	29	20	19	7	4	1	550

Diagram I. shows the regression lines of Measurement No. 1 on the carapace for these two depths; the dotted line is for the deep water forms, the continuous line for the shallow water forms. It will be noticed that one carapace length is associated with a much smaller value of Measurement No. 1 in the deep water forms than in the shallow.

DIAGRAM I. *Measurement No. 1.*



Comparison between Regression Lines of Measurement No. 1 on Carapace Length between Shallow Water ♂♂ (BB) and Deep Water ♂♂ (DD).

Shallow Water ♂♂.
Mean Measurement No. 1, 10·2718 mm.
Mean Carapace Length 8·4063 mm.
Regression of Measurement No. 1 on Carapace Length 1·6212.

Deep Water ♂♂.
Mean Measurement No. 1, 9·7078.
Mean Carapace Length 8·5854.
Regression of Measurement No. 1 on Carapace Length 1·5628.

Table XI. shows the correlation between the length of the right chela and the length of the carapace for deep, Table XII. for the shallow water ♂ forms.

For the deep water forms	$r = \cdot 9389 \pm \cdot 0036$,
For the shallow water forms	$r = \cdot 9503 \pm \cdot 0028$,
Difference	$= \cdot 0114 \pm \cdot 0046$.

TABLE XIII.

Deep Water Forms. ♂. *Correlation between Carapace Length and Chela Index.*

Chela Index.

Carapace Length (millimetres).	*386–395*	*396–405*	*406–415*	*416–425*	*426–435*	*436–445*	*446–455*	*456–465*	*466–475*	*476–485*	*486–495*	*496–505*	*506–515*	Totals
12·1—12·5	—	—	—	1	1	—	—	—	—	—	—	—	—	2
11·6—12·0	—	1	2	1	3	4	3	1	—	—	—	—	—	15
11·1—11·5	—	—	2	1	1	6	6	1	1	—	—	—	—	18
10·6—11·0	1	1	—	1	5	8	10	3	4	—	—	—	—	33
10·1—10·5	—	—	1	1	3	6	7	6	1	2	—	—	—	27
9·6—10·0	—	—	—	—	2	7	11	13	11	5	2	1	—	52
9·1— 9·5	—	—	—	3	—	6	10	16	11	4	1	—	—	51
8·6— 9·0	—	—	—	—	—	6	13	11	10	5	4		—	49
8·1— 8·5	—	—	—	—	—	—	11	9	15	9	7	—	—	51
7·6— 8·0	—	—	—	—	—	1	5	8	13	7	2	—	1	37
7·1— 7·5	—	—	—	—	—	1	3	16	19	7	3	—	—	49
6·6— 7·0	—	—	—	—	—	1	8	6	12	7	5	1	—	40
6·1— 6·5	—	—	—	—	—	—	—	4	11	8	—	—	—	23
5·6— 6·0	—	—	—	—	—	1	1	2	8	7	3	—	1	23
5·1— 5·5	—	—	—	—	1	1	1	2	4	1	—	—	—	10
4·6— 5·0	—	—	—	—	—	—	—	1	1	1	—	—	—	3
Totals	1	2	5	8	15	48	89	99	122	63	27	2	2	483

TABLE XIV.

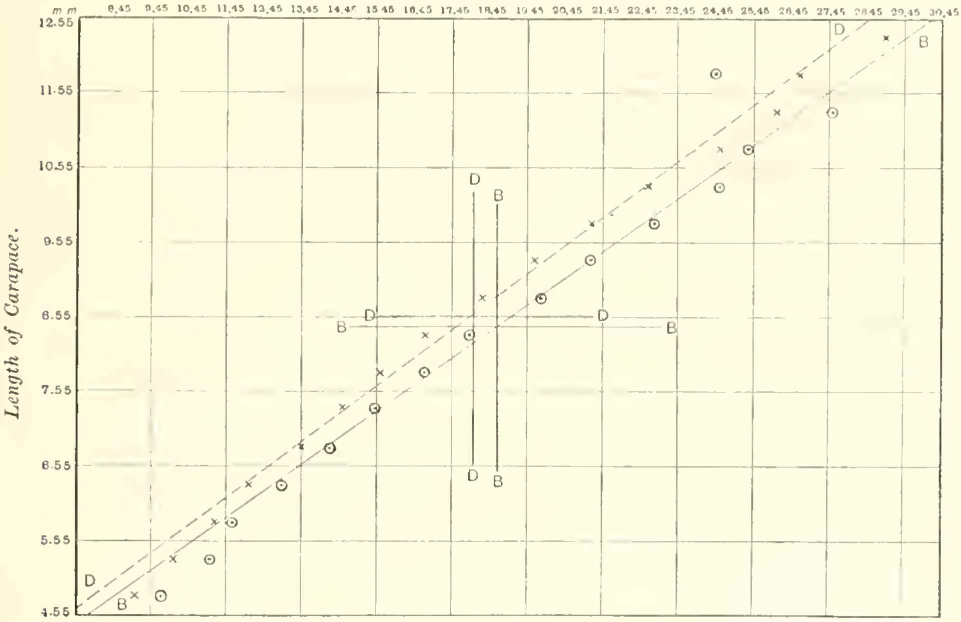
Shallow Water Forms. ♂. *Correlation between Length of Carapace and Chela Index.*

Chela Index.

Carapace Length (millimetres).	*396–405*	*406–415*	*416–425*	*426–435*	*436–445*	*446–455*	*456–465*	*466–475*	*476–485*	*486–495*	*496–505*	Totals
11·6—12·0	—	—	1	1	—	—	—	—	1	—	—	3
11·1—11·5	1	2	4	2	—	—	—	—	—	—	—	9
10·6—11·0	—	2	4	3	9	4	5	1	—	—	—	28
10·1—10·5	1	2	4	6	14	10	6	—	—	—	—	43
9·6—10·0	—	2	5	9	14	11	6	4	—	1	—	52
9·1— 9·5	—	1	1	7	14	16	9	8	2	—	—	58
8·6— 9·0	1	—	4	14	8	17	12	2	3	2	—	63
8·1— 8·5	—	1	—	4	18	22	19	7	4	2	—	77
7·6— 8·0	—	—	—	4	5	17	17	11	3	—	—	57
7·1— 7·5	—	—	1	—	6	13	26	8	3	1	1	59
6·6— 7·0	—	—	—	1	7	13	12	5	1	—	—	39
6·1— 6·5	—	—	—	—	2	5	8	6	2	1	—	24
5·6— 6·0	—	—	—	1	2	6	5	3	3	—	—	20
5·1— 5·5	—	—	—	1	1	1	2	4	2	1	—	12
4·6— 5·0	—	—	—	—	—	—	1	1	1	—	—	3
4·1— 4·5	—	—	—	—	—	1	1	1	—	—	—	3
Totals	3	10	24	53	100	136	129	61	25	8	1	550

Diagram II. shows the regression lines of the chela length on the carapace for deep and shallow water forms, the dotted line representing that of the deep water forms as in Diagram I. The diagram shows that the same carapace length is associated with a smaller chela length in the deep water forms than in the shallow, but the difference between deep and shallow is not so marked as in Diagram I.

DIAGRAM II. *Length of Right Chela.*



Comparison between Regression Lines of Right Chela Length on Carapace Length between Shallow Water ♂♂ (BB) and Deep Water ♂♂ (DD).

Shallow Water ♂♂.
Mean Length of Right Chela 18·6773 mm.
Mean Length of Carapace 8·4063 mm.
Regression of Chela on Carapace 1·405.

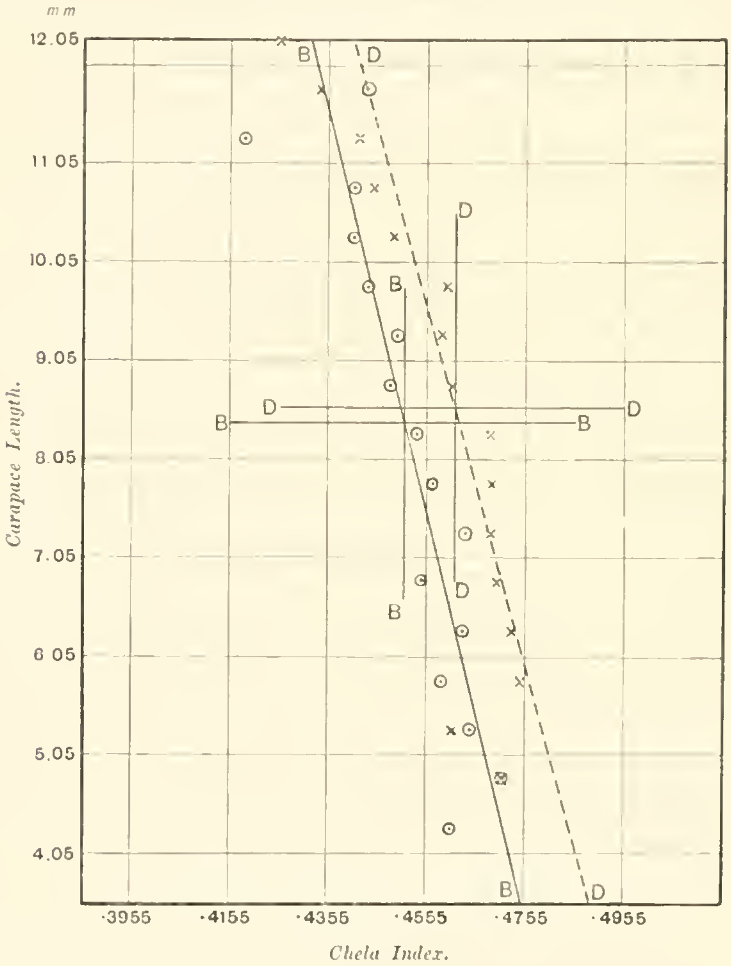
Deep Water ♂♂.
Mean Length of Right Chela 17·9676.
Mean Length of Carapace 8·5854.
Regression of Chela on Carapace 1·329.

Tables XIII. and XIV. show the correlation between carapace length and chela index for deep and shallow water groups respectively.

Deep water $r = -\cdot 5224 \pm \cdot 0223$,
Shallow water $r = -\cdot 4448 \pm \cdot 0231$,
Difference $\cdot 0776 \pm \cdot 0321$.

Diagram III. shows the regression lines of the chela index on the carapace length. The same carapace length is associated with a smaller chela index for the shallow water than for the deep water forms.

Diagram III. *Regression Lines of Chela Index on Carapace Length in ♂♂ from Shallow Water* (BB) *and from Deep Water* (DD).



With regard to the females, Tables XV. and XVI. correspond to Tables IX. and X. of the males and show the correlation between the carapace length and Measurement No. 1 for the deep and shallow water forms respectively.

Deep water	$r = \cdot 8466 \pm \cdot 0088$,
Shallow water	$r = \cdot 8428 \pm \cdot 0083$,
Difference	$\cdot 0037 \pm \cdot 0121$.

The difference being less than its probable error is quite negligible.

TABLE XV.

Deep Water Forms. ♀. Correlation between Length of Carapace and Measurement No. 1.

Measurement No. 1 (millimetres).

Length of Carapace (millimetres).	4·1–4·5	4·6–5·0	5·1–5·5	5·6–6·0	6·1–6·5	6·6–7·0	7·1–7·5	7·6–8·0	8·1–8·5	8·6–9·0	9·1–9·5	9·6–10·0	10·1–10·5	Totals
9·6—10·0	—	—	—	—	—	—	—	—	—	—	1	1	—	2
9·1— 9·5	—	—	—	—	—	—	—	2	2	3	6	—	—	13
8·6— 9·0	—	—	—	1	—	—	—	2	6	18	7	—	2	36
8·1— 8·5	—	—	1	—	2	—	3	29	51	9	2	1	—	98
7·6— 8·0	—	—	3	2	1	4	35	47	9	5	1	—	—	107
7·1— 7·5	—	1	2	—	2	21	47	18	2	—	—	—	—	93
6·6— 7·0	—	1	—	—	13	30	10	2	—	—	—	—	—	56
6·1— 6·5	—	—	—	10	19	—	—	—	—	—	—	—	—	29
5·6— 6·0	1	1	7	9	4	—	—	—	—	—	—	—	—	22
5·1— 5·5	—	1	3	1	—	—	—	—	—	—	—	—	—	5
4·6— 5·0	—	6	1	—	—	—	—	—	—	—	—	—	—	7
4·1— 4·5	1	1	—	—	—	—	—	—	—	—	—	—	—	2
Totals	2	11	17	23	41	55	95	100	70	35	17	2	2	470

TABLE XVI.

Shallow Water Forms. ♀. Correlation between Length of Carapace and Measurement No. 1.

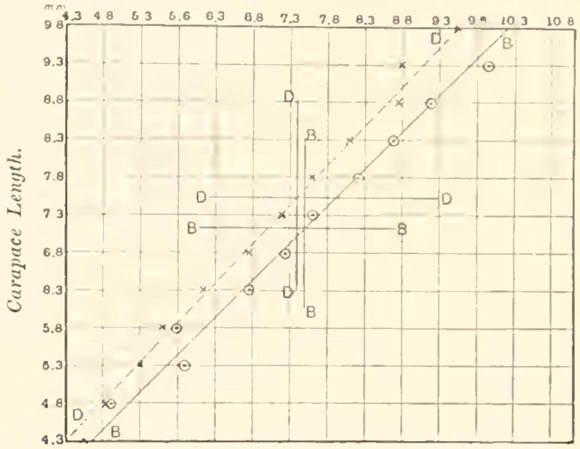
Measurement No. 1 (millimetres).

Length of Carapace (millimetres).	4·1–4·5	4·6–5·0	5·1–5·5	5·6–6·0	6·1–6·5	6·6–7·0	7·1–7·5	7·6–8·0	8·1–8·5	8·6–9·0	9·1–9·5	9·6–10·0	10·1–10·5	10·6–11·0	11·1–11·5	Totals
9·1—9·5	—	—	—	—	—	—	—	—	—	—	1	1	—	1	—	3
8·6—9·0	—	—	—	—	—	—	—	—	1	5	8	2	—	—	—	16
8·1—8·5	—	—	—	—	1	—	1	3	12	30	9	1	—	—	—	57
7·6—8·0	—	—	—	1	1	2	8	25	46	22	2	1	—	—	—	108
7·1—7·5	—	2	—	—	—	9	34	54	19	1	—	—	—	—	—	119
6·6—7·0	—	3	—	—	4	25	62	19	2	—	—	—	—	—	—	115
6·1—6·5	—	—	1	1	19	38	14	—	—	—	—	—	—	—	—	73
5·6—6·0	2	2	3	7	25	3	—	—	—	—	—	—	—	—	—	42
5·1—5·5	—	—	—	11	2	—	—	—	—	—	—	—	—	—	—	13
4·6—5·0	1	4	3	—	—	—	—	—	—	—	—	—	—	—	—	8
Totals	3	11	7	20	52	77	119	101	80	58	20	5	—	1	—	554

Diagram IV. corresponds to Diagram I. It represents the regression lines of Measurement No. 1 on carapace length, the dotted line being that of the deep water females, the continuous line that of the shallow water females.

It will be seen that the relation these two lines bear to one another is very much the same as that between the corresponding lines of the males. The diagram shows therefore that in the females as well as the males the same carapace length is associated with a smaller value of Measurement No. 1 in the deep water forms than in the shallow water forms.

Diagram IV. *Measurement No. 1.*



Comparison between Regression Lines of Measurement No. 1 on Carapace between Shallow Water ♀♀ (BB) and Deep Water ♀♀ (DD).

Shallow Water ♀♀.
Mean Carapace Length 7·1222 mm.
Mean Measurement No. 1 7·4850 mm.
Regression ·999. $r = \cdot 8428$.

Deep Water ♀♀.
Mean Carapace Length 7·5405.
Mean Measurement No. 1 7·4000.
Regression ·9535. $r = \cdot 8466$.

Tables XVII. and XVIII. show the correlation between the chela length and the carapace length in the deep and shallow water females, corresponding to Tables XI. and XII. of the males.

Deep water	$r = \cdot 8626 \pm \cdot 0080$,
Shallow water	$r = \cdot 8841 \pm \cdot 0063$,
Difference	$= \cdot 0215 \pm \cdot 0106$.

TABLE XVII.

Deep Water Forms. ♀. *Correlation between Length of Right Chela and Length of Carapace.*

Length of Right Chela (millimetres).

Length of Carapace (millimetres).

	8·0–8·9	9·0–9·9	10·0–10·9	11·0–11·9	12·0–12·9	13·0–13·9	14·0–14·9	15·0–15·9	16·0–16·9	17·0–17·9	18·0–18·9	Totals
9·6—10·0	—	—	—	—	—	—	—	—	—	—	2	2
9·1— 9·5	—	—	—	—	—	—	—	3	4	4	2	13
8·6— 9·0	—	—	—	1	—	—	1	6	15	11	2	36
8·1— 8·5	—	—	1	—	2	1	15	51	25	3	—	98
7·6— 8·0	—	—	4	1	3	9	50	34	4	2	—	107
7·1— 7·5	—	1	2	—	2	43	43	2	—	—	—	93
6·6— 7·0	—	1	—	1	23	28	3	—	—	—	—	56
6·1— 6·5	—	—	1	20	8	—	—	—	—	—	—	29
5·6— 6·0	1	2	10	9	—	—	—	—	—	—	—	22
5·1— 5·5	—	2	3	—	—	—	—	—	—	—	—	5
4·6— 5·0	1	6	—	—	—	—	—	—	—	—	—	7
4·1— 4·5	2	—	—	—	—	—	—	—	—	—	—	2
Totals	4	12	21	32	38	81	112	96	48	20	6	470

TABLE XVIII.

Shallow Water Forms. ♀. *Correlation between Length of Right Chela and Length of Carapace.*

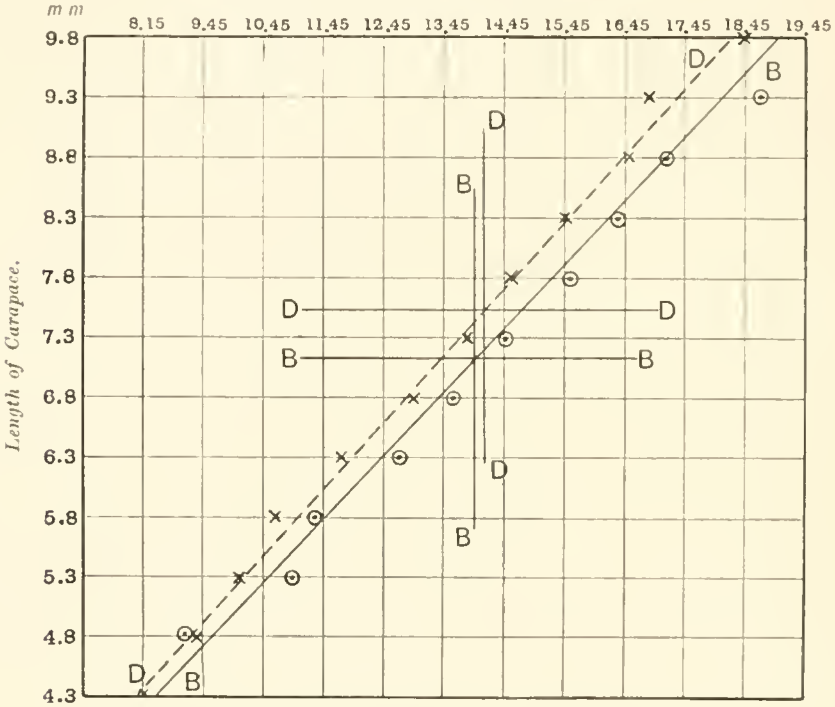
Length of Right Chela (millimetres).

Length of Carapace (millimetres).

	7·0–7·9	8·0–8·9	9·0–9·9	10·0–10·9	11·0–11·9	12·0–12·9	13·0–13·9	14·0–14·9	15·0–15·9	16·0–16·9	17·0–17·9	18·0–18·9	19·0–19·9	Totals
9·1—9·5	—	—	—	—	—	—	—	—	—	—	—	2	1	3
8·6—9·0	—	—	—	—	—	—	—	—	1	4	9	2	—	16
8·1—8·5	—	—	—	—	—	1	—	2	11	32	10	1	—	57
7·6—8·0	—	—	—	—	1	1	2	21	56	25	2	—	—	108
7·1—7·5	—	—	2	—	—	1	23	63	29	1	—	—	—	119
6·6—7·0	—	—	1	2	—	7	71	31	2	1	—	—	—	115
6·1—6·5	—	—	—	—	8	40	24	1	—	—	—	—	—	73
5·6—6·0	—	2	2	5	22	10	1	—	—	—	—	—	—	42
5·1—5·5	—	—	—	7	6	—	—	—	—	—	—	—	—	13
4·6—5·0	—	3	4	1	—	—	—	—	—	—	—	—	—	8
Totals	—	5	9	15	37	60	121	118	99	63	21	5	1	554

Diagram V. shows the regression lines of the right chela length on the carapace length for deep and shallow water females; it corresponds to Diagram II. The two lines bear much the same relation to one another as in that diagram. And in the same way as in Diagram II. the regression lines run closer together than in Diagram I. thus those of Diagram V. run closer together than those in Diagram IV.,

DIAGRAM V. *Length of Right Chela.*



Comparison between Regression Lines of Right Chela Length on Carapace Length of Shallow Water ♀♀ (BB) and Deep Water ♀♀ (DD).

Shallow Water ♀♀.
Mean Carapace Length 7·1222 mm.
Mean Chela Length 13·9735 mm.
Regression ·9329. $r = \cdot 8841$.

Deep Water ♀♀.
Mean Carapace Length 7·5405 mm.
Mean Chela Length 14·1415 mm.
Regression ·9013. $r = \cdot 8626$.

showing that these points in relation to the chela and carapace for the deep and shallow water forms which are illustrated by these diagrams are much the same in the males as they are in the females. Namely that the same carapace length is associated with a smaller value for Measurement No. 1 in the deep water forms than in the shallow water forms, also with a smaller value of Measurement No. 2, but that neither in the male nor in the female is the difference of the values of Measurement No. 2 so great as of Measurement No. 1.

It was found that in the females the value of r for the correlation between chela index and carapace length was less than its probable error and the regression practically nothing and so neither the correlation tables nor the diagram of the regression lines are given here.

V.

Comparison of Regression and Correlation Coefficients.

TABLE XIX.

Regression Coefficients of First Order.

Regression Coefficients	Male		Female	
	Deep Sea	Shallow	Deep Sea	Shallow
Measurement No. 1 on Carapace Length	1·5628	1·6212	·9535	·9994
Carapace Length on Measurement No. 1	·5723	·5378	·7516	·7108
Right Chela Length on Carapace Length	1·3293	1·4056	·9013	·9329
Carapace Length on Right Chela Length	·6633	·6477	·8255	·8379
Chela Index on Carapace Length	– ·2744	– ·2447	—	—
Carapace Length on Chela Index	– ·9947	– ·8084	—	—

Now at first sight it might appear that the greatest differences here between deep sea and shallow forms occur when we consider the regressions of the other organs on carapace length. But if we are dealing only with the question of significant differences, we must note that the probable errors are much larger in these cases. Thus the regression of right chela length on carapace length for deep sea males must be read 1·3293 ± ·0298, while for the inverse regression of carapace length on right chela length we have ·6633 ± ·0037. Hence the change in the latter regression from shallow to deep sea forms although so much smaller is really more significant. If, however, we suppose the changes between the regression coefficients for the two forms to be due, not to the random character of the samples, but to the direct action of natural selection, the greatest absolute differences are clearly in the regressions of the other organs on the carapace length, or the suggestion naturally follows that the differentiation between the deep sea and shallow forms is due more to a direct selection of the chela than of the carapace length. It would be interesting to determine whether this suggestion is or is not strengthened when the regression coefficients of higher orders are investigated.

Exhibiting all the correlation coefficients in a single table we have

TABLE XX.

Correlation Coefficients.

Characters	Male		Female	
	Deep Sea	Shallow	Deep Sea	Shallow
Measurement No. 1 and Carapace Length	·9458	·9338	·8466	·8428
Right Chela Length and Carapace Length	·9389	·9503	·8626	·8841
Chela Index and Carapace Length ...	– ·5224	– ·4448	Insensible	Insensible

Or, we conclude that the male has the parts investigated far more highly correlated than the female. We have thus the males more variable and more highly correlated than the females. It is remarkable that in man the female is on the whole more variable, and more highly correlated than the male. There would appear therefore to be a relation between high variability and high correlation. Or, perhaps, it would be better to say that the more stringent the selection the smaller is the variability, and as a rule the correlation. Interpreted in this manner we might hazard the suggestion that the female crabs of both forms are subjected to more stringent selection than the males.

VI.

General conclusions. (*a*) We have seen that with respect to the three characters measured certain differences do exist between those animals which have been living in deep water and those which have been living in shallow water. These differences may be congenital or they may be produced afresh in any generation. The latter alternative is more likely than the former as the larvae are pelagic and those produced by deep water parents must get mixed up pretty effectually with those that spring from shallow water. It is of course possible that the former find their way to the deep water in settling down to the bottom and the latter to the shallow water, but this is rather difficult to imagine; or that they sink down without regard to the depth of the water, but die off immediately if they are not in approximately the same depth in which their parents lived. It is a problem of the same kind as is connected with the existence of local races in many animals with pelagic larvae.

In the case of the difference arising separately in each generation, it must be produced either by selection or by the direct action of the environment and we have no data for determining which of these is the case.

(*b*) The male is certainly more variable and more highly correlated than the female in both deep sea and shallow forms.

(*c*) In both sexes the deep sea forms are more variable than the shallow water forms.

These facts are consonant with a greater selection of female than male and of shallow water than deep sea forms.

THE LAW OF ANCESTRAL HEREDITY*.

BY KARL PEARSON, F.R.S.

(1) To any one who has made a close study of heredity, whether it be in men, mice or plants, there are one or two conclusions which must be accepted at once. The first of these conclusions is:

(*a*) *That a knowledge of the characters of the parents does not accurately define the character in the offspring.*

Every one is familiar with the fact that given two pairs of parents possessing the like character in the same degree the offspring of one pair will differ from those of the other in respect of this character, and further that all the offspring of the same pair are not alike in the character; and this variation in the offspring of the same pair or of like pairs of parents may be very great indeed.

This point although familiar is frequently neglected. It is so important that it seems worth while to illustrate it by material at my disposal. The pedigrees I give are only samples of many hundreds in my possession, but they are conclusive. I turn first to coat-colour in thoroughbred horses.

(i) The mare *Ally* (bay) covered with *Little John* (bay) gave a roan colt. There are no black coats in the ancestry of either *Ally* or *Little John* back to their great-grandsires and great-grandams. *Jest Ally's* dam has, however, the *Delpini* grey colour, which accounts for the roan. *Ally* again covered, this time by *Interpreter* (bay), gives a *black* filly. There is no black blood in *Ally*, but *Interpreter's* paternal grandsire was the well-known black horse, *Socerer.* Thus two bay horses may give a roan, or two bays may give a black foal. I can cite any number of instances, of course, in which they give bay, brown or chestnut offspring.

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* The following paper is put together not as a reply to Mr W. Bateson's rhetorical attack on the Law of Ancestral Heredity (published in his recent book *Mendel's Principles of Heredity*), but simply to indicate to those interested in the matter what are really the fundamental assumptions involved in the Law of Ancestral Heredity, and how far it enables us to describe actually observed experience in man, horse and dog, which I am unable under any hypothesis to bring under Mendel's "Principles." In the course of this year various investigations on heredity in both plant and animal life will I hope be published, and these will tend to throw further light on the laws of heredity.

(ii) *Socerer* was a black horse, son of the black *Trumpator*, out of *Young Giantess*, a bay. But the four grandparents of *Socerer* were two chestnuts, a brown and a bay, and of the eight great-grandparents seven were bays and of one, the *Snap Mare*, I have not been able to ascertain the colour. Where did *Socerer* and *Trumpator* get their blackness from? *Snap* was brown, but his maternal grandsire was the black horse *Gypsy*. In the same way *Rinaldo* black was born of the chestnut *Whiskey Mare* covered by the brown stallion *Milo*, but in the fourth and fifth generation backwards comes in the black *Gypsy* blood through the brown stallion *Snap*.

(iii) A bay filly is the result of crossing the chestnut stallion *General Graham* and the grey *Beningborough Mare*. The chestnut stallion *Prophet* and the grey mare *Virago* give a grey filly. The chestnut stallion *Woodpecker* and the grey *Herod Mare* give a chestnut filly *Chestnut Skin*. Clearly the coat-colour of the parents will not define that of the offspring. The bay filly, however, of the *General Graham* and *Beningborough Mare* cross is elucidated when we know that while it had one chestnut and one grey grandparent, it had two bays for grandparents.

I could multiply these pedigrees indefinitely, but the above will be sufficient to demonstrate the point for horses, i.e. the coat-colour of a horse may be unintelligible unless we examine the ancestry. We may easily find offspring of all shades from grey to black, whose parents had the same colour, say both bays.

I now pass to dogs. Let us first take Basset Hounds. Here the colours are lemon, white and black, and if they all occur the hound is termed tricolour.

(i) In 1885 the tricolour bitch *La Fanfare* put to the tricolour dog *Bourbon* gave the tricolour pup *Bluette*,—melanism appeared in the offspring. In the same year the tricolour bitch *Queen Dido* covered by the tricolour dog *Bourbon* gave the lemon and white pup *Blonde*, or melanism disappeared. In 1887 the lemon and white bitch *Jessie* crossed with the lemon and white dog *K. Bendigo* gave the tricolour hound *Bendigo II*. Thus when melanism is present in the parents it may be absent in the children, or when absent in the parents, it may reappear in the children.

(ii) *His Lordship* was tricolour, but his parents *Scipio* and *Fama* were only lemon and white. His black becomes quite explicable when we note that of his four grandparents two were tricolour.

(iii) Or, in the same litter, the tricolour bitch *Iris* by the tricolour dog *Count* casts five hounds of which three are tricolour and two lemon and white only. It is thus impossible to predict from the character of the parents whether black will or will not appear in the offspring. But here again ancestry throws light on the matter. *Count's* parents *Ugly* and *Rosalind II.* were both lemon and white and *Count* got his black from his paternal grandparents, who were both tricolour. Of *Iris*' parents one showed melanism and the other did not.

What we are dealing with here is not a slight variation in shade, but the presence or total absence of black pigmentation. This point must be borne in mind.

These cases might be enough, perhaps, for dogs, but I cite the following instances out of many for greyhounds*.

I would first state that I have sought in vain for any single colour or combination of colours which breeds true to itself. There is as far as I have been able to ascertain absolutely nothing that can correspond to Mendel's "recessive" character. At first it seemed to me possible that black would be such. *Bessie Hay*, a black bitch covered by *Grinlaw* a black dog, had a litter of entirely black offspring in 1896, and repeated this performance with four instead of five whelps in 1897. But the black bitch *Finest Fury* covered by the black dog *Petronius* in a litter of eight had five black and three red offspring. Black *Middleton* and black *Raven* gave three black, one blue, a red and a fawn offspring, while the black bitch *Mayfly* crossed by the black *Hermas* produced four black and two fawn offspring. In fact almost every possible greyhound colour will be found in the offspring of two black parents,—black, blue, either of these and white, fawn, red, brindled! If we next take such a cross as black and red we find red *Recoil* covered by black *Fortuna Favente* giving six black offspring and we at once consider whether black may be "dominant." But the red bitch *Maid of all Work* threw in a litter to the black dog *Black Marauder* six brindled, one black, one blue offspring and a white bitch with one eye and two ears black. Black *Nebula* crossed by red *Herschel* gave five fawn offspring, one black and one blue and white. Parents of the same colours will be found to give litters of red, and black and white offspring. A blue sire (*Blue Peter VIII.*) and a black dam (*Flotilla*) will produce all the offspring red, while a black bitch covered by a black and white sire will give pure black dogs, pure white dogs, or mixtures of black, red or fawn with white. The black bitch *Queen of the Colonies* covered by the red fawn dog *Dodger* gave beside five black and three red offspring a white bitch. No single colour breeds true to itself, and while all these results are explicable on the basis of ancestry, the Mendelian can only pass them over by asserting that the greyhound is an incorrigible mongrel.

We may next pass to eye-colour in man and I give the following instances†. Father and mother both blue, two out of six children dark brown. Father and mother both blue, one out of six children dark brown (colour of father's maternal grandfather). Father and mother both light grey, four children light grey, and five black or very dark brown (colour of maternal grandmother). Now turn to the opposite end of the scale: Father and mother black or very dark brown, one of eight children light blue (colour of paternal grandfather). Father black and mother dark brown, one child out of three hazel (the colour of paternal grandmother). Father and mother very dark brown or black, two children blue and one black. Father and mother very dark brown or black, two out of four children blue (colour of maternal grandmother). It is clear that neither end of the eye-colour scale gives a recessive character which breeds true to itself. Now note what we get if we attempt to select for several generations *paying attention to ancestry*. Father and mother blue, all four grandparents blue, and the five recorded great-grandparents blue, all four children blue. Parents and grandparents all blue, all eight children blue. Father and mother light grey, and the four grandparents all light grey or blue, all seven children grey. Father and mother grey and all the known ancestry of both grey or blue, all four children grey or blue. Father and mother both dark brown, all known ancestry brown for both, out of nine children seven dark brown, one brown and one light brown. Father and mother hazel, all ancestry to great-grandparents hazel with the exception of a blue-eyed grandfather, all five children hazel.

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* Pedigrees taken from data collected by Mr Howard Collins.

† I gave the substance of such pedigrees to Prof. Weldon, who made use of it in his article on Mendel's Law in *Biometrika* (Vol. I. p. 242) in a passage criticised by Mr Bateson in his *Defence of Mendel*, p. 192. I first tested Mendel's theory of dominance on eye-colour in man when I read Mendel's paper in 1900 and then communicated the result to Professor Weldon.

Lastly let us look at crosses between blue and dark eyes. Father light blue and mother dark brown, three children blue, four children dark brown. Father dark brown and mother blue, two children blue and four brown-eyed. Father dark brown and mother blue, three brown and six blue-eyed children; here all mother's known ancestry was blue. If blue were "dominant" such results are impossible unless we suppose every dark brown parent above recorded was a hybrid. If blue be "recessive" it ought to breed true, but we have seen that it does not. But as a matter of fact we can pick out cases from the record of one blue and one very dark parent giving almost every conceivable result, e.g. father light blue and mother black, four children blue and one light grey. Father brown and mother blue, eight children ranging from brown to dark brown. In all cases an examination of ancestry throws light, if it does not fully explain in each case what has taken place. It is clear from these cases that parentage will not enable us to predict offspring definitely. But what is also equally clear is that if all the ancestry be the same the offspring may differ among themselves considerably, e.g. by the presence or total absence of black pigmentation. Hence we reach our second conclusion:

(*b*) *That a knowledge of the whole ancestry while it certainly limits the range of variation does not absolutely define the character of the offspring.*

(2) Before we leave the cases above it is just worth reiterating that nothing corresponding to Mendel's principles appears in these characters for horses, dogs, or men. If black or grey coat-colour in horses were "recessive," when two blacks were mated we should expect only black offspring, but black can disappear for a generation or even two and then reappear. Or, take a case like that of a grey horse *Viscount*, where grey remained dominant for three generations only to disappear before the chestnut of the mare *Blue Stocking* in the *Viscount* and *Blue-Stocking* filly *Miss Johanna*! It is the same with every coat-colour taken, its

relative constancy depends largely on the extent to which it has appeared in the ancestry, and one by one black, bay, chestnut, grey must be dismissed by the Mendelian as neither "recessive" nor "dominant," but as marking "permanent and incorrigible mongrels." The same remark applies to coat-colour in hounds, black must be the mark of incorrigible mongrelism for it is neither recessive nor dominant. It is the same with red and white, no colour in the parents gives for every case consistent offspring.

Nor again do we find that two black-eyed human beings nor two blue-eyed human beings mated together give any sign of the dominance of one or other of these extremes; any more than do intermediate tints to which the description of "incorrigible mongrelism" may by some be applied. It will be clear that for man (and this applies not only to eye-colour, but to hair-colour, and to a whole series of measurable characters* of which we have in each case more than a thousand instances in our Family Records), for horses in coat-colour, and for dogs in coat-colour† nothing approaching Mendelian principles holds. We have therefore to classify large portions of the animal kingdom as exceptions to Mendel's Laws,—and these are cases where the evidence is not based on five or ten individual crosses followed perhaps for two or at most three generations—but on 1000's of crosses, and where the pedigree has been or can be investigated for some of the material for five to ten generations. What may happen in the case of plant hybrids, I am not able from personal observation to assert, but there is enough weight of evidence here to make one pause before one is prepared to admit that Mendel, or his followers, can change each conception of life in which heredity bears a part!

(3) Taking our stand then on the observed fact that a knowledge neither of parents nor of the whole ancestry will enable us to predict with certainty in a variety of important cases the character of the individual offspring we ask: What is the correct method of dealing with the problem of heredity in such cases? The causes A, B, C, D, E, ... which we have as yet succeeded in isolating and defining are not always followed by the effect X, but by any one of the effects U, V, W, X, Y. We are therefore not dealing with causation but correlation, and there is therefore only one method of procedure possible; we must collect statistics of the frequency with which U, V, W, X, Y, Z respectively follow on A, B, C, D, E.... From these statistics we know the most *probable* result of the causes A, B, C, D, E and the frequency of each deviation from this most probable result. The recognition that in the existing state of our knowledge the true method of approaching the problem of heredity is from the statistical side, and that the most that we can hope at present to do is to give the *probable* character of the offspring of a given ancestry, is one of the great services of Francis Galton to biometry.

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* Mendelianism fails also for skin-colour in crosses between the black and white races of man.

† Other characters in pedigree stock are being taken into consideration at present.

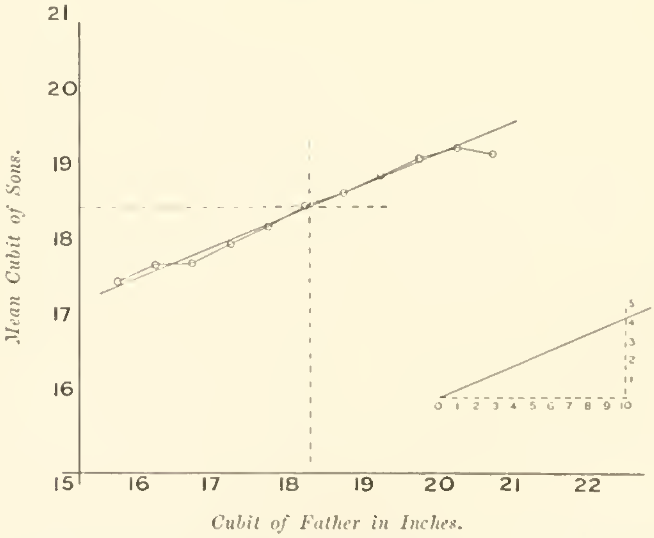
Yet since the publication of Francis Galton's *Natural Inheritance* in 1889 much progress has been made in the theory of multiple correlation, and also in our treatment of characters not quantitatively measurable, but which we may reasonably suppose to be based on quantitative factors,—as the degree of yellow pigment in the human eye. Further larger series of observations have been collected and reduced, and we are much clearer than we were ten years ago on the nature of regression and 'blend.'

Given two relatives *P* and *Q* the general theory of statistics tells us how to dress a table from which we can read off at once the most probable character of *P* when we know the character of *Q*. There is no biological theory in this, it is merely an arrangement of statistical information into classes. Such classes may be arranged on an actual quantitative scale, or as in the case of eye or hair colour on a qualitative scale, which with a high degree of probability corresponds to a true quantitative scale in some one or other pigment, whose amount cannot be actually determined. In both cases the methods of statistics allow us to determine:

(*a*) The variability in character of both *P* and *Q* in terms of the range of one of these arbitrary classes.

(*b*) The average variability in the same terms of an array of *P*'s corresponding to a given *Q*, or of an array of *Q*'s corresponding to a given *P*.

(*c*) The first two terms of the closest linear function which expresses the probable character of *P* or *Q* in terms of the known character of *Q* or *P*.



Inheritance of Cubit.

Probable value of cubit of son *Q*, for a known cubit in father *P*. A marked deviation from linearity only occurs in the extreme case of father's cubit, where there were only a few instances to base the son's mean upon.

All this has nothing in it peculiar to heredity, it is simply an application of the higher theory of statistics.

If we ask how closely the above linear function gives the probable Q-value from the known P-value; the answer is: Admirably, for those characters which have been actually tested. I have 21 tables involving three characters for diverse relationships in man and each embracing upwards of 1000 pairs. I choose one, the cubit, perfectly at random and it will be seen at once that, within the limits of random sampling, no function could give the probable character of Q for a given value of P with greater efficiency.

The line here dealt with is the so-called regression line; it is a purely statistical result, and has no relation to any biological theory or hypothesis as to heredity. Its determination depends on the so-called coefficient of correlation of P and Q, which we will write r_{pq}, and their variabilities σ_p and σ_q. In addition we must know the mean, the modal or, what is often sufficient, the median values of P and Q.

Now suppose that instead of one relative P we have any number $P_1, P_2, P_3 \dots$, and let the corresponding statistical constants be $r_{p_1q}, r_{p_2q}, r_{p_3q} \dots \sigma_{p_1}, \sigma_{p_2}, \sigma_{p_3} \dots$. Then the theory of statistics shows us that, if p_q be the most probable deviation of Q from the type of its generation, and $h_{p_1}, h_{p_2}, h_{p_3} \dots$ be the observed deviations of $P_1, P_2, P_3 \dots$ from the types of their respective generations, then:

$$p_q = J_1 \frac{\sigma_q}{\sigma_{p_1}} h_{p_1} + J_2 \frac{\sigma_q}{\sigma_{p_2}} h_{p_2} + J_3 \frac{\sigma_q}{\sigma_{p_3}} h_{p_3} \dots\dots\dots\dots\dots\dots\text{(i)},$$

where $J_1, J_2, J_3 \dots$ are known expressions involving only the r_{pq}'s, and can be calculated as soon as the latter have been found from observation.

Further the variability of Q about its most probable value, when we know the P's, takes on the average the value $\sigma_q \times \Delta$, where Δ is another well-known and calculable function of the r_{pq}'s.

The above is in no sense a biological theory, it is based on no data whatever except the actual statistics: it is merely a convenient statistical method of expressing the observed facts. If the facts are there it expresses them up to a certain point,—the most probable or most frequent value of the individual given his relatives. It cannot possibly within this range be upset by any hypothesis of heredity, Mendelian or otherwise, for it is based on no biological assumptions whatever. It is merely a convenient description of statistical tabulations.

I now proceed to limitations based on actual experience.

(i) The correlation coefficients between relatives are positive.

There is overwhelming evidence in favour of this in insects, animals and plants, for direct relationship back to great-great-grandparents and in a considerable range of collateral relationships—fraternal and avuncular.

But it may be shewn that if the r's are positive, increasing the number of relatives used as a basis of prediction reduces the value of Δ. The rate of reduction decreases considerably as we increase the number of correlations, but for the first increase from one on to two, three ... six, etc. it is very sensible.

We therefore conclude that:

There will be less average deviation from the probable character of an individual when we increase the number of relatives on which the prediction is based.

As a special case we say that:

Prediction will be closer when we use the ancestry as well as the parents in forming it, than when we use the parents alone.

As soon as we accept the positive character of the correlation coefficients between offspring and ancestry this is an absolutely certain result. It could only be invalidated if the parents defined the offspring absolutely, i.e. if Δ for two parents was zero, or the correlation with parentage perfect. But this is completely negatived by the fact [*] that the same pair of parents will produce offspring with very different characters,—e.g. two bay horses may give a black horse and not only a bay. Thus it seems to me that any hypothesis of inheritance which neglects ancestry is foredoomed to failure in the wide range of vital types for which we have shewn already that parentage does not define offspring uniquely. The variation in offspring of the same parents is not merely confined to slight differences in size of organs, but in many covers differences in colour, for example, as great as those upon which the Mendelians have based their discrimination between dominant and recessive characters.

(ii) In actual measurement on many series the r's come out less than unity and there are theoretical reasons for holding that this must always be the case.

Hence if the variability of the offspring generation be not much larger than that of the generation of any relative, the probable deviation from type of any individual will only receive from any relative a fraction of his observed deviation from his type.

This is Mr Galton's principle of regression, the exceptionality of P_1 is on the average only exhibited in part by his relative Q. This purely statistical and legitimate conclusion was seized upon as a biological law, and all life, but for constant selection, was assumed to be in a state of regression to some distant ancestry. The expression (i) does not warrant this assertion at all. To begin with there is no reason to suppose:

(a) That $\sigma_q = \sigma_{p_1} = \sigma_{p_2} = \sigma_{p_3} = \dots$ or that the variability of all generations is the same. In numerous and large series I have not found it so[†].

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[*] For all the characters yet dealt with whether quantitative or qualitative in insects, animals and plants, there is no approach to $\Delta = 0$.

[†] The offspring generation tends almost universally to be more variable than that of any ancestral generation—parents are a selected portion of the community.

(*b*) $h_{p_1}, h_{p_2}, h_{p_3} \dots$ are deviations from the type of each generation. I have never found the types of each generation identical. They often differ very sensibly.

(*c*) Each individual ancestor contributes only a fraction of his deviation from type to the probable deviation in type of the offspring. But if the ancestry be maintained for two or three generations at a given deviation from type the contributions of the different terms of (i) provide a probable deviation in type of offspring which may be equal to or may even exceed that of the ancestry.

This point is over and over again forgotten when biologists talk of regression as if it were a persistent retrogressive factor.

As a matter of fact with the numbers we already have for man, breeding true for three or four generations gives a value of p_q within a small percentage of the selected h_p, and this is only slowly modified, if the stock continues to breed true to itself. There is no such thing in statistical theory as a *necessary* regression if the selected stock pair with selected stock. The rapid establishment of breeds is not evidence therefore against the present view of heredity. On the contrary it flows at once from it.

(4) So far there is practically no assumption in our treatment of heredity, which has not been justified by ample experience, e.g. the close linearity in distribution of the probable value of a character in one relative for a known value of a character in a second. Further it enables us to predict probable values from any group of known relatives. When however we predict from direct ancestors only we state more particularly the law of ancestral heredity. If we are content with parents, or possibly in some cases with grandparents also, we have material for making a fairly close prediction, but if we want to deal with whole lines of ancestry, we are met at once by the difficulty of collecting statistical material for the correlation of the offspring's character with that of the higher antecedents. In man few observations or measurements have been made on a higher than the grandparental generation, and even in pedigree animals, where we can go much further back, the characters recorded are never quantitative, but concern colour or markings. To get over this absence of material Mr Galton originally proposed that we should correlate not with each individual ancestor but with the mean of each ancestral generation, females being reduced to a male standard. There is no assumption in this because, of course, correlation can statistically be worked for any such group of ancestry. But as we do not know the values of the correlation coefficients of the higher groups, Mr Galton suggested that we should take the $J_1, J_2, J_3, \dots$ of equation (i) on p. 217 equal to $\frac{1}{2}, \frac{1}{4}, \frac{1}{8} \dots$ respectively. This was undoubtedly an assumption, although not an unreasonable one *à priori*. That the intensity of ancestral heredity diminishes as we go backwards is demonstrated by both experiment and observation; and a geometrical series naturally first arises as a measure of such diminishing influence. But the law could only be demonstrated on the base of the first few terms of the J-series, and

Mr Galton's evidence from stature only gave a rough test of the series $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$ having the right trend. He did not himself claim more for it than this; thus after suggesting that $J_1 = \frac{1}{2}$ and $J_2 = \frac{1}{4}$, he writes: "It would, however, be hazardous to extend this sequence with confidence to more distant generations*."

Referring again to this point, Mr Galton says in 1897: "I stated it briefly and with hesitation in my book on *Natural Inheritance*†."

When I personally first came to investigate the matter from the more purely mathematical side, it seemed to me better to start with the coefficients of correlation and make no assumption as to the values of the regression coefficients in the J-series of p. 217‡. Each coefficient found would serve as the basis of a limited amount of prediction, and we must plod away finding coefficients for as many generations for as many characters in as many races as we possibly could. When these were known, we should possibly be able to predict the relationship between the correlation coefficients of successive generations even beyond the limits of observation and experiment. If we had to make an hypothesis at all, it seemed to me most reasonable to suppose the correlation not the regression coefficients of each generation of ancestry to diminish in a geometrical series. This view I kept steadily before me, but it was very difficult to find material going sufficiently far back to test it. Theoretically I shewed that if these correlations formed a geometrical series then the regression coefficients, or J's, would also form a geometrical series, if we neglected the effects of assortative mating§, but the data I had collected did not as far as they went justify Mr Galton's proposed $\frac{1}{2}, \frac{1}{4}, \frac{1}{8} \ldots$ series.

Taking the matter broadly we may say that the Law of Ancestral Heredity implied two ideas:

(*a*) That the proper method to proceed in heredity within the race is by the statistical theory of multiple correlation—this does not exclude the truth of any physiological theory, although it may serve to confirm or refute such a theory. The correlation between parent and offspring in man will remain about ·45 to ·5, whatever theory of gametes may eventually be accepted, and the prediction of probable character in son from actual character in father remains equally valid.

(*b*) That a knowledge of the nearer coefficients of correlation, i.e. those between offspring, and parents, grandparents, and possibly great-grandparents, will suggest the more distant ones, and that probably these, and consequently the multiple regression coefficients, are expressible as a geometrically decreasing series.

This is the hypothesis involved in my own expression of the Law of Ancestral

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* *Natural Inheritance*, p. 136.

† *R. S. Proc.* Vol. XLI. p. 401. I have elsewhere shewn that the evidence from Basset Hounds admits of other interpretations. *R. S. Proc.* Vol. 66, p. 140 *et seq.*

‡ "Regression, Panmixia and Heredity." *Phil. Trans.* Vol. 187, p. 253 *et seq.*

§ "On the Law of Ancestral Heredity." *R. S. Proc.* Vol. 62, p. 394.

Heredity, and I believe it is a generalisation, which Mr Galton might, perhaps, be willing to accept in place of his much more definite series of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, etc.

(5) Assuming we had the data for quantitatively measurable characters one would prefer to deal with such, but failing this I have asked myself how far the successive correlation coefficients for ancestry in qualitative characters (which we may reasonably suppose to have a quantitative scale behind them) fall into a geometrical progression. In eye-colour for man I have previously given* the four parental and the eight grandparental correlations. Mr F. Lutz has completed† the series, as far as it is possible to do on Mr Galton's material, by investigating the correlation of offspring with the great-grandparents, arranged in four groups. Mr L. Bramley-Moore and I had given the four parental and two of the grandparental coat-colour correlations for horses in 1899‡. Mr N. Blanchard published two further grandparental correlations recently§, and he has now completed the whole series of eight, and they will be found in a note attached to this paper. Dr Alice Lee worked out the great-grandparental and the great-great-grandparental correlation from the data provided by Mr Blanchard's MS. pedigree books; these are given in a second note below.

We have reached the following results:

Mean Correlation	Eye-colour in Man	Coat-colour in Horse
Parental	·4947	·5216
Grandparental	·3166	·2976
Great-grandparental	·1879	·1922
Great-great-grandparental	—	·1469

When we note that the probable error of these coefficients is about the order ·03, one is at once struck with their substantial agreement. We are forced to the conclusions:

(i) That eye-colour in man and coat-colour in horses are inherited in sensibly the same manner.

(ii) That the original series proposed by Mr Galton $\frac{1}{3}$, $\frac{1}{9}$ ... for these correlations‖ will not fit them at all.

It might be supposed that pigmentation followed some other rule than measurable characters. But in recent work on measurable characters for 12 series, each numbering upwards of 1000 cases, I have found for man the mean parental value ·46. This is the largest inheritance series for man yet worked out, being based on measurements of upwards of 1100 families, so I am pretty confident that parental inheritance for man lies between ·45 and ·5, and is quite incompatible with a value of $\frac{1}{3}$*.

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* *Phil. Trans.* A, Vol. 195, p. 106.

† See Note on the Influence of Change in Sex on Inheritance. *Biometrika*, Vol. II. p. 237.

‡ *Phil. Trans.* A, Vol. 195, p. 93.

§ *Biometrika*, Vol. I. p. 361.

‖ *Natural Inheritance*, p. 133.

I now turn to the problem of how far the data can be represented by a geometrical series. I fitted the best geometrical series first to the horse only, then to man, and finally to both with the following results.

A. Horse, observed	Best Geometrical Series	Man, observed
·52	·48	·49
·30	·32	·32
·19	·21	·19
·15	·14	—

B. Man, observed	Best Geometrical Series	Horse, observed
·49	·50	·52
·32	·31	·30
·19	·19	·19
—	·12	·15

In the first case, the series being a, ar, ar^2, ... we have:

$$a = {\cdot}4809 \text{ and } r = {\cdot}6602.$$

In the second case:

$$a = {\cdot}5007 \text{ and } r = {\cdot}6167.$$

If we take the best geometrical series embracing the data for both races we have:

C. Horse, observed	Man, observed	Best Geometrical Series
·52	·49	·49
·30	·32	·32
·19	·19	·20
·15	—	·13

Here we have:

$$a = {\cdot}4921 \text{ and } r = {\cdot}6428.$$

Lastly, if we take $a = {\cdot}5$, $r = \frac{2}{3}$ as close round numbers, we have:

D. Horse, observed	Man, observed	Close Series
·52	·49	·50
·30	·32	·33
·19	·19	·22
·15	—	·15

Within the errors of observation this close series represents excellently the observed results for either eye-colour in man or coat-colour in the horse. In other words:

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* As long as the only data for a measurable character in man—Mr Galton's stature data—gave a parental correlation of about $\frac{1}{3}$ it seemed needful to emphasise the distinction between the results for eye-colour and stature. In the light of my present knowledge, the distinction between the two classes of characters seems not so clear or needful. A large series of pedigree dogs at present in hand gives a parental correlation almost identical with that of horse and man.

As far as the available data at present go inheritance coefficients for ascending ancestry are within the limits of observational error represented by a geometrical series and by the same series.

From this it follows that[*]: *The contributions of the ancestry also follow a geometrical series, although not that originally proposed by Mr Galton.*

(6) Mr Galton has assumed, that if the relatives include all the ancestry, and if all these ancestry had the same deviation h, the offspring will have a probable deviation of h. This is really the introduction of a biological hypothesis, the truth of which can only be tested by observation. Mr Galton deduces this result in the following manner: he supposes a stable population, i.e. one in which the mean and variability of each generation remain the same, and the parents in each generation are the whole or at any rate a random sample of that generation; there must also be no reproductive selection, or fertility must not be correlated with the character of which the inheritance is under consideration. Further there must be no assortative mating. Under these circumstances we have the following form of (i) p. 217:

$$p_q = (J_1 + J_2 + J_3 + \dots)\, h,$$

and since Mr Galton holds that p_q will then equal h, we have

$$J_1 + J_2 + J_3 + \dots = 1 \quad \dots\dots\dots\dots\dots\dots\dots\dots(2).$$

Any geometrical series $\epsilon(1 + \rho + \rho^2 + \dots)$ for the J's will satisfy this condition if $\epsilon = 1 - \rho$; the series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$ is not the only one satisfying (2)[†].

But it is doubtful[‡] how far this conclusion is justifiable. Statistically it is of course unnecessary. (i) is a relation between the probable deviation of Q from its mean and the actual deviations of each P from their individual means. There is no reason why the means of Q and of all the P's should be the same. They may be different owing to environment or to selection. Further there is no reason why the variabilities should be the same; parents may be a selection out of the general community in each generation. As soon as we realise that the h's are deviations from the generation means, and these are not all the same, and that the variability in each generation differs, the need for the relation

$$J_1 + J_2 + J_3 + \dots = 1$$

ceases to be apparent.

In data like eye-colour in man and coat-colour in thoroughbred horses there has been undoubtedly a secular change going on; the proportion of blue eyes in the latest offspring is considerably less than in the great-great-grandparents, while the early grey horses have largely disappeared from the stud-book. Further we find each ancestral generation is roughly speaking less variable than the one below it. Men who live to be parents are a more select group than their offspring and this is still more true of grandparents and higher ancestry. To be a great-great-grandparent means to have a healthy stock, and such men form a selection only of the community in each generation. Hence neither for man nor the thoroughbred horse need we *à priori* expect to find, for the correlation coefficients as actually determined, the relation $J_1 + J_2 + J_3 + \ldots = 1$ or $\epsilon = 1 - \rho$ satisfied. The actual statistics for man and the horse conclusively prove that it is not satisfied. If we had a population in which all members were randomly mated and were parents of equal fertility; further if every generation had the same mean and variability, then we might have $\epsilon = 1 - \rho$, but then the correlations of ancestry and offspring would probably take very different values from what we find them to have in the cases of man and horse under existing circumstances.

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[*] *R. S. Proc.* Vol. 62, p. 394.

[†] *R. S. Proc.* Vol. 62, p. 402, and Vol. 66, p. 147.

[‡] It is hardly consonant, for example, with what we know of eye-colour, where there is an assortative mating coefficient of ·1002: see *Phil. Trans.* A, Vol. 195, p. 113.

(7) Supposing all mating to be random* we can find ϵ and ρ and all the J's by the method of my first paper on the "Law of Ancestral Heredity†." In the notation of the present paper, if ar^n be the correlation coefficient of "the $n+1$th mid-parent" with the offspring, and if $J_q = \epsilon'\rho'^{q-1}$, we find:

$$r = \rho' + \epsilon' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3),$$

$$-a + \epsilon'\rho' a/(1 - \rho' r) = -\epsilon' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4);$$

whence:

$$\rho'^2 - \rho' \frac{1 + r^2 - 2ar}{r - a} + 1 = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5).$$

Hence taking $a = \cdot 5$ and $r = \frac{2}{3}$ as close enough to the observed values we have:

$$\rho' = \cdot 2251, \qquad \epsilon' = \cdot 4415.$$

Formula (i) of p. 217 using "mid-parents" now becomes:

$$p_q = \cdot 4415 \left\{ \frac{\sigma_0}{\Sigma_1} h_{p_1} + \cdot 2251 \frac{\sigma_0}{\Sigma_2} h_{p_2} + (\cdot 2251)^2 \frac{\sigma_0}{\Sigma_3} h_{p_3} + \ldots \right\}.$$

Here $\Sigma_1, \Sigma_2, \Sigma_3 \ldots$ are the standard deviations of the mid-parental groups, and $\Sigma_s = \sigma_s/(\sqrt{2})^s$, if σ_s be the standard deviation of the sth generation of ancestors. Thus:

$$\begin{aligned} p_q &= \cdot 4415 \times \sqrt{2} \left\{ \frac{\sigma_0}{\sigma_1} h_{p_1} + \cdot 2251 \sqrt{2} \frac{\sigma_0}{\sigma_2} h_{p_2} + (\cdot 2251 \sqrt{2})^2 \frac{\sigma_0}{\sigma_3} h_{p_3} + \ldots \right\} \\ &= \cdot 6244 \left\{ \frac{\sigma_0}{\sigma_1} h_{p_1} + \cdot 3184 \frac{\sigma_0}{\sigma_2} h_{p_2} + (\cdot 3184)^2 \frac{\sigma_0}{\sigma_3} h_{p_3} + \ldots \right\} \\ &= \epsilon \left\{ \frac{\sigma_0}{\sigma_1} h_{p_1} + \rho \frac{\sigma_0}{\sigma_2} h_{p_2} + \rho^2 \frac{\sigma_0}{\sigma_3} h_{p_3} + \ldots \right\}. \end{aligned}$$

Clearly $\epsilon = \cdot 6244$ is not equal to $1 - \rho$ or $\cdot 6816$, or Mr Galton's hypothesis is certainly not satisfied either for man or horse. But since we know in these cases that $\sigma_1 = \sigma_2 = \sigma_3 = \ldots = \sigma_0$ is not true, and further that $h_{p_1}, h_{p_2}, h_{p_3} \ldots$ are measured from different means, we have no *à priori* right to suppose it ought to be satisfied.

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* *R. S. Proc.* Vol. 66, p. 149.

† *R. S. Proc.* Vol. 62, p. 394.

(8) It is not without interest to follow up what would happen in man and horse if, with our present numbers, we supposed the conditions for a stable population to be really satisfied. Let us select about the deviation h from the common mean for s generations, then the offspring would have a probable deviation from that mean of

$$\epsilon \frac{(1-\rho^s)}{1-\rho} h.$$

If we then ceased to select, but bred the offspring among themselves for t generations, the offspring of the tth generation would have a probable deviation from the mean of

$$\epsilon \cdot \frac{1-\rho^s}{1-\rho} \cdot (\sqrt{2}\, r)^t\, h.$$

Let us turn these results into numbers, remembering that $\epsilon = \cdot6244$ and $\rho = \cdot3184$.

Result of selecting all ancestry for one generation $= \cdot62h$
,, ,, ,, two generations $= \cdot82h$
,, ,, ,, three ,, $= \cdot89h$
,, ,, ,, innumerable ,, $= \cdot92h.$

In other words indefinitely long selection would only raise us to ·92 of the desired character. We should therefore not get the whole of the desired character *without selecting beyond it.* But on the other hand by merely selecting for two generations we should get within ten, and for three generations within four per cent. of the possible maximum of indefinitely protracted selection. Hence selection for a very few generations would raise the selection rapidly to within a small percentage of the maximum result.

Generations after which selection is stopped and simple breeding-in of selected stock follows.

	First	Second	Third	Innumerable
Last offspring of selection ...	$\cdot62h$	$\cdot82h$	$\cdot89h$	$\cdot92h$
First generation of in-bred stock	$\cdot59h$	$\cdot78h$	$\cdot84h$	$\cdot86h$
Second ,, ,, ,,	$\cdot56h$	$\cdot73h$	$\cdot79h$	$\cdot81h$
Third ,, ,, ,,	$\cdot52h$	$\cdot67h$	$\cdot74h$	$\cdot77h$
Tenth ,, ,, ,,	$\cdot35h$	$\cdot46h$	$\cdot49h$	$\cdot51h$

Next let us suppose selection stopped and the stock to breed-in or mate with its likes. The above table shows how it would slowly degenerate. The essential point to be noted is that while two or three generations of selection would carry us up to 80 or 90 per cent. of the desired character, ten generations following this of merely in-breeding without any selection would not have cost us 50 per cent. of the character so acquired. Quick influence of selection, slow effect of regression would be the result of combining the actually observed values with Mr Galton's theory as to what should hold for a stable population.

The reader must not forget that the illustration here given is absolutely hypothetical; the statistical constants obtained are deduced from material to which Mr Galton's conditions hardly apply even as a rough approximation. Yet it is possible that something of the kind here indicated may occur in special cases. But if so, we ought to be very cautious of using vague categories in problems of heredity. If the mean tint of a seed, say, be yellow, and h would carry us well into the green end of the scale, $\cdot 4h$ might still be green, and certainly for three and possibly for a good many more generations we might consider the stock arising from a *single* selection to be breeding true to itself, although actually it might be slowly regressing to the original tint of the early ancestry. It seems absolutely necessary in all such cases to have some colour standard and determine quantitatively whether successive generations do or do not tend to slowly approach or depart from it. The statement that ancestry has no influence might well be deduced by the use of a rough category, which would still class $\cdot 62h$ with $\cdot 52h$.

(9) *Conclusions.*

(*a*) In all cases as those of man, horse and dog, where parents of identical character do not produce identical offspring, the theory of statistics shows us that closer prediction may be obtained when we predict from many instead of few relatives. This follows from the consideration that all the heredity coefficients are *positive.*

Attention is therefore properly paid to ancestry in such cases, and it is very misleading to suggest that any law of heredity can be universal which neglects ancestry.

(*b*) The law of ancestral heredity in its most general form is not a biological hypothesis at all, it is simply the statement of a fundamental theorem in the statistical theory of multiple correlation applied to a particular type of statistics. If statistics of heredity are themselves sound the results deduced from this theorem will remain true whatever biological theory of heredity be propounded.

(*c*) The law of ancestral heredity as founded on the theory of multiple correlation involves no *biological* theory of regression. The term regression has unfortunately been taken from statistical theory and interpreted in a biological sense. In statistics the regression is always to the mean of the forecasted character, but no assumption is made that this mean is identical with that of the foreknown character*. Further, if there be a number of cognates, we can *à priori*, i.e. before quantitative analysis, not state whether the total amounts they will contribute to the predicate will or will not indicate a biological regression*.

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* There is a "regression" for example if we predict breadth of skull from its length. I think it might be useful to adopt the word "predicate" for the forecasted and "cognate" for the foreknown character.

(*d*) The law of ancestral heredity as a pure statistical statement has been supplemented by hypotheses which need verification and are semi-biological. These may (i) either be hypotheses as to the multiple regression coefficients, or (ii) as to the individual coefficients of correlation.

Mr Galton has assumed that the former are the geometrical series $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ .... I have assumed that the *mean* correlation coefficients for each ancestral generation form some geometrical series. It follows from my assumption that the regression coefficients would also form a geometrical series, but not necessarily Mr Galton's.

(*e*) In eye-colour in man and coat-colour in horses the mean ancestral coefficients of regression form within the limits of errors of random sampling a geometrical series, but it is not Mr Galton's series:

$$\cdot 5000, \quad \cdot 2500, \quad \cdot 1250, \quad \cdot 0625 \ldots,$$

but more nearly:

$$\cdot 6244, \quad \cdot 1988, \quad \cdot 0630, \quad \cdot 0202 \ldots.$$

In other words actual statistics show that in man and horse the parents are much more and the grandparents and higher ancestry less influential than on Mr Galton's hypothesis.

Thus the law of ancestral heredity (by which we are to understand the theory of multiple correlation together with the hypothesis that the mean ancestral correlations or the regression coefficients form a geometrical series) fits the data for horse and man remarkably well.

(*f*) In man and horse we find the means of each generation differ, and further the variabilities of each generation differ. It is an assumption to suppose under these circumstances that the sum of the regression coefficients (or rather that part of them which we have represented by J_1, J_2, J_3, ...) is unity.

Any geometrical series for the regression coefficients which satisfies the condition $\epsilon = 1 - \rho$ (like Mr Galton's does) would give on the supposition of equal means and variabilities for each generation *no regression whatever* after a stock began to in-breed.

If we may apply (which is very doubtful) our values for the J's in man and horse to cases in which the means and variabilities of each generation remained the same, there would result the following principle:

Two or three generations of selection would produce a stock of upwards of 90 per cent. of the selected character, but no amount of selection, unless of a greater

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[*] It is curious that the original numbers selected by Mr Galton for the regression coefficients $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{8}$, etc. indicate no regression whatever towards the predicate mean, after the first generation, if the stock in-breeds or breeds with its likes. It is characteristic of how conceptions are misunderstood, that this point of "regression" is what the majority of biologists have seized as the one easily comprehended principle out of the whole of Mr Galton's work!

than the desired amount of character, would give us more than 92 per cent. of it. After selection ceases a very slow regression sets in, which would be hardly perceptible without very definite quantitative measurement for the first three or four generations of in-breeding.

No rough classification like a Mendelian category would enable us to test whether such slow regression is really taking place, i.e. whether "recessives" are really breeding true to their stock*. And above all no Mendelian theory can (*a*) replace the statistical treatment unless it shows that offspring are absolutely determined by their parents, for if the correlation be not perfect, a better prediction must be obtainable by using the positively correlated ancestry, or (*b*) be proved without careful quantitative analysis of the variations falling within the wide categories adopted.

Appendix I. Note on a paper by G. U. Yule.

This paper was written before I was aware that Mr G. U. Yule was writing some account of the law of ancestral heredity, and was sent to press before I had seen his paper (*The New Phytologist*, November and December, 1902). With much of his paper, I agree, for example, with his insistence on the point that the laws of intra-racial heredity are not incompatible with Mendelian principles holding for hybridisation. I should lay, however, far less stress than he does on the value of the existing evidence in favour of those principles. That evidence certainly justifies the making of new and crucial experiments, but these will have to be made with much greater caution and closer quantitative definition of the categories employed before we can say with certainty how far either of the two principles of dominance and segregation actually applies even to the cases of hybridisation already dealt with by the Mendelians. I shall be surprised if the laws which govern intra-racial and inter-racial heredity are not more closely related than appears to me possible under the recent formulation of Mendelian Principles by Mr Bateson. Further Mr Yule does not seem to me to have clearly expressed my personal position with regard to the law of ancestral heredity. I believe that my memoir of 1896 was the first in which the equations of multiple regression were worked out and applied to the problem of heredity. Such equations are, I presume, what Mr Yule refers to and says may be termed the law of ancestral heredity. But the objection to such form of the law is simply the impossibility of working it out for all the ancestry, and for all characters of all races. The problems that arise at once in our minds are: Is there any relationship between the correlations of offspring and successive ancestors? Are these correlations in cases, where direct selection of the characters is small or can be allowed for, the same or approximately the same for different characters of the same race, or for divers characters of different races? Is there any relation between the correlations, which appears fairly in accordance with observation and yet will simplify the otherwise appalling complexity of the multiple regression equations? The memoir of 1896 gave the general regression equations, and indicated what results would flow supposing one or other of a variety of hypotheses due to Galton, Weismann and others were true. The memoir of 1898 adopted the simpler hypothesis that the correlation coefficients decrease in geometrical progression, *it did not involve the fixity of the numerical constants of heredity* which Mr Yule tells us has not stood the test of time. This simpler hypothesis, discussed in the present paper, still seems to me to stand the test of time. Lastly, I think, the reader of Mr Yule's paper would believe from the words: "Selective mating, natural selection, reproductive selection, the effect of circumstance, had all in turn to be recognised as causes affecting the values of the constants of heredity," that these factors were introduced *post hoc*, i.e. after observation had failed to give fixity of constants, whereas almost the reverse is the true case. The definition of selective mating was given in my first paper to the Royal Society on biparental inheritance in 1895[*] before I had applied the formulæ to actual data at all. Reproductive selection was considered in 1896[†], and the influence of natural selection on correlation (including the coefficients of heredity) was given in College Lectures of the session 1896–7, and put into a form identical with that of the memoir just issued[‡] in August 1897. The memoir of 1898 on the law of ancestral heredity was written with a knowledge of the influence of the three factors of selective mating, natural selection and reproductive selection on the constants of heredity. As a matter of fact they had been biometrically defined in or before the memoir of 1896, and were not "all in turn recognised as causes affecting the values of the constants of heredity," or used to bolster up some theory which had not stood the test of time.

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* For example, if we selected extreme chestnut (towards roan-chestnuts) for two generations only in the horse, yet if the progeny in-bred for 10 generations, the offspring would still be well in the chestnut range of coat-colour, and no category like chestnut would tell us whether the horses were breeding true or regressing.

What I felt in 1896, I still feel, namely: that the problem of heredity from the purely statistical standpoint is an extremely difficult one, that the factors of selective mating, natural selection, reproductive selection, environmental influence are extremely complex, especially when we place plants or insects in very artificial surroundings for the purpose of experimenting on their laws of reproduction. Still these factors do not invariably tend in one direction, and when we take long series, as free from their influence as is practically possible, we shall be able to judge from the clustering of the heredity coefficients round certain values—a phenomenon which I think is obvious in existing results—what in broad lines is the quantitative intensity of heredity for different relationships, and for different characters in different species. Mr Yule indicates that he has a theory which allows him to determine the influence of the environmental, reproductive, selective and mating factors; I can only say that I, as well as most biometricians, will heartily welcome its publication, if it not only allows, in the case of any heredity statistics which it is feasible to collect, for the correcting of the heredity constants for all these influences, but achieves this by some hypothesis which is *à priori* as reasonable, and *à posteriori* as justified as the simple one that ancestral correlations diminish in a geometrical progression.

Appendix II. On Inheritance (Grandparent and Offspring) in Thoroughbred Racehorses.

BY NORMAN BLANCHARD, B.A.

(1) AT the suggestion of Professor Karl Pearson I have recently worked out the remaining four cases for the inheritance of coat-colour in thoroughbred horses, viz. those between foals—colts or fillies—and their paternal and maternal grandams. The work was done in a somewhat different manner to that on the four earlier tables. In order to consider the bearing of coat-colour inheritance on Mendel's theory, actual colour pedigrees were formed going back to the great-great-grandparents and in some cases to the fifth and eighth generation of ancestry. Further, in order to get more striking colour contrasts than are now-a-days possible, the first three volumes of the stud-book were dealt with, I thus went back to the earliest records of coat colour in thoroughbreds. Here we find 2 to 3 per cent. of black horses and 9 to 10 per cent. of

[*] "On Regression and Inheritance in the case of two Parents," *R. S. Proc.* Vol. 58, 1895.

[†] "On Reproductive Selection," *R. S. Proc.* Vol. 59, p. 301, 1896.

[‡] *Phil. Trans.* Vol. 200, pp. 1—66, 1902.

greys, while in the recent studbooks these percentages are hardly 2 per cent. and ·5 to 1 per cent. respectively. In the early colour records the finer distinctions, e.g. "black or brown," "bay or chestnut," etc., of the modern terminology rarely appear, but this is not of importance. A thousand cases were taken for each series, and the correlation Tables I. to IV. were constructed as in Pearson and Bramley-Moore's memoir on *Inheritance of Coat-Colour in Horses**, in fact the method there used was followed identically, the fourfold division between chestnut and bay being used for the short tables. The following results were obtained, the notation being that of Professor Pearson's memoir: *On the Correlation of Characters not quantitatively measurable*†.

(2) Table I. reduces to

Paternal Grandams.

		Bay and darker	Chestnut and lighter	Totals
Colts.	Bay and darker ...	486	188	674
	Chestnut and lighter	172	154	326
	Totals	658	342	1000

This gives:

$$h = \cdot 4070105, \quad H = \cdot 3672283,$$
$$k = \cdot 4509851, \quad K = \cdot 3603667,$$

and the equation:

$$\cdot 3212093 = r + \cdot 0917778 r^2 + \cdot 1107746 r^3 + \cdot 0606283 r^4$$
$$+ \cdot 0308582 r^5 + \cdot 0443421 r^6 + \cdot 0101909 r^7 + \cdot 0339243 r^8 + \text{etc.}$$

the root of which is

$$r = \cdot 3085.$$

From Table II. we have

Maternal Grandams.

		Bay and darker	Chestnut and lighter	Totals
Colts.	Bay and darker ...	539	149	688
	Chestnut and lighter	198	114	312
	Totals	737	263	1000

giving:

$$h = \cdot 6341100, \quad H = \cdot 3300624,$$
$$k = \cdot 4901880, \quad K = \cdot 3537792,$$

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* *Phil. Trans.* A, Vol. 195, pp. 92 *et seq.*

† *Phil. Trans.* A, Vol. 195, pp. 1—47.

and the equation:

$$\cdot 2735648 = r + \cdot 1554165r^2 + \cdot 0631895r^3 + \cdot 0928545r^4$$
$$+ \cdot 0100882r^5 + \cdot 0608670r^6 - \cdot 0007343r^7 + \cdot 0412678r^8 + \text{etc.}$$

the root of which is

$$r = \cdot 2614.$$

From Table III.

Paternal Grandams.

		Bay and darker	Chestnut and lighter	Totals
Fillies.	Bay and darker ...	525	189	714
	Chestnut and lighter	175	111	286
	Totals	700	300	1000

giving:

$$h = \cdot 5243986, \quad H = \cdot 3517190,$$
$$k = \cdot 5689150, \quad K = \cdot 3393338,$$

and the equation:

$$\cdot 2111434 = r + \cdot 1491612r^2 + \cdot 0817246r^3 + \cdot 0906581r^4$$
$$+ \cdot 0138143r^5 + \cdot 0606133r^6 + \cdot 0014633r^7 + \cdot 0420798r^8 + \text{etc.}$$

the root of which is

$$r = \cdot 2041.$$

From Table IV.

Maternal Grandams.

		Bay and darker	Chestnut and lighter	Totals
Fillies.	Bay and darker ...	542	163	705
	Chestnut and lighter	187	108	295
	Totals	729	271	1000

giving:

$$h = \cdot 6097900, \quad H = \cdot 3312572,$$
$$k = \cdot 5388334, \quad K = \cdot 3380886,$$

and the equation:

$$\cdot 2505039 = r + \cdot 1750152r^2 + \cdot 0701968r^3 + \cdot 0976633r^4$$
$$+ \cdot 0088602r^5 + \cdot 0630885r^6 + \cdot 0001216r^7 + \cdot 0419921r^8 + \text{etc.}$$

the root of which is

$$r = \cdot 2392.$$

(3) The eight grandparental correlations for the horse are given below together with the corresponding ones for eye-colour in man for the sake of comparison. The whole series has now been determined by Pearson and Bramley-Moore* or myself†.

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* *Loc. cit.* p. 93.

† *Biometrika*, Vol. I. p. 361.

		Horse. Coat-colour	Man. Eye-colour
1	Pat. Grandfather and Son ...	·3238	·4213
2	Pat. Grandfather and Daughter	·3609	·3802
3	Mat. Grandfather and Son ...	·3590	·3717
4	Mat. Grandfather and Daughter	·3116	·2969
5	Pat. Grandmother and Son ...	·3085	·2722
6	Pat. Grandmother and Daughter	·2011	·2205
7	Mat. Grandmother and Son ...	·2614	·2623
8	Mat. Grandmother and Daughter	·2392	·3180
	Mean	·2961	·3179

It will be seen at once that the *mean* result for men is substantially in agreement with that for horses. We can hardly suppose grandparental correlation for the characters in man and horse dealt with to differ much from ·3. The mean value of grandparental correlation in the horse for cases 1 to 4 drawn from the modern studbooks is ·3388 and for cases 5 to 8 drawn from the older records ·2533. I believe this is due to the close relationship of so many of the horses in the earliest period of breeding. The difference may also to some extent depend on the somewhat rougher appreciation and description of colour. If there be any influence of change of sex in the intensity of heredity for coat-colour in the horse, it is not manifested in the above statistics, which thus differ widely from those of eye-colour in man.

TABLE I.

Colts and Paternal Grandams.

Paternal Grandams.

Colts.	bl.	bl./br.	br./bl.	br.	br./b.	b./br.	b.	b./ch.	ch./b.	ch.	ch./ro.	ro./ch.	ro.	ro./gr.	gr./ro.	gr.	Totals
bl.	8	—	—	6	—	—	11	—	—	12	—	—	—	—	—	3	40
bl./br.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
br./bl.	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1	2
br.	8	—	—	39	—	—	75	—	—	20	—	2	—	—	—	7	151
br./b.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
b./br.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
b.	13	—	—	95	—	—	230	—	—	101	—	2	1	—	—	39	481
b./ch.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
ch./b.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
ch.	2	—	—	27	—	—	113	—	—	82	—	1	—	—	—	16	241
ch./ro.	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
ro./ch.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
ro.	—	—	—	2	—	—	2	—	—	1	—	—	—	—	—	2	7
ro./gr.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
gr./ro.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
gr.	—	—	—	10	—	—	16	—	—	13	—	—	—	—	—	37	76
Totals	31	—	—	180	—	—	447	—	—	230	—	5	1	—	—	106	1000

TABLE II.

Colts and Maternal Grandams.

Maternal Grandams.

		bl.	bl./br.	br./bl.	br.	br./b.	b./br.	b.	b./ch.	ch./b.	ch.	ch./ro.	ro./ch.	ro.	ro./gr.	gr./ro.	gr.	Totals
Colts.	bl.	—	—	—	6		—	13	—	—	4	—	—	—	—	—	3	31
	bl./br.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	br./bl.	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	—	2
	br.	3	—	—	34	—	—	89	—	—	17	—	1	1	—	—	10	155
	br./b.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	b./br.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	b.	6	—	—	73	—	—	309	—	—	76	—	1	—	—	—	35	500
	b./ch.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	ch./b.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	ch.	3	—	—	29	—	—	132	—	—	58	—	—	2	—	—	14	238
	ch./ro.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	ro./ch.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	ro.	—	—	—	1	—	—	1	—	—	1	—	—	1	—	—	1	5
	ro./gr.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	gr./ro.	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
	gr.	2	—	—	7	—	—	22	—	—	7	—	—	—	—	—	30	68
	Totals	14	—	—	150	—	—	573	—	—	164	—	2	4	—	—	93	1000

TABLE III.

Fillies and Paternal Grandams.

Paternal Grandams.

		bl.	bl./br.	br./bl.	br.	br./b.	b./br.	b.	b./ch.	ch./b.	ch.	ch./ro.	ro./ch.	ro.	ro./gr.	gr./ro.	gr.	Totals
Fillies.	bl.	—	—	—	5	—	—	13	—	—	9	—	—	—	—	—	1	28
	bl./br.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
	br./bl.	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
	br.	8	—	—	38	—	—	83	—	—	31	—	—	—	—	—	6	166
	br./b.	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
	b./br.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	b.	15	—	—	91	—	—	270	—	—	109	—	—	—	—	—	32	517
	b./ch.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	ch./b.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	ch.	2	—	—	25	—	—	113	—	—	60	—	—	—	—	—	14	214
	ch./ro.	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
	ro./ch.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	ro.	—	—	—	1	—	—	4	—	—	—	—	—	—	—	—	—	5
	ro./gr.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	gr./ro.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	gr.	1	—	—	12	—	—	17	—	—	11	—	—	—	—	—	25	66
	Totals	26	—	—	172	—	—	502	—	—	221	—	—	—	—	—	79	1000

TABLE IV.

Fillies and Maternal Grandams.

Maternal Grandams.

Fillies.	bl.	bl./br.	br./bl.	br.	br./b.	b./br.	b.	b./ch.	ch./b.	ch.	ch./ro.	ro./ch.	ro.	ro./gr.	gr./ro.	gr.	Totals
bl.	1	—	—	4	—	—	9	—	—	5	—	—	—	—	—	6	25
bl./br.	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
br./bl.	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
br.	1	—	—	33	1	—	84	—	—	29	—	—	1	—	—	7	156
br./b.	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
b./br.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
b.	6	—	—	80	—	—	322	—	—	73	—	—	—	—	—	40	521
b./ch.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
ch./b.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
ch.	2	—	—	17	—	—	138	—	—	60	—	—	—	—	—	15	232
ch./ro.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
ro./ch.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
ro.	—	—	—	1	—	—	2	—	—	—	—	—	—	—	—	—	3
ro./gr.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
gr./ro.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
gr.	1	—	—	9	—	—	17	—	—	8	—	—	—	—	—	25	60
Totals	11	—	—	144	1	—	573	—	—	177	—	—	1	—	—	93	1000

Appendix III. On Inheritance (Great-grandparents and Great-great-grandparents and Offspring) in Thoroughbred Racehorses.

By ALICE LEE, D.Sc.

From Mr Blanchard's racehorse coat-colour pedigrees, I have, paying no attention to sex, been able to extract 1155 cases of great-grandparent and offspring and 978 cases of great-great-grandparent and offspring. When it is noted that there are 16 types of great-grandparental and 32 types of great-great-grandparental relationship, so that 48 correlation tables would be required for the full working out of these cases, it will be noted why in this preliminary study, I have not differentiated between the sexes.

Tables I. and II. reproduce my data.

From Table I. I find in the fourfold division hitherto adopted:

Great-Grandparents.

		Bay and darker	Chestnut and lighter	Totals
Offspring.	Bay and darker ...	601	264	865
	Chestnut and lighter	166	124	290
	Totals	767	388	1155

Hence

$$h = \cdot 671{,}102, \quad k = \cdot 423{,}602,$$

and the equation is:

$$\cdot 198{,}115 = r + \cdot 141{,}813 r^2 + \cdot 075{,}166 r^3 + \cdot 085{,}182 r^4 + \text{etc.}$$

giving as root

$$r = \cdot 1922.$$

From Table II. we have:

Great-great-Grandparents.

		Bay and darker	Chestnut and lighter	Totals
Offspring.	Bay and darker ...	497	252	749
	Chestnut and lighter	130	99	229
	Totals	627	351	978

I find

$$h = \cdot 725{,}258, \quad k = \cdot 361{,}420,$$

and the equation

$$\cdot 149{,}992 = r + \cdot 131{,}061 r^2 + \cdot 068{,}681 r^3 + \cdot 077{,}532 r^4 + \text{etc.}$$

which gives

$$r = \cdot 1469.$$

We see from these results that the ancestral relationships in coat-colour for the horse are far more intense than was originally supposed for the like grades of relationship. We have in round numbers:

Parental correlation ... ... $= \cdot 50$.
Grandparental correlation ... ... $= \cdot 30$.
Great-grandparental correlation ... $= \cdot 20$.
Great-great-grandparental correlation $= \cdot 15$.

As far as this series goes, not the coefficient itself, but the difference appears halved at each ascent, and it is clear that the $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, ... series cannot possibly hold for the regression coefficients of the multiple correlation series.

TABLE I.

Great-Grandparents and Offspring.

Great-Grandparents.

Offspring.	bl.	bl./br.	br./bl.	br.	br./b.	b./br.	b.	b./ch.	ch./b.	ch.	ch./ro.	ro./ch.	ro.	ro./gr.	gr./ro.	gr.	Totals
bl.	4	—	—	17	—	—	46	—	—	27	—	—	—	—	—	2	96
bl./br.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
br./bl.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
br.	6	—	—	35	—	—	97	—	—	49	—	—	—	—	—	9	196
br./b.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
b./br.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
b.	17	—	—	82	—	—	297	—	—	149	—	—	—	—	—	28	573
b./ch.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
ch./b.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
ch.	1	—	—	34	—	—	103	—	—	75	—	—	—	—	—	22	235
ch./ro.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
ro./ch.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
ro.	—	—	—	2	—	—	8	—	—	4	—	—	—	—	—	2	16
ro./gr.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
gr./ro.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
gr.	—	—	—	5	—	—	13	—	—	13	—	—	—	—	—	8	39
Totals	28	—	—	175	—	—	564	—	—	317	—	—	—	—	—	71	1155

TABLE II.

Great-great-Grandparents and Offspring.

Great-great-Grandparents.

Offspring.	bl.	bl./br.	br./bl.	br.	br./b.	b./br.	b.	b./ch.	ch./b.	ch.	ch./ro.	ro./ch.	ro.	ro./gr.	gr./ro.	gr.	Totals
bl.	2	—	—	14	—	—	40	—	—	24	—	—	—	—	—	7	87
bl./br.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
br./bl.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
br.	3	—	—	21	—	—	69	—	—	43	—	—	—	—	—	6	142
br./b.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
b./br.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
b.	5	—	—	70	—	—	273	—	—	143	—	—	—	—	—	29	520
b./ch.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
ch./b.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
ch.	1	—	—	17	—	—	89	—	—	61	—	—	—	—	—	19	187
ch./ro.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
ro./ch.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
ro.	—	—	—	2	—	—	7	—	—	5	—	—	—	—	—	1	15
ro./gr.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
gr./ro.	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
gr.	—	—	—	6	—	—	8	—	—	10	—	—	—	—	—	3	27
Totals	11	—	—	130	—	—	486	—	—	286	—	—	—	—	—	65	978

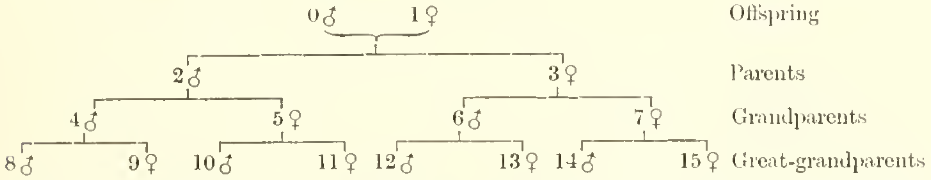
MISCELLANEA.

Note on the Influence of Change in Sex on the Intensity of Heredity.

By FRANK E. LUTZ, M.A.

(1) In Professor Karl Pearson's memoir, *On the Inheritance of Eye Colour in Man*[*] it is stated on page 117 "that the younger generation is more highly correlated with an ascendant or collateral of the same than of the opposite sex," and again page 115, "that change of sex weakens the intensity of heredity." It seemed worth while pursuing this point further and grouping together all the available evidence as far as eye-colour in man is concerned. Accordingly, Mr Galton having again most kindly put his eye-colour data at Prof. Pearson's disposal, I have worked out the correlation between great-grandparents and offspring, which completes as far as ancestral correlation is concerned the whole of the available records of this sort. I hope shortly to examine in the same way a large mass of data which I have gathered concerning coat-colour in dogs of the Pointer breed.

I use the following scheme of notation for pedigree:



This only differs from Mr Galton's widely accepted scheme in the introduction of 0 for the male offspring to distinguish them from 1, the female offspring. $r_{ss'}$ will mark the correlation between a pair of relatives one of whom is in class s, the other in class s'; and a reference to the above scheme will at once show us which pair of relatives is being dealt with. Thus $r_{0,\,12}$ is the correlation of male great-grandchildren with the father of the maternal grandfather, and so on.

(2) Unfortunately the data for great-grandparents is not very extensive. We had altogether only 2996 cases, or about 187 for each of the 16 possible great-grandparental correlations. This is distinctly too few considering the magnitude of the probable errors which occur when we deal with quantities not quantitatively measurable[†]. Accordingly I paid no attention to the separation of sex in the offspring but grouped my great-grandparental data into four classes: (*a*) those in which there is no change of sex between great-grandparent and offspring, (*b*) those in which there is one change of sex, (*c*) those in which there are two changes, (*d*) those in which

---

[*] *Phil. Trans.* A, Vol. 195, p. 102 *et seq.*

[†] *Phil. Trans.* A, Vol. 195, p. 42.

there are three changes. I further classed Professor Pearson's constants for parental and grandparental data* in the same manner, taking the average of the coefficients obtained by him for each class.

The results are as follows:

Parental Inheritance.

Mean $r_{0,2}, r_{1,3} = \cdot 5300$ ... ... No change of sex.
Mean $r_{0,3}, r_{1,2} = \cdot 4594$ ... ... One change of sex.

Grandparental Inheritance.

Mean $r_{0,4}, r_{1,7} = \cdot 3697$ ... ... No change of sex.
Mean $r_{0,5}, r_{1,4}, r_{1,6}, r_{0,7} = \cdot 3004$ ... ... One change of sex.
Mean $r_{0,6}, r_{1,5} = \cdot 2961$ ... ... Two changes of sex.

Great-grandparental Inheritance.

†Groups 8 and 0, 15 and 1 give $r = \cdot 3471$ ... ... No change of sex.
Groups 9 and 0, 11 and 0, 15 and 0, 8 and 1, 12 and 1, 14 and 1 give $r = \cdot 2217$ ... ... One change of sex.
Groups 10 and 0, 12 and 0, 14 and 0, 9 and 1, 11 and 1, 13 and 1 give $r = \cdot 1452$ ... ... Two changes of sex.
Groups 13 and 1, 10 and 0 give $r = \cdot 0375$ ... ... Three changes of sex.

The mean result of the four great-grandparental coefficients here found is ·1879, or if all the material be clubbed together and one correlation table formed for it, it is ·1824, the two results being practically identical, considering the probable error of their difference. At the end of this note I give the actual correlation tables, remarking that the classes are those of Mr Galton's eye-colour classification, i.e.

1 = Light blue.	5 = Light brown.
2 = Blue, dark blue.	6 = Brown.
3 = Grey, blue-green.	7 = Dark brown.
4 = Dark grey, hazel.	8 = Very dark brown, black.

For purposes of calculation the fourfold table was formed by classing together 1–3, and 4–8.

Now, although the probable error of these results runs from ·033 to ·055, their invariable decrease with an increasing number of changes of sex is highly significant. It can hardly be said that there is a constant factor of reduction for each change of sex; but, if it existed, a more elaborate system of measurements would probably be required to satisfactorily determine it. Still we can conclude with absolute safety for eye-colour in man and probably therefore for many other characters in other forms of life that:

Every change of sex in the line of ancestry sensibly weakens the intensity of inheritance.

It is desirable accordingly in future heredity experiments to record carefully such changes for they can obviously be of much significance. We may even see in them some justification for refusing to admit inheritance of title through the female line, although the objection would apply equally to the transfer from the female through the male line.

---

[*] *Phil. Trans.* A, Vol. 195, p. 106.

[†] The results here are obtained, not by taking the means of correlation coefficients, but by clubbing groups together.

TABLE I.—*No Change of Sex.*

Offspring.

Great-grandparents.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	Totals
1	—	1	—	—	—	—	—	—	1
2	1	51	35	29	2	15	7	5	115
3	5	33	31	13	—	6	13	—	101
4	—	7	4	9	1	1	3	—	28
5	—	—	—	—	—	—	—	—	—
6	—	7	8	6	3	10	9	—	43
7	—	8	8	7	—	2	8	5	38
8	1	2	4	3	—	—	—	2	12
Totals	7	109	90	67	6	37	40	12	368

Offspring.

Great-grandparents.	*1—3*	*4—8*	Totals
1—3	157	49	206
4—8	90	72	162
Totals	247	121	368

$$*h = \cdot 44322, \quad k = \cdot 15042,$$
$$\cdot 02319 r^4 + \cdot 13090 r^3 + \cdot 03333 r^2 + r - \cdot 35688 = 0,$$
$$r = \cdot 3471.$$

TABLE II.—*One Change of Sex.*

Offspring.

Great-grandparents.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	Totals
1	4	3	8	5	—	1	—	—	21
2	8	177	95	76	5	39	31	17	448
3	1	69	85	52	2	20	26	1	256
4	6	30	21	27	2	7	15	1	109
5	—	4	—	—	—	—	—	—	4
6	2	37	27	17	3	30	20	4	140
7	—	15	20	24	3	4	9	9	84
8	—	10	13	12	2	2	7	5	51
Totals	21	345	269	213	17	103	108	37	1113

Offspring.

Great-grandparents.	*1—3*	*4—8*	Totals
1—3	450	275	725
4—8	185	203	388
Totals	635	478	1113

$$h = \cdot 38909, \quad k = \cdot 17772,$$
$$\cdot 02436 r^4 + \cdot 13697 r^3 + \cdot 03458 r^2 + r - \cdot 22496 = 0,$$
$$r = \cdot 2217.$$

TABLE III.—*Two Changes of Sex.*

Offspring.

Great-grandparents.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	Totals
1	5	4	9	1	2	—	1	3	25
2	6	175	86	80	4	27	34	12	424
3	3	68	94	57	2	27	27	4	282
4	5	28	21	14	3	5	13	1	90
5	—	4	—	—	—	—	1	—	5
6	1	34	30	34	1	20	14	5	139
7	1	37	34	25	1	16	14	4	132
8	1	4	8	6	—	4	7	2	32
Totals	22	354	282	217	13	99	111	31	1129

Offspring.

Great-grandparents.	*1—3*	*4—8*	Totals
1—3	450	281	731
4—8	208	190	398
Totals	658	471	1129

$$h = \cdot 37852, \quad k = \cdot 20911,$$
$$\cdot 02785 r^4 + \cdot 13654 r^3 + \cdot 03958 r^2 + r - \cdot 14642 = 0,$$
$$r = \cdot 1452.$$

[*] The notation is the same as that used by Pearson, *Phil. Trans.* A, Vol. 195, pp. 1—47.

TABLE IV.

Three Changes of Sex.

Great-grandparents.	Offspring.								
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	Totals
1	—	3	1	—	—	—	3	1	8
2	1	49	33	18	1	8	11	2	123
3	1	25	28	27	2	10	6	—	99
4	—	14	7	8	—	4	4	—	37
5	—	1	—	—	—	—	2	—	3
6	5	17	8	6	2	3	2	5	48
7	1	24	13	9	—	4	11	2	64
8	—	1	1	—	—	—	1	1	4
Totals	8	134	91	68	5	29	40	11	386

Great-grandparents.	Offspring.		
	1—3	*4—8*	Totals
1—3	141	89	230
4—8	92	64	156
Totals	233	153	386

$$h = \cdot 24264, \quad k = \cdot 26275,$$

$$\cdot 02290 r^4 + \cdot 14603 r^3 + \cdot 03188 r^2 + r - \cdot 03758 = 0,$$

$$r = \cdot 0375.$$

TABLE V.

Total Material.

Great-grandparents.	Offspring.								
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	Totals
1	9	11	18	6	2	1	4	4	55
2	16	452	249	203	12	89	83	36	1140
3	10	195	238	149	6	63	62	5	738
4	1	79	53	58	6	20	35	2	264
5	—	9	—	—	—	—	3	—	12
6	8	95	73	63	9	63	45	14	370
7	2	84	75	65	4	26	42	20	318
8	2	17	26	21	2	6	15	10	99
Totals	58	942	732	565	41	268	299	91	2996

Great-grandparents.	Offspring.		
	1—3	*4—8*	Totals
1—3	1198	735	1933
4—8	534	529	1063
Totals	1732	1264	2996

$$h = \cdot 19705, \quad k = \cdot 37238,$$

$$\cdot 02591 r^4 + \cdot 13798 r^3 + \cdot 03669 r^2 + r - \cdot 18455 = 0,$$

$$r = \cdot 1824.$$

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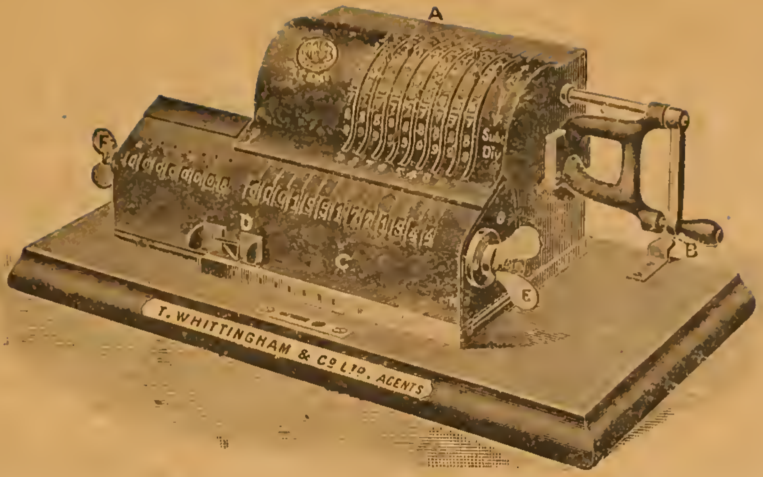
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VOLUME II JUNE, 1903 No. 3

ACTINOSPHAERIUM EICHORNI.

A BIOMETRICAL STUDY IN THE MASS RELATIONS OF NUCLEUS AND CYTOPLASM*.

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BY GEOFFREY SMITH, New College, Oxford.

Introduction.

THE following study is concerned with what appears at present to be the critical problem of cytology, the inter-relation of nucleus and cytoplasm, or more strictly of chromatin and cytoplasm. Much knowledge of chemico-physiological significance is possessed by naturalists on this head, but concerning the mass relations that hold between chromatin and cytoplasm under different physiological conditions, a knowledge of which one would consider fundamental from a chemico-physiological point of view, strangely little is known. How important this mass relation may be is brought home to one who remembers that the most obvious distinction between the male and female sexual cells throughout organic nature is this very one of the mass relations between chromatin and cytoplasm.

In order that the following observations and experiments on *Actinosphaerium* may be easily grasped, I will shortly recapitulate the history of its encystment and reproduction.

Encystment may be induced by starvation for a longer or shorter period. The creature about to encyst becomes much vacuolated, and fixes itself to the vessel in which it is confined, withdrawing its pseudopodia. The endoplasm becomes exceedingly opaque, appearing conspicuously white to the naked eye, and shortly afterwards it breaks up into a varying number of discoid bodies lying in the gelatinous exoplasm, each of which contains a single nucleus. During the course of the next twenty-four hours each mother cyst divides to form two daughter cysts; the nucleus of each daughter cyst now divides twice, giving off two polar bodies; the daughter cysts then fuse, about twenty-four hours after their formation, sister with sister, to form circular discoid conjugation cysts each containing a conjugation nucleus. The conjugation cysts form thick siliceous coats and remain in this condition for several weeks.

---

* By the permission of the authorities of New College, Oxford, I was enabled to work in Professor R. Hertwig's laboratory at Munich during part of the winter of 1902. Through his attention and guidance it has been possible to bring this research to some result.

Preliminary Experiments and Observations.

The free living *Actinosphaeria* possess a great many more nuclei than the few that are preserved for this process of encystment: my first object therefore was to determine roughly how many nuclei are thrown away in the process of encystment. For this purpose I took several hundred individuals of various sizes, measured them*, put three-quarters to starve and in consequence to encyst (at room temperature 15°—19° C.), and killed and stained the remaining quarter in order to count their nuclei. The results of these two experiments are set forth in Tables I. and II.

TABLE I.

Diameter of Individuals in Micrometer Units.

Number of cysts built at 15°—19° C.	*15*	*17*	*19*	*21*	*23*	*25*	*27*	*29*	*31*	Totals
1	1	—	—	—	—	—	—	—	—	1
2	2	3	—	—	—	—	—	—	—	5
3	—	4	3	4	2	—	—	—	—	13
4	—	—	2	3	5	1	1	—	—	12
5	—	—	—	4	3	2	1	1	2	13
6	—	—	—	4	5	1	1	2	—	13
7	—	—	—	1	3	5	—	7	—	16
8	—	—	—	—	1	1	3	3	1	9
9	—	—	—	—	2	—	3	4	—	9
10	—	—	—	—	2	—	—	1	4	7
11	—	—	—	—	—	—	1	1	1	3
12	—	—	—	—	—	—	—	2	3	5
13	—	—	—	—	—	—	—	—	1	1
14	—	—	—	—	—	—	—	—	1	1
15	—	—	—	—	—	—	—	—	1	1
Totals	3	7	5	16	23	10	10	21	14	109

---

* Every precaution was taken to ensure accuracy in the measurement. Each specimen was measured with a micrometer under Zeiss AA, 2, the unit of measurement being one division of an eye-piece micrometer, which was found with the objective used to correspond to 1·81 μ. The creatures were selected for measurement only when the pseudopodia were fully expanded, the contour of the body was circular, and the creature was suspended freely in a drop of water. The diameter of the body was then taken from edge to edge, ignoring the pseudopodia. The difficulty of determining the correlation between the size of *Actinosphaeria* and the number of cysts built was encountered chiefly in the liability of the creatures to divide or fuse one with another (plastogamy) after they had been placed in the vessels to starve and to encyst. To avoid mistakes from this cause the vessels were kept as constantly under supervision as possible, and discretion has been used in various ways: but absolute accuracy in cases of such difficulty can hardly be ensured.

TABLE II.

Diameter of Individuals in Micrometer Units (unit of measurement = 1·81 μ).

Number of Nuclei.	*19*	*21*	*23*	*25*	*27*	*29*	*31*	Totals
50	1	—	—	—	—	—	—	1
100	3	4	—	—	—	—	—	7
150	2	8	11	—	1	—	—	22
200	—	3	7	8	2	—	—	20
250	—	—	1	10	4	1	—	16
300	—	—	—	1	7	2	—	10
350	—	—	—	—	—	2	1	3
400	—	—	—	—	—	3	3	6
450	—	—	—	—	—	2	1	3
500	—	—	—	—	—	3	1	4
550	—	—	—	—	—	1	—	1
600	—	—	—	—	—	1	1	2
650	—	—	—	—	—	—	1	1
Totals	6	15	19	19	14	15	8	96

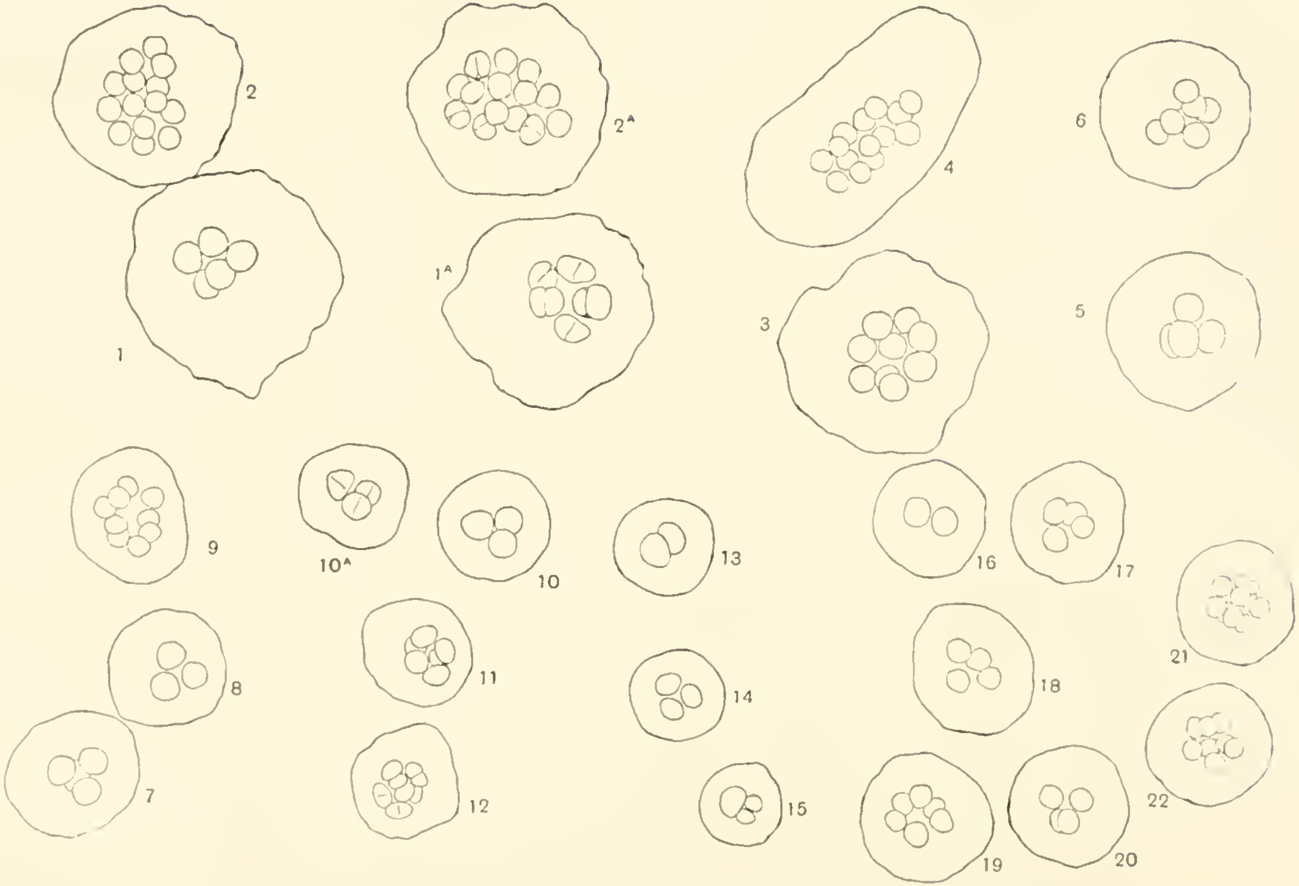
Table I. records the correlation between size of *Actinosphaerium* and the number of cysts built. The correlation coefficient =0·7692 ± ·0264. The standard deviation of the sizes is 4·4372 units =8·031 μ; that of the numbers of cysts is 2·9478.

Table II. records the correlation between size of *Actinosphaerium* with number of nuclei. Correlation coefficient = ·8540 ± ·0168. The standard deviation of the sizes is 3·4094 units = 6·171 μ, that of the numbers of nuclei is 2·5181 × 50. The two correlation coefficients here obtained are exceedingly high, as high as those we find between corresponding bones of two fingers of the same hand, etc. It would be interesting to see whether equally high coefficients obtain for multinucleate *Protozoa* in general.

The above correlation tables had shown that great variability existed in *Actinosphaeria* of similar bulk as to the number of nuclei they contained and as to the number of cysts they built during encystment, and that these two variables corresponded in a certain degree. In the course of counting and observing the cysts tabulated in Fig. B. I became gradually impressed with a further fact, namely, that the cysts built by different individuals were often of very different sizes, and moreover that the size of the cyst seemed to bear a certain relation to their number: i.e. when an *Actinosphaerium* had built very numerous cysts for its size then those cysts were generally small, and *vice versâ*. On Plate I. Figs. 1A—15 are collected the most striking examples of this fact, which I could gather from these first experiments. Here it will be seen that in *Actinosphaeria* of all kinds of sizes individuals occurred, some of which built numerous and small cysts, while others built fewer and larger.

The next preliminary experiments were directed towards finding out what effect would be produced on the number and size of the cysts by subjecting the animals to different temperatures. Animals of a certain size (36·2—39·8 μ) were selected: half were placed in an incubator at 24°—26° C., the other half were placed in an unheated room where the temperature varied between 7° and 12° C.

PLATE I.


2
2ᴬ
4
6
1
1ᴬ
3
5
9
10ᴬ
10
13
16
17
21
8
11
14
18
7
12
15
19
20
22

Figs. 1ᴬ, 2ᴬ. Two individuals forming daughter cysts.
Figs. 1, 2. Same as above, having formed conjugation cysts.
Figs. 3, 4, 5, 6, 7, 8, 9, 10, 11. Individuals of various sizes in conjugation cyst stage.
Fig. 10ᴬ. Daughter cysts not yet fused.

Figs. 12, 13, 14, 15. Mother and daughter cysts.
Figs. 16, 17, 18, 19, 20. Individuals with conjugation cysts made at 24°—26° C.
Figs. 21, 22. Conjugation cysts at 7°—12° C.
All figures drawn to scale under Zeiss AA, 2.

In 60 hours the individuals of the warm culture were already in the conjugation-cyst stage, while the similar cold culture was not in a similar stage of encystment till a week and four days after it was prepared. The two cultures built cysts in the following numbers:—

No. of cysts	2	3	4	5	6	7	8	9
Frequency in warm culture	2	4	6	1	3	0	0	0
Frequency in cold culture	0	0	1	1	2	1	3	2

The reason why fewer reached the conjugation cyst stage in the cold culture was that owing to the delayed encystment the culture had to be more frequently weeded of individuals that had divided etc. This experiment suggested that the higher temperature caused fewer cysts, and the cold caused more numerous cysts to be built, while on Plate I. Figs. 16—22 show that the rule adumbrated by the previous experiment is confirmed by these, since the cysts formed in the cold being more numerous are at the same time smaller than the comparatively few cysts built in the warm.

The preliminary experiments now described had suggested the following facts and problems:

A. An *Actinosphaerium* of a given size may contain a number of nuclei varying between wide extremes and in consequence—

B. It may build a varying number of cysts.

C. When it builds few cysts for its size then the cysts tend to be large; when it builds many they tend to be small.

D. An increased temperature hastens on encystment, a low temperature retards it.

E. An increased temperature causes fewer and larger cysts to be built, a low temperature causes many and smaller, i.e. an increased temperature causes a greater percentage of nuclei to be thrown away.

F. In what way does the temperature act upon the number of nuclei rejected? That is, does the temperature affect the number of nuclei gradually, or only at the period when they are being rejected?

G. In what way does the size of the cyst affect the nucleus contained? That is, does the nucleus vary in correspondence to the varying size of the cyst or does it remain the same so that a different mass-relation between nucleus and cytoplasm occurs in the different cases?

Further Researches.

The following researches were directed towards confirming, correcting and answering the facts and queries lettered above.

A and B. To test these observations more narrowly several hundred individuals were selected of a certain size (34·39—41·63 μ). A part of these were killed, stained and their nuclei counted. Fig. A. The other part was put to encyst at room temperature 15°—19° C. with the result shown in Fig. B. From these figures the proportion of nuclei retained for encystment at room temperature by individuals of the above size can be fairly accurately gauged. The number is somewhat lower than previous observers have found; but it will be shown that the number of nuclei retained is greatly dependent on temperature.

C. When an *Actinosphaerium* builds few cysts for its size then the cysts tend to be small, and *vice versâ*.

If reference be made to Table III. which refers to the cysts built at room temperature, a complete confirmation of this rule will be obtained. This table was made up from those individuals used for constructing Fig. B whose cysts could be conveniently measured: the measurements refer to the diameter of the conjugation cysts. This table expresses the correlation between the number of cysts and the size of the cysts. The correlation coefficient $= -0{\cdot}6689 \pm {\cdot}0402$. The standard deviation of the numbers of cysts $= 1{\cdot}7599$; that of the sizes of the cysts $= 2{\cdot}3487 \times {\cdot}3\mu$. It will be seen that we have a most substantial negative correlation.

TABLE III.

Number of Cysts at 15°—19° C.

Diameter of Conjugation Cysts in μ.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	Totals
5·2	—	—	—	—	—	—	—	—	—
5·5	—	—	—	—	—	1	—	—	1
5·8	—	—	—	—	—	—	—	—	—
6·1	—	—	—	1	—	—	—	—	1
6·4	—	—	—	—	—	—	—	1	1
6·7	—	—	—	—	—	—	—	—	—
7·0	—	—	—	—	—	1	1	2	4
7·3	—	—	1	1	—	2	4	—	8
7·6	—	—	3	2	8	2	2	1	18
7·9	—	—	6	4	2	3	1	—	16
8·2	—	2	8	4	5	—	—	—	19
8·5	—	8	4	2	—	—	—	—	14
8·8	—	—	1	—	—	—	—	—	1
9·1	—	1	—	—	—	—	—	—	1
9·4	—	—	—	—	—	—	—	—	—
9·7	—	—	—	—	—	—	—	—	—
10·0	—	—	—	—	—	—	—	—	—
10·3	1	—	—	—	—	—	—	—	1
10·6	—	—	—	—	—	—	—	—	—
10·9	1	—	—	—	—	—	—	—	1
Totals	2	11	23	14	15	9	8	4	86

Since the cysts built by one and the same individual are not always of exactly the same size I have had to average the size of the cysts in each individual. This applies to the whole table. The difference in size of the cysts built by any one individual was frequently nothing and only in two or three cases more than

Fig. B.

128 Individuals encysted at temperature of room.

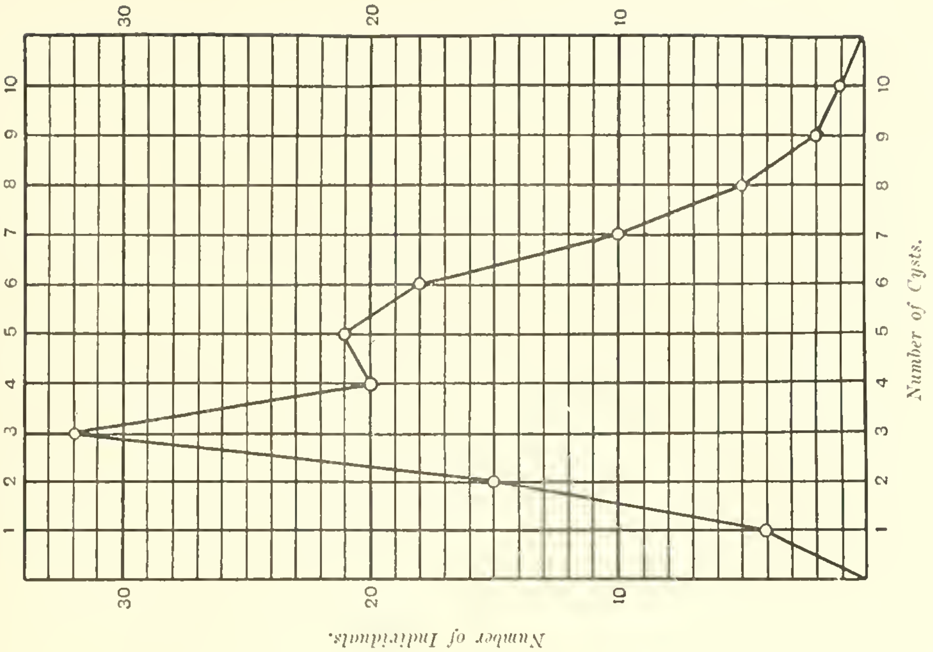
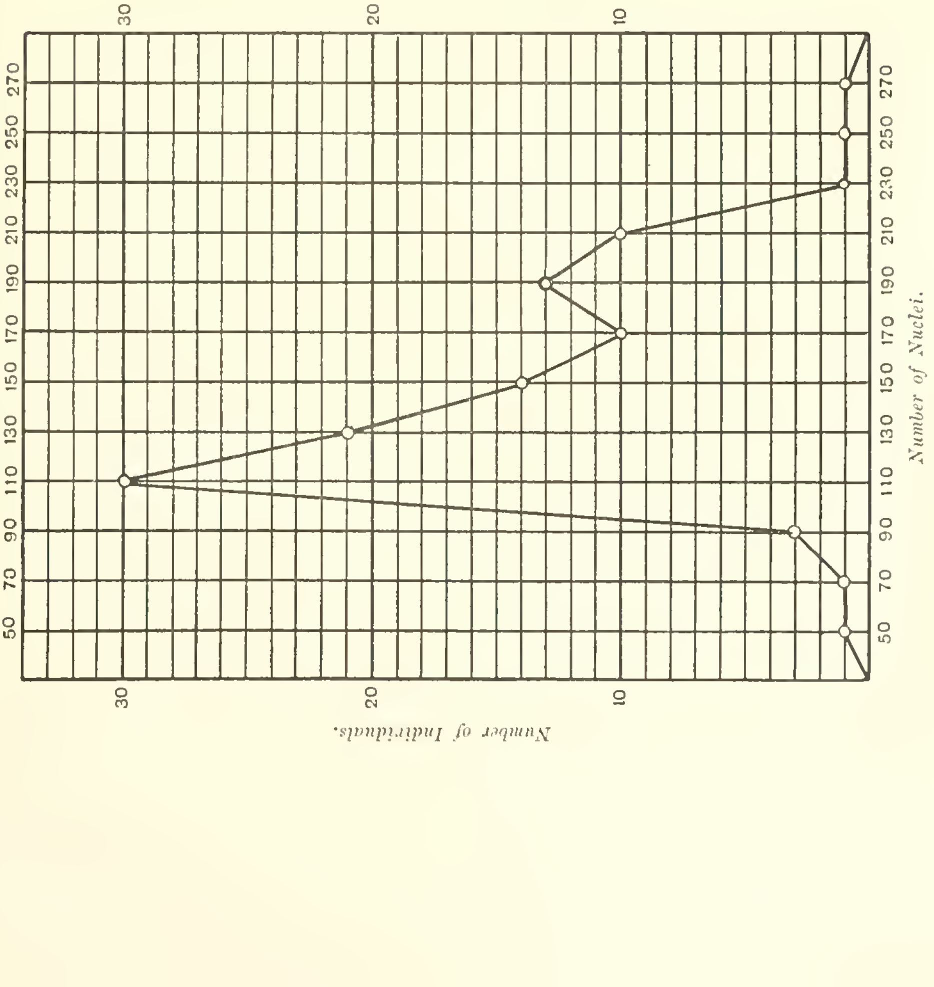


Fig. A.

106 Individuals.



half an unit of the scale, so that the process of averaging was only employed to cover small fluctuations.

D. An increased temperature hastens on encystment, a low temperature retards it.

I have proved the truth of this general statement by numerous instances but I have not determined the facts with the accuracy which the question deserves. The following rough observations must at present suffice:—

1. Individuals measuring 34·39—41·63μ were placed in the warm temperature (24°—26° C.) on Saturday evening 8th; on following Wednesday afternoon, 12th, conjugation cysts were formed in most cases.

2. Similar culture to above was set on 8th at room temperature (15°—19° C.); encystment began on 12th, conjugation cysts formed on 17th.

3. Similar culture to above placed in the cold temperature (7°—12° C.) on the 8th; encystment began on 19th, conjugation cysts formed on 23rd.

4. Culture at 24°—26° C. set on Saturday, all conjugated on following Thursday.

5. Culture at 24°—26° C. set on Sunday, all conjugated following Saturday.

6. Culture at 24°—26° C. set on 23rd, all conjugated 30th.

7. Culture placed in cold temperature (7°—12° C.) from Nov. 23rd—Dec. 2nd. Put in warm Dec. 2nd, all conjugated Dec. 8th.

8. Culture set at 7°—12° C. on Dec. 2nd, conjugation cysts formed on Dec. 16th.

From these facts it may be concluded that in starvation cultures the time of onset of encystment is directly dependent on the temperature, being quickest in the heat, intermediate at room temperatures, and much protracted in the cold.

E. An increased temperature causes fewer and larger cysts to be built, and *vice versâ.*

The reader must refer to Table IV. Here I have tabulated the correlation between the number of cysts and the size of cysts at 24°—26° C. with thin Arabic figures, while the Arabic figures in brackets refer to creatures that encysted in the cold 7°—12° C. It will be seen that the majority of the Arabic figures fall in the lower left-hand quarter of the table, i.e. the quarter referring to few and large cysts, while the figures in brackets fall in the upper right-hand corner, the quarter referring to numerous and small cysts. The figures in Table III. would form a kind of bridge between the two series in Table IV. The somewhat wide deviation from the mean exhibited by certain individuals that encysted at a high temperature suggests further problems: but the small number of instances do not permit of generalization.

TABLE IV.

Number of Cysts.

Diameter of Conjugation Cysts in μ.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	Totals
5·2	—	—	—	—	—	—	—	—	—	—	—	—	(1)	—	1
5·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
5·8	—	—	—	—	—	—	—	—	—	(1)	—	—	—	(1)	2
6·1	—	—	—	1	—	—	—	—	—	—	(1)	—	—	—	2
6·4	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
6·7	—	—	—	—	—	—	(2)	(1)	—	—	—	—	—	—	3
7·0	—	—	—	5	(1)	(1)	(2)	—	(1)	—	—	—	—	—	10
7·3	—	—	—	I.	I. 1	(1)	—	—	—	—	—	—	—	—	4
7·6	—	II. 2	3	1	I. 1	I. 1	—	—	—	—	—	—	—	—	12
7·9	—	4	I. 1	(1)	1	—	—	—	—	—	—	—	—	—	8
8·2	—	I. 3	IV. 2	1	—	—	—	—	—	—	—	—	—	—	11
8·5	I. 1	I. 5	2	—	—	—	—	—	—	—	—	—	—	—	10
8·8	1	I. 7	—	—	—	—	—	—	—	—	—	—	—	—	9
9·1	1	1	—	—	—	—	—	—	—	—	—	—	—	—	2
9·4	2	—	—	—	—	—	—	—	—	—	—	—	—	—	2
9·7	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
10·0	—	I. 1	—	—	—	—	—	—	—	—	—	—	—	—	2
10·3	3	—	—	—	—	—	—	—	—	—	—	—	—	—	3
10·6	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
10·9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	9	29	13	10	6	4	4	1	1	1	1	0	1	1	81

Arabic figures refer to individuals encysted at 24°—26° C., Arabic figures in brackets to individuals encysted at 7°—12° C., Roman letters to individuals starved in the cold and introduced into the warm.

F. In what way does the temperature act upon the number of nuclei rejected?

It is plain that the temperature may act in one of two ways: either it may gradually increase or decrease the number of nuclei before encystment sets in, or else it may have no effect during this period but only at the actual onset of encystment, when the nuclei are being rejected. That this latter alternative is the truth is shown by the following experiments.

1. A culture was put to starve in the cold (7°—12° C.) on Nov. 23rd. It was introduced into the warm temperature (24°—26 C.) on Dec. 2nd, none of the creatures having as yet begun to encyst. On Dec. 8th conjugation cysts had been formed by all the individuals. The result is shown in Table IV. by the Roman letters. Here we see that whatever effect the application of cold under starvation for nine days may have had is obliterated by the subsequent application of warmth, for the Roman letters fall indifferently among the Arabic which refer to ordinary warm encystments.

2. Individuals of various sizes were well fed with Stentor in the cold. Half of them were killed and stained and found to be very rich in nuclei. The other half were introduced into the warmth on Dec. 7. On the following day they

had begun to encyst. The three individuals which were allowed to reach the conjugation stage had cysts of the following dimensions:

(*a*) 1 cyst = 8·31 μ, 15 cysts = 8·75 μ.
(*b*) 2 cysts = 8·75, 1 cyst = 8·31.
(*c*) 4 cysts = 8·31.

The cysts built by these individuals, then, were all large (cf. Table III.): from which we may safely judge that they were few in relation to the organisms' size. And yet we have seen that the organisms possessed many nuclei after being fed in the cold: but this seems to have had no effect in determining how many nuclei should be preserved for the process of encystment. This experiment shows in a striking manner that it is the temperature at the time of encystment which determines to an overmastering degree the number and in consequence the size of the cysts to be formed.

3. As the corollary to the above two experiments I may mention that I killed and stained the non-encysted weedings from both hot and cold starvation cultures in numerous instances, but was never able to find the slightest evidence that an increase of nuclei had taken place in the cold or a decrease in the warmth, in such a way that by this means the different numbers of nuclei retained for encystment in the two cases might be accounted for.

G. In what way does the size of the cyst affect the nucleus (chromatin) contained?

I must preface the following section, which constitutes the crucial part of the investigation, by drawing a distinction between the nucleus as a morphological entity and the chromatin as a physiological entity characterizing the former. We are not concerned here with a change of size of the nucleus as a morphological structure, a change which could be brought about by an alteration in the conditions of tension in the cell without any deeper changes in the physiological relations. But we are concerned with a change in the relative proportions of chromatin, the distinctive property of the nucleus, to the cytoplasm; and the question to be answered is, In the different sized cysts does the chromatin remain of a constant mass so that a different relation of chromatin and cytoplasm is to be observed in the different sized cysts, or has the cell the power of readjusting the relation between chromatin and cytoplasm? The reader must turn to Plate II. for an answer. These figures, drawn to scale, represent sections* through conjugation cysts built in the warmth, Plate II., in the cold, Plate III., and at room temperature, Plate IV.

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* Owing to the opacity of the cysts I was obliged to cut sections of them in all cases. The thickness of the cyst coats makes them very difficult to preserve. I first used Picroacetic acid which distorts the cytoplasm (Figs. α—γ), subsequently I used Perenni's fluid which is more satisfactory (Figs. δ—ζ'), the fact that the cytoplasm in Figs. α—γ is distorted does not affect the relation of chromatin to cytoplasm; the size of the cyst may be roughly judged by the outline of the cyst coat and the mass of chromatin would not be altered by the preserving fluid used. Purists in Histology may neglect Figs. δ—ζ', and convince themselves of the main truth of the argument by the similarly ill-preserved figures.

A comparison of Figs. *a′*—*g‴* with Figs. 1—4′ shows that the mass of chromatin does not alter in correspondence to the different sizes of the cysts. In Figs *α*—*ζ′* which represent cysts built in the warmth, it appears that the amount of chromatin is absolutely as well as relatively less than in the other cases. It also tends to be more scattered.

(N.B. Where the chromatin lumps extended over more than one section they have been included in the figure.)

From the evidence that can be gleaned from Plates II. III. IV. the following rule may be formulated:—*Actinosphaeria* encysting under different temperatures exhibit a different mass relation between chromatin and cytoplasm, in the cold the chromatin being in excess, in the warmth the cytoplasm being in excess.

Conclusion.

Attention has been called to certain facts in the encystment of *Actinosphaerium Eichorni*. Of these the most important is that a different mass relation of chromatin and cytoplasm may be induced in the unicellular cysts by the application of heat and cold.

From the different sizes of the cysts formed under similar conditions of temperature, and from the widely different number of nuclei contained by free living *Actinosphaeria* of similar sizes and living under similar conditions of temperature, it must be judged that temperature is not the sole cause of this relation between chromatin and cytoplasm: analogy and a few facts given above point to nutrition as constituting the other chief cause. Whether the rule that warmth diminishes the amount of chromatin in proportion to the cytoplasm and that cold increases it will prove to be of at all wide application it is impossible to say from the absence at present of parallel experiments: experiments on this head are now being made by Professor Hertwig at Munich which seem to point to similar but wider conclusions. It is permissible to suggest that a deeper research into this and kindred problems may reveal facts which will throw a light upon the physiological differentiation of sex.

EXPLANATION OF PLATES II. III. AND IV.

Plate II. Figs. *α*—*ζ′*. Sections through conjugation cysts built in the warm (24°—26° C.). Note that in *δ* the nuclei are not yet fused.

Plate III. Figs. *a′*—*g‴*. Sections through the conjugation cysts built in the cold (7°—12° C.).

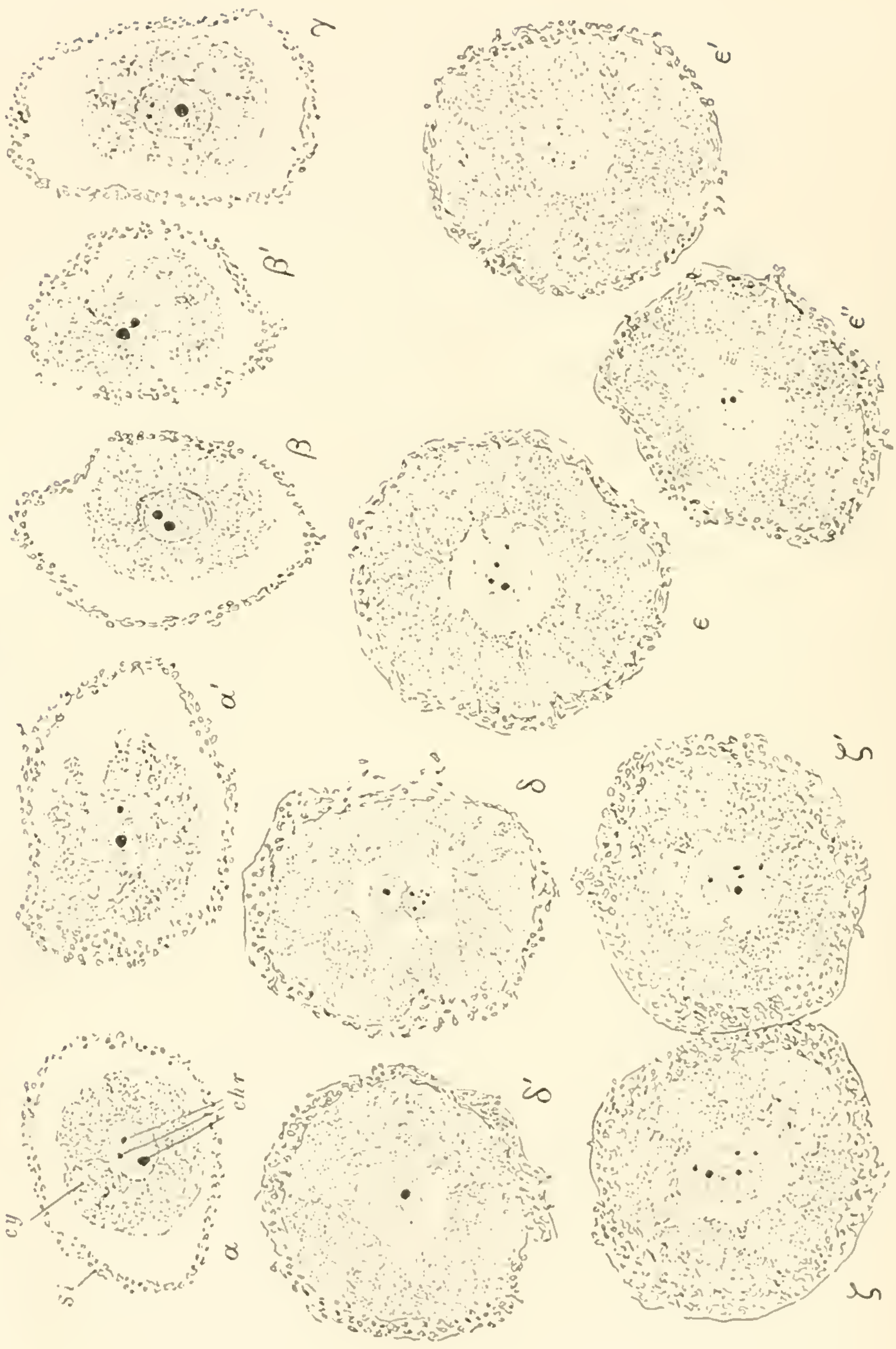
Plate IV. Figs. 1—4′. Sections through conjugation cysts built at room temperature (15°—19° C.).

In all cases the figures bearing the same letter or number are from cysts produced by the same *Actinosphaerium*.

All figures drawn accurately to scale under Zeiss objective DD, eye-piece 4.

si = siliceous cyst wall.
cy = cytoplasm.
chr = chromatin.

PLATE II.


γ
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β
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α
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δ
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ζ'
ζ

Plate III.

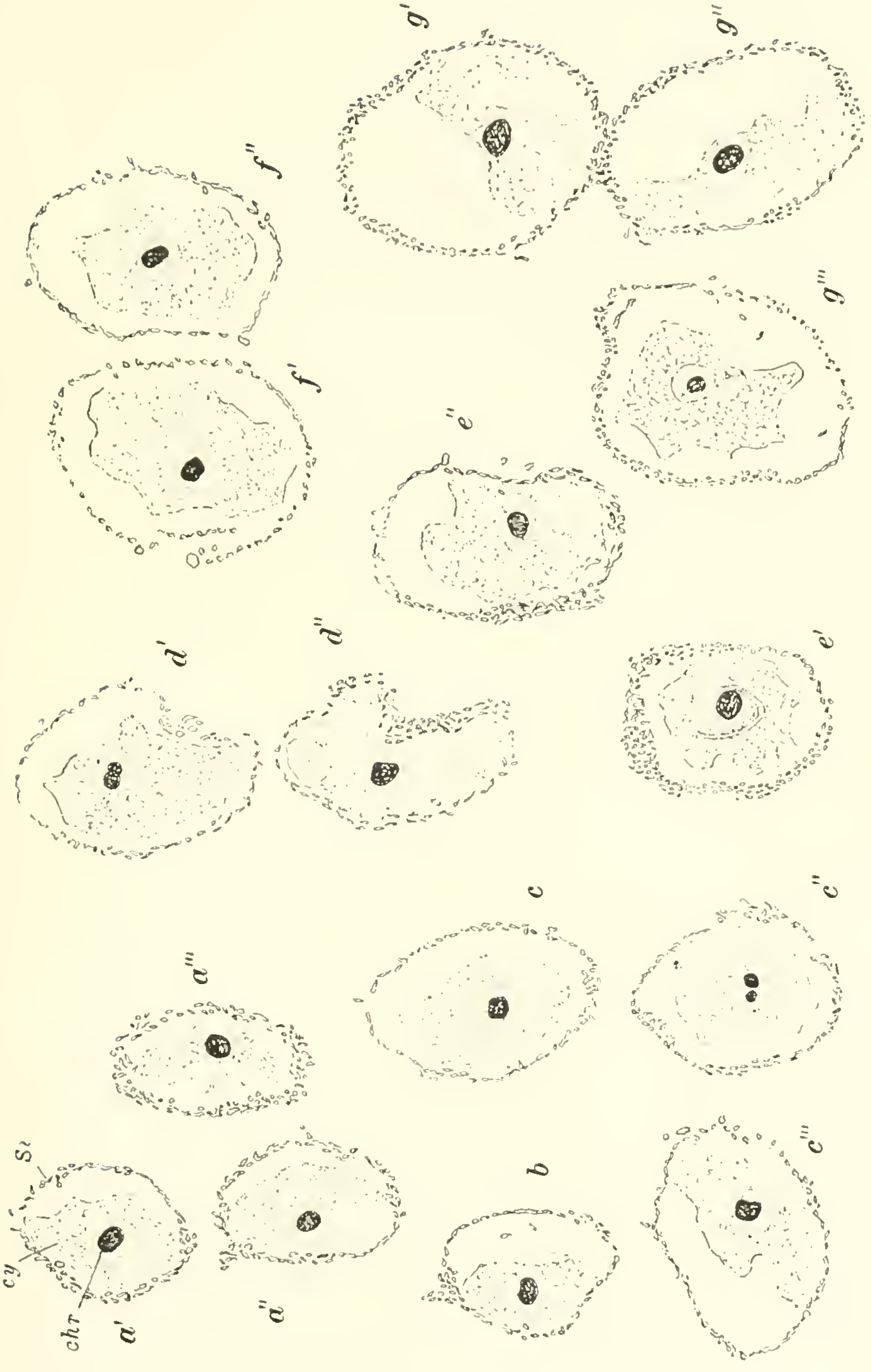
a'
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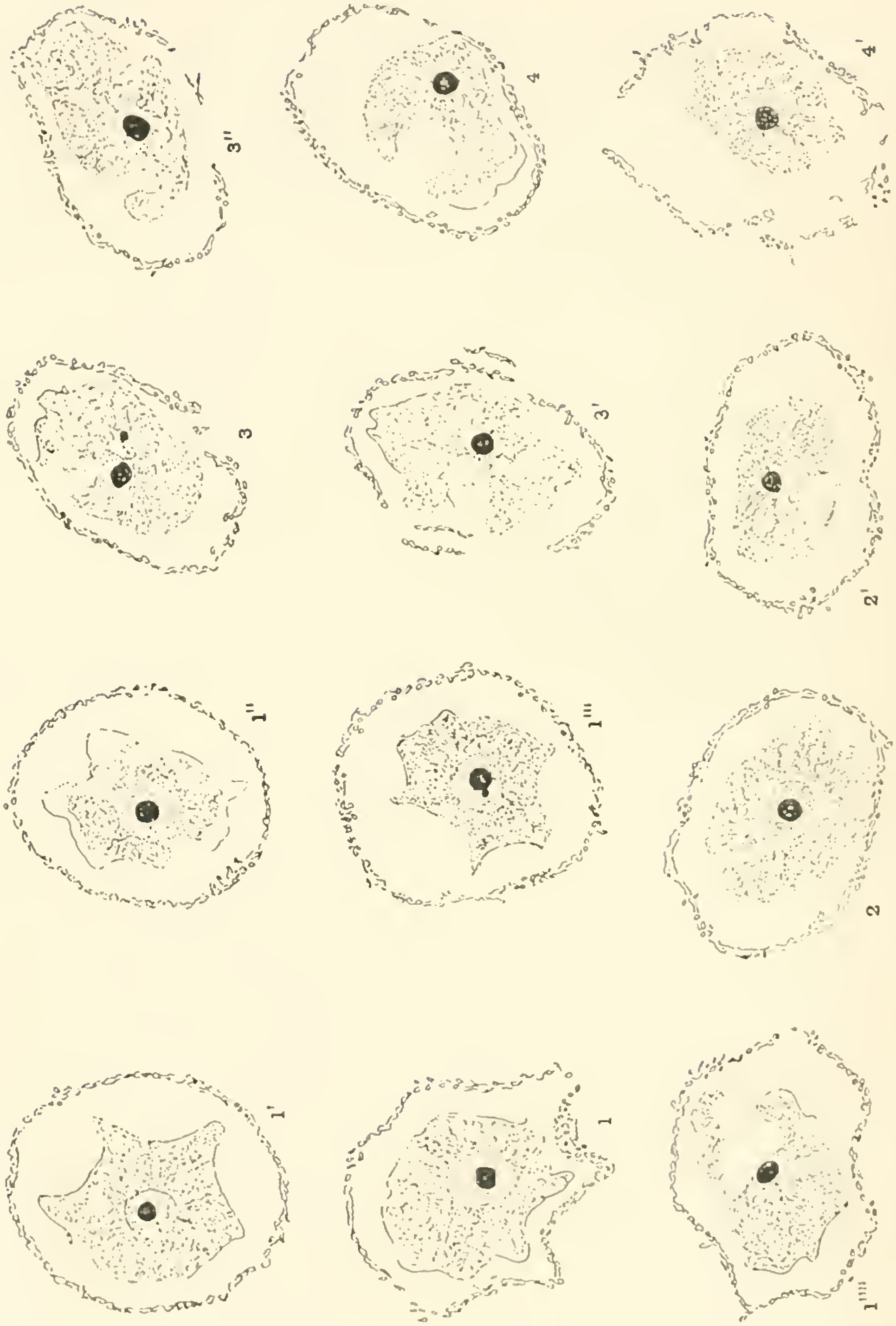
Plate IV.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A PRELIMINARY ATTEMPT TO ASCERTAIN THE RELATIONSHIP BETWEEN THE SIZE OF CELL AND THE SIZE OF BODY IN *DAPHNIA MAGNA* STRAUS.

By ERNEST WARREN, D.Sc., Zoological Laboratory, University College, London.

In the course of investigating the development of the trematode of the crayfish (*Distomum corrigerum*) it was observed that the nuclei of the general body cells of the developing animal were very much smaller than those of the fully-grown creature. On comparing animals of different ages it could be readily seen that as the body grew as a whole so the individual cells became gradually larger.

According to the usual view that cells are units of very great individuality, growth of the body would imply *à priori* simply an increase in the *number* of these units and not a change in their mean size or general character.

Although in the case of the trematode there was no doubt about the intimate relationship between the size of the body and the size of the cell during a certain period of growth, yet it was desirable to ascertain whether there was a similar relationship in an actively growing animal of practically adult structure and not merely in a developing embryo.

For this purpose *Daphnia* appeared to be a suitable subject if some measurable cell character could be found in the fresh animal.

The surface of the carapace of *Daphnia magna* is covered with diamond-shaped areas which correspond with considerable exactitude to the cell-outlines of the underlying epidermal cells. A number of daphnia of various sizes were treated with picrocarmine. The free picric acid fixed the tissues to a certain extent, and the nuclei of the epidermal cells were sometimes, although by no means always, stained a very pale pink. It was then seen that typically one nucleus occurred under each area. Fairly frequently, however, but especially under any irregularly shaped areas two nuclei could be seen. In such cases the polygonal area would correspond to two epidermal cells. It was particularly noticed that the occurrence of two nuclei under an area could be observed as frequently in young and small individuals as in older and larger specimens.

The diameters of the diamond-shaped areas of the carapace are therefore regarded as giving us a conception of the sizes of the epidermal cells of that region. Only regular diamond-shaped areas were measured and in this way most of the measures would strictly correspond to cells. The fact that in some cases there may have been unavoidably included areas corresponding to two cells will not affect the comparative results because such would occur as often in the young as in the fully-grown animals.

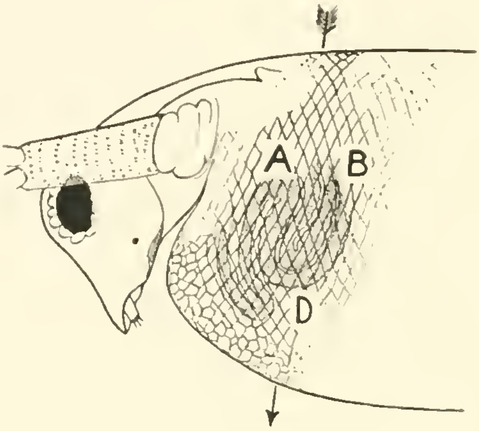

Fig. 1.

The dimensions taken were:

(1) The length of the body, measured from the anterior convexity of the head in front of the compound eye to the base of the dorsal spine.

(2) The diameters of 10 diamond-shaped areas over the middle lobe of the maxillary gland, and measured in the vertical direction. (See Fig. 1, *ABD*, the arrow indicates the direction of measurement.)

The daphnia were taken from a large tank at University College, London, and 201 animals of varying sizes were measured.

The measurements were made with an ocular micrometer, and under such magnification that a division of the micrometer scale in measuring the body-length was = 0·0320 mm., and the cell-length = 0·00385 mm.

The accompanying table gives the correlation surface for the length of daphnia and diameter of carapace cells. For the working scale the unit of length for the body was five divisions of the ocular micrometer = 0·160 mm., and for the cell dimension = 0·01925 mm.

The line of means for the arrays is represented in the annexed diagram. There is clearly a considerable correlation between the size of the body and the size of the cells, and this correlation is much greater at the younger periods of growth than it is at any later period.

Correlation Table of Body and Cell Lengths.

Size of Cell		50—54	55—59	60—64	65—69	70—74	75—79	80—84	85—89	90—94	95—99	100—104	105—109	110—114	115—119	120—124	125—129	130—134		
Body Length		Working Scale																		
		1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	Totals	Cell Means
32— 36	Working Scale *1*	—	—	1	2	9	6	2	—	—	—	—	—	—	—	—	—	—	20	5·3000
37— 41	*2*	1	2	8	19	37	34	23	15	10	1	—	—	—	—	—	—	—	150	5·8333
42— 46	*3*	—	—	1	2	5	13	23	25	13	11	7	—	—	—	—	—	—	100	7·7900
47— 51	*4*	—	—	—	3	7	13	13	11	6	15	8	4	—	—	—	—	—	80	8·0500
52— 56	*5*	—	—	2	2	5	9	11	29	28	16	13	13	9	7	3	3	—	150	9·4733
57— 61	*6*	—	—	—	2	6	9	23	33	29	17	20	—	1	—	—	—		140	8·4357
62— 66	*7*	—	—	—	3	12	19	34	43	41	37	26	11	4	—	—	—	—	230	8·5957
67— 71	*8*	—	—	—	—	—	3	8	11	18	28	20	16	11	2	3	—	—	120	10·2667
72— 76	*9*	—	—	—	1	3	6	8	12	24	26	29	26	22	11	3	6	—	180	10·7611
77— 81	*10*	—	—	—	—	1	3	11	17	16	17	21	19	12	17	8	5	3	150	11·0267
82— 86	*11*	—	—	—	—	—	1	8	13	27	25	20	14	17	10	8	4	3	150	10·9533
87— 91	*12*	—	—	—	2	4	2	17	25	31	30	16	9	4	—	—	—	—	140	9·1000
92— 96	*13*	—	—	—	—	6	8	11	19	27	21	17	10	1	—	—	—	—	120	9·0000
97—101	*14*	—	—	—	—	1	4	22	19	34	49	45	27	11	5	3	—	—	220	10·0364
102—106	*15*	—	—	—	—	—	1	2	6	6	15	15	13	2	—	—	—	—	60	10·3167
Totals		1	2	12	36	96	131	216	278	310	308	257	162	94	55	28	18	6	2010	

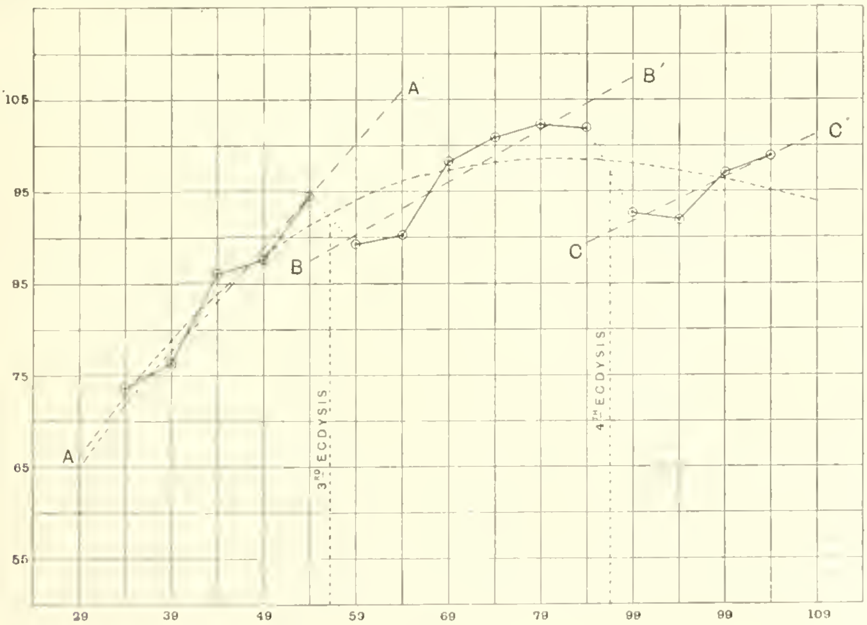

FIG. 2.

In this line of means two pronounced irregularities can be observed, one in the neighbourhood of a body length of 57—66 and one between 87—96. These mark the positions of the 3rd and 4th ecdyses. In these daphnia the body length at the 1st ecdysis was about 36 units, at the 2nd about 48, at the 3rd 57—61, at the 4th 77—91; after the 4th shedding the animal generally produces eggs in the brood pouch and there is an indefinite number of ecdyses varying according to the length of life.

It might perhaps be objected that the measurements of the polygonal areas of the carapace simply indicates the amount of stretching of the exoskeleton between successive moults and that the cells divide underneath quite independently of them. But if this were so the number of nuclei under each area would gradually increase as the animal approached its next moult. An examination of a number of individuals did not, however, support this view and it appears that not until the creature is on the point of moulting do the cells divide to any extent, and then it is clear from a glance at the diagram that the divided cells are larger than the cells which occurred immediately after the previous ecdysis, i.e. the mean of the cells in array 87—91 is higher than that of array 57—61.

In the diagram the lines of correlation are drawn through three successive periods of growth (1) the period from the time of hatching to the time when the 3rd moult is about to take place, (2) the period from the 3rd moult to the 4th moult, (3) the period after the 4th moult has occurred.

Since the individuals are grouped simply according to the size of the body these periods will considerably overlap, for the size of the animal at which the different ecdyses take place varies very considerably from animal to animal. This overlapping of the different periods of growth produces a disturbing element which is especially noticeable in the array 82—86 and it obliterates the destruction which otherwise should be observed between the periods of the 1st and 2nd ecdyses.

If we make allowance for this disturbing element it must be agreed that the means of the arrays cluster fairly well around the three lines of correlation; the constants are [*]

(1) Correlation $r = \cdot 5505$; Regression $= 1\cdot 1337$;
Regression line: $y - 3\cdot 4200 = 1\cdot 1337\,(x - 7\cdot 6500)$[†].

(2) Correlation $r = \cdot 3930$; Regression $= \cdot 5851$;
Regression line: $y - 8\cdot 4329 = \cdot 5851\,(x - 9\cdot 9217)$.

(3) Correlation $r = \cdot 2477$; Regression $= \cdot 4813$;
Regression line: $y - 13\cdot 3704 = \cdot 4813\,(x - 9\cdot 5944)$.

It will be seen that the regression is greatest at the first period of growth and least at the third period, that is the size of the constituent cells is less closely related to the size of the body in the older than in the younger periods of growth.

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[*] I am greatly indebted to Dr A. Lee for most kindly calculating these constants; and also those of the parabola to be mentioned below.

[†] y is the mean cell-length, x the body-length, both measured in the working scale units of the correlation table on p. 257.

If we now disregard the effect of ecdysis we can fit a continuous curve to the observations. A parabola of the 3rd order is shown in the diagram, the equation to it is:

$$y = 9{\cdot}0807\,\{1{\cdot}09787 + {\cdot}024{,}125x - {\cdot}005{,}992x^2 + {\cdot}000{,}228x^3\},$$

where the origin is at body-length 8 on working scale (observed cell mean 10·27), the unit of x, or the body-length, is five divisions of ocular micrometer used for body-length, and of y, or the mean cell-length, five of the divisions of the ocular micrometer used for cell-length (see p. 256).

An examination of this parabola shows us that the regression between size of body and size of cell, i.e. the slope of this curve, is very considerable at the younger periods but that this regression rapidly lessens as the animal grows older; there is also a distinct tendency for a negative regression to occur after the array 72—76*. This would mean that up to a certain period of life after the animal has been hatched, the cells of the carapace grow larger as the body grows, and that after this period has been passed the cells on dividing never again attain to such large dimensions. It leads to the question how far the period of life of maximum cell-size is to be regarded as the period of maximum vitality.

In the development of an animal (excluding the influence of yolk) the egg-cell divides into smaller and smaller cells and as the morula as a whole grows the constituent cells become smaller and the regression between the size of body and the size of cell is consequently negative. The cells reach a *minimum* size, and then in the developing distomum after a pause the regression becomes positive and the cells attain their *maximum* dimensions. Afterwards, in the case of daphnia and perhaps generally, there follows a slight negative regression.

It would be interesting to ascertain in a series of forms whether these are general phenomena, and if so, to find the point in the developing embryo at which the positive correlation becomes changed into a negative one, also the point of maximum cell-size in the actively growing animal. Perhaps these points might vary for different tissues or organs of the body or they might be nearly uniform throughout the whole body.

The essential feature of this preliminary note is to demonstrate the fact of the intimate relationship which obtains between the body as a whole and its constituent cells, the mean diameters of the cells change *pari passu* with the growth of the body.

There can be no doubt that a wider statistical study into the subject of cellular characters would yield results of fundamental importance both with regard to the laws of growth, and to our conceptions of the individuality of the cell-unit in the body of the living organism.

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* Other curves were tried for graduating the observations, but the graduated values found for them were remarkably close to the present, and they shewed in the same way the like tendency to an ultimate reduction in cell size. The term regression is here used for the slope of the tangent to the curve which graduates the relation between body-length and mean cell-size, when we ignore the ecdysis effects.

GRADUATION AND ANALYSIS OF A SICKNESS TABLE.

BY W. PALIN ELDERTON, *Actuary.*

"The whole problem of evolution is a problem in vital statistics—a problem of longevity, of fertility, of health and of disease." *Biometrika*, Vol. I. p. 320.

I.

Graduation by Frequency Curves.

THE Report by the late William Sutton to the Chief Registrar of Friendly Societies in 1896 contains between one and two thousand pages of statistics dealing with the rates of sickness and mortality experienced by Friendly Societies in England and Wales between the years 1856 and 1880. On pages 1134—1151 a table of the rates of sickness for 1876—1880 (excluding the first three years of membership) is given which, Professor Pearson suggested to me, might afford an interesting study in graduation. Some graduated figures are given on pages 1167—1171 of the report but though they are referred to on p. xiv. of the Introduction the method by which they were obtained is not described. The ungraduated rates of sickness are obtained by dividing the number of weeks' sickness for the year of age by the number of persons of the same age exposed to the risk of being sick.

When an attempt is made to graduate a rate of sickness or mortality by a formula we may consider that we are trying to fit a curve to the actual number of weeks' sickness or deaths on the assumption that the number exposed to risk is unity at each age. Of course, this is an imaginary state of affairs and there seems little reason for adopting it in preference to taking some algebraic or trigonometric curve to represent the exposed to risk and then working on the amount of sickness or number of deaths obtained from the hypothetical series by multiplying its successive numbers by the ungraduated rates of sickness or mortality for the appropriate ages; or, we might adopt the opposite process and assume a curve for the sickness or deaths and graduate the exposed to risk. To illustrate this proposition we may consider Gompertz' "Law of Mortality," which assumes that the force of mortality ($-\frac{1}{l_x}\frac{d}{dx}l_x$, where l_x is the number of persons living at age x in a stationary population) is of the form Ac^x and from the point of view taken above we can describe the "law" as assuming that if the exposed to risk be represented by the normal curve of error then the deaths will be represented by a curve of the same type having the same standard deviation. For the force of mortality is equal to the ordinate of the deaths divided by the ordinate of the exposed; this gives us $\kappa e^{-(x^2-x\cdot+hx-h^2)/2\sigma^2} = Bc^x$.

If possible, I think it would be a good principle in graduation to work on the exposed to risk and deaths (or weeks' sickness), for these functions are actual frequencies, but it is often difficult to do this in practice. In the present case, for instance, the exposed to risk for ages 18—22 are 3114; 4029; 5350; 23485; 33534 and this sudden change in the size of the numbers is awkward to deal with; nor is this the only difficulty, for though the exposed curve rises to a maximum at age 28 and then steadily falls, the sickness curve has two distinct maxima at ages 38 and 67.

A first examination of Sutton's table was made by assuming the normal curve of error as a hypothetical exposed to risk. The reason for choosing a frequency curve was that I have found it possible to represent (approximately) the exposed and deaths statistics in actual mortality investigations by such curves. The values for the integral $\left(\frac{2}{\sqrt{\pi}}\int_{\tau}^{\tau'} e^{-t^2}dt\right)$ were found by taking equal intervals of ·06; this used the whole table given in Galloway's *Probability*. It was then necessary to decide on the origin for this curve. I wished to obtain, if possible, a frequency curve for the number of weeks' sickness derived from the hypothetical exposed and to avoid the two maxima of the sickness distribution referred to above. It was desirable, therefore, to have the mode of the exposed curve near the minimum of the sickness curve and this was obviously convenient for it was near the middle of the range. Age 52 was chosen as the origin. The amount of sickness was obtained by multiplying the areas of the normal curve $y = 67{\cdot}703\, e^{-x^2/2(11{\cdot}785)^2}$ by the rates of sickness and the resulting figures were grouped about every fifth age to render the calculation of the moments less laborious*. The grouped figures ran smoothly and were fitted closely by the Type I. frequency curve

$$y = 248{\cdot}34\left[1+\frac{x}{88{\cdot}67125}\right]^{13{\cdot}8230}\left[1-\frac{x}{37{\cdot}6809}\right]^{5{\cdot}8741},$$

with an origin at 64·269406. The graduated rates were unsatisfactory above age 85, as they decreased very rapidly owing to the sickness curve ending at 101·950.

The rates of sickness given in Fig. 1 were obtained by dividing the areas of the above-mentioned Type I. curve corresponding to each age by the areas of the

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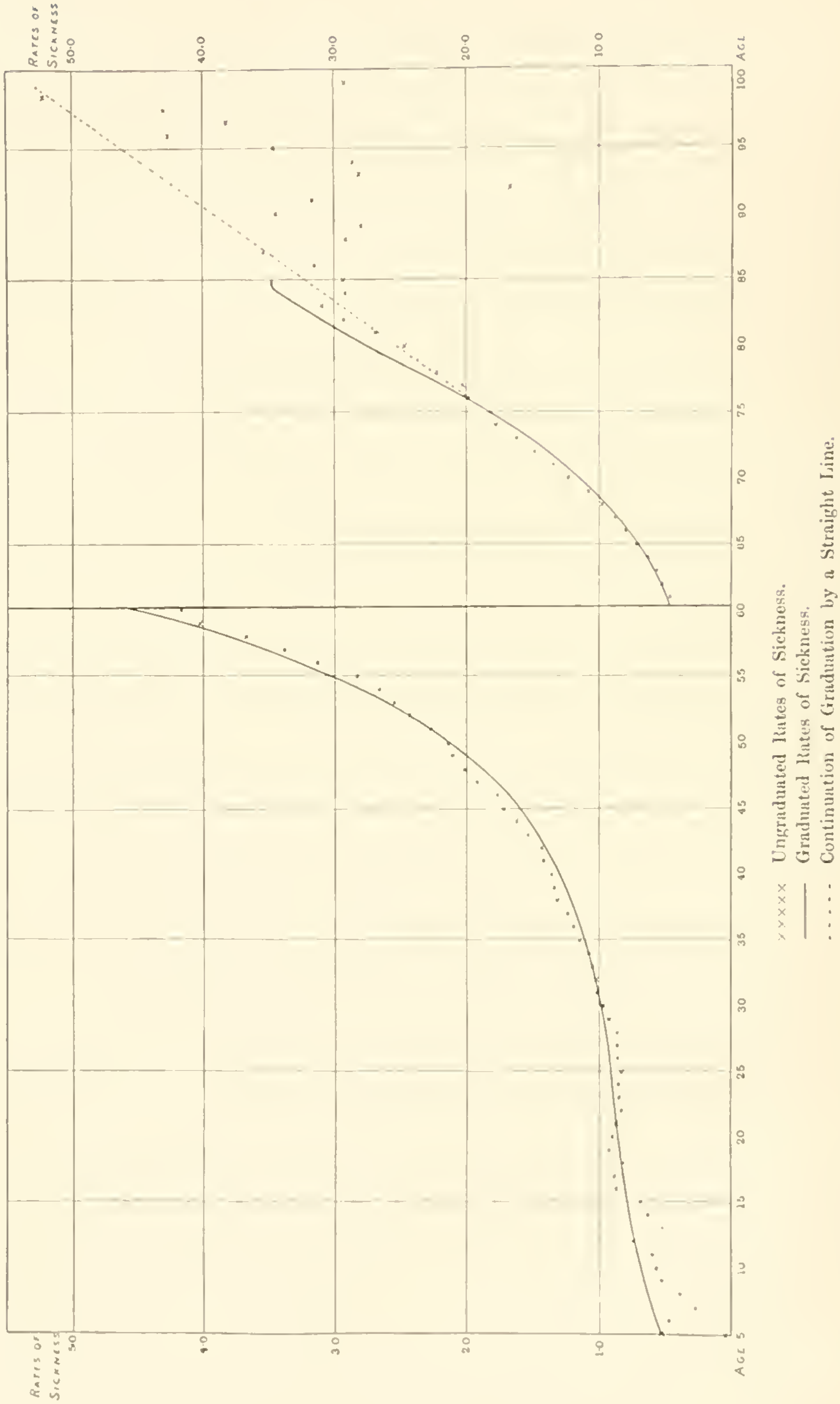
* An attempt to split up the distribution into two normal curves failed to give satisfactory results.

Fig. 1.


Rates of Sickness
50·0
40·0
30·0
20·0
10·0
Age
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
Rates of Sickness
5·0
4·0
3·0
2·0
1·0
× × × × × Ungraduated Rates of Sickness.
——— Graduated Rates of Sickness.
· · · · · Continuation of Graduation by a Straight Line.

normal curve which had been already used for finding the actual amount of sickness to which the Type I. curve was fitted. Strictly speaking, I think it would have been more correct to read off ordinates in the denominators, but the analysis of the point can hardly be dealt with in a few lines, and would be of a technical character. The use of areas makes it impossible to write down an algebraic expression for the rates of sickness; the quotient of the ordinates of the two curves would not give these rates, but a function that might be called the "force of sickness," for it is analogous to the force of mortality mentioned above.

This result was so good that I decided to make some further trials in the hope of obtaining other interesting representations. Symmetrical limited-range curves were tried with the same origin but though nearly all the Type I. curves I obtained represented the amount of sickness closely they failed at the ends of the table to give a satisfactory graduation of the sickness rates. A few trials were also made with different origins.

The next attempts were made from the opposite point of view, that is, a hypothetical curve was assumed for the sickness. The same normal curve was tried and the exposed to risk were then represented by a Type IV. curve

$$y = 273{\cdot}3649\,[1 + x^2/(66{\cdot}9576)^2]^{-21{\cdot}06072}\, e^{-4{\cdot}4504 \tan^{-1}(x/66{\cdot}9576)},$$

with an origin at 52·5043939. This curve is very nearly symmetrical but on trying a normal curve it was found to fail above age 66 (i.e. for 34 ages. Compare Makeham's "Law of Mortality" which is unsuitable for the first 25 ages). In order to see if the quotient of two normal curves would graduate the rates I tried to fit the parabola $y = a + bx + cx^2$ * to their logarithms but the result was unsatisfactory. Trials were then made with limited-range curves and the corresponding exposed to risk represented by Type I. curves. The results were unsatisfactory as regards the rates for at least 15 consecutive ages, in the case of the Type IV. and normal curve, for instance, the rates were considerably too low from age 67 to age 82. A trial with the Type I. curve to the rates of sickness also gave a poor result.

Viewed as processes of graduation the trials mentioned above are, as a rule, satisfactory for the main part of the sickness table and the practical conclusion to be drawn is that a rapid graduation can be made with reasonable results as far as age 80 or 85. The assumption of a hypothetical curve for the exposed is preferable to the opposite course, for, if the latter be adopted, there is some doubt as to the correct values to be given to the exposed to risk at ages where no persons have been sick. In the present case this only occurred at age 4, but in small experiences it would be a matter of greater importance. Another reason for the preference is that the old age end of the experience is relatively smaller when the hypothetical curve is used for the sickness than when it is used for the exposed; that is to say, the former method gives less weight to the old age values than the

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* Rate of sickness $= ke^{-(x-h)^2/s_1} \div e^{-(x-l)^2/s_2}$, therefore log rate $= ax^2 + bx + c$ (see Pearson, "On the Systematic Fitting of Curves," *Biometrika*, Vol. II. Pt. I.).

latter method, and, as we wish to graduate our rates as nearly as possible to the end of the table, the point is worth bearing in mind.

The first method I have described gave the best graduation and it could be improved by calculating the figures for the sickness to a few more places of decimals and using every term for finding the moments instead of groups of five. We might also treat the rate of sickness beyond age 100 as 52·167*; for by using an infinite-range curve as the exposed we assume that some live beyond the last age occurring in the experience and it is not unreasonable to suppose that they would be permanently sick. The rates for the last ten or fifteen ages if unsatisfactory might be altered by graduating the end of the experience by a parabolic curve or they might be supplied by drawing a straight line such as the dotted one shewn in the figure. I may remark, however, that for practical purposes it is unnecessary to obtain graduated rates of sickness right up to the limit of old age; in fact Bowser's tables† end (as regards sickness) at age 80 and he says on p. xv. of his Introduction that "if the rules of a society permit 'sick pay' to be "continued for the whole of life the valuer should regard the sickness benefit "after age 80, or still better age 75, in the light of a *Pension for life*" and his pension tables provide for the chronic sickness of old age.

For the present purpose it seems unnecessary to discuss the advantages of the method of graduation as it would involve a somewhat lengthy comparison with other methods which would have to be explained for non-actuarial readers; but it is obvious that the graduated rates must run evenly and will agree with the observed rates closely if the amount of sickness can be represented accurately by a frequency curve.

II.

Analysis of Sickness Table.

My object in continuing the work beyond a mere graduation of Sutton's Table was to see if sickness may be referred to certain broadly marked periods of life corresponding to those which Professor Pearson shewed were characteristic of mortality.

Before giving the work done in connection with sickness it will be well to recall the results obtained by Professor Pearson from his investigation of mortality. The function on which he worked was that known to actuaries as the "deaths" or "d_x" column of the mortality table‡. This column gives the number of persons who would die at each year of age in a stationary community supported by, say, l_0 births, or it may be considered as the number of persons who would die in each year of age out of l_0 persons born at the same time. When a mortality table is constructed the first function obtained is either the probability of living a year (p_x) or of dying during a year (q_x) at age x, and, since d_x is obviously equal to l_x (the number living at the exact age x) $\times q_x$ it is easy to see how the column d_x can be formed continuously from q_x.

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* 52·167 weeks was taken as a full year by Sutton; on p. xii. he says "it should be added that I "have invariably considered as regards broken weeks that there are six working days in a week."

† "Friendly Societies' Valuation and other tables deduced from the experience of the Independent "Order of Oddfellows, Manchester Unity Friendly Society, 1866—1870" by Wilfred A. Bowser. London, C. and E. Layton, 1896. The Manchester Unity table is the standard for Friendly Society Valuations.

‡ The reader must be careful to distinguish between the *d* column of the mortality table and the number of deaths in a mortality experience; the latter is the actual number who die, the former is a

In *The Chances of Death* (London, 1897)* Professor Pearson gave a description of his analysis of the "deaths" column of the mortality table (English, No. IV. Table) constructed by Dr Ogle from the Registrar-General's Returns for the years 1871—1880 and shewed that the old age part of the deaths curve could be represented accurately by a Type III. frequency curve having its origin at 71·5, that is, at the middle of the d_{71} group, and when this "old age" curve was deducted from the deaths curve the end of the remainder could be fitted by a normal curve having its origin at 41·5. This curve was called the "Middle life" curve and when it was deducted a "Youth" curve of the normal type with an origin at 22·5 was found. The rest of the deaths curve was fitted by a "Childhood" curve of Type III. with origin at age 3 and an "Infancy" curve which started 9 months before birth at infinity and was of the type $y_0 x^{-p} e^{-\gamma x}$.

Turning from Professor Pearson's work to the table we have graduated we notice at once that from a statistical point of view there is a great difference between mortality and sickness, for in the case of the former if a person dies he must, of necessity, pass out of observation, but a person may be sick, and, especially in the earlier periods of life, recover—that is, he does not cease to be a part of the experience under observation. For this reason we are obliged to alter our conception of a sickness table in order to obtain a function analogous to that used in the analysis of the mortality table. The rate of sickness at any age, as it is now used, is the average number of weeks' sickness falling to the lot of each person under observation at that age and if we divide this rate by 52·167† we get the average number of years' sickness, or *the probability* (r_x *say*) *that an individual will be sick for the whole year* under certain hypothetical circumstances arising from the assumption that persons are either sick for a whole year or not at all, and, having been sick for the whole year, pass out of observation altogether just as if they had died. The assumption is subsequent to the calculation of the sickness rates and it will not affect any results already obtained; our change of view merely accentuates the fact that we deal with the community as a whole and not with its individual members.

If we take h_0 persons at birth, then $h_0 r_0$ ($= f_0$ say) will represent the amount of sickness in years or the number of persons who are sick for the year of age (0—1)

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purely hypothetical function tabulated for convenience, and they can only be connected by means of the rates of mortality. The same distinction must be made between the number of weeks' sickness and the f column.

[*] See also *Phil. Trans.* A, Vol. 186, p. 343.

[†] Number of weeks in the year assumed by Sutton (see footnote, p. 264).

on the assumptions involved in our altered arrangement of the table. If we continue the calculations remembering that $h_x = h_{x-1} - f_{x-1}$ and $f_x = h_x r_x$ we shall form an "*f*" column for sickness equivalent to the "*d*" column for death.

An "*f*" column was formed from the graduated rates of sickness and, as it appeared to shew some similarities to the "*d*" column, I proceeded to split it up into component parts beginning, of course, with the old age end of the curve. The equations found for the various curves and the corresponding numbers are given in Table II. and the curves drawn out (Fig. 2) but it would be unwise to place much reliance on the "Childhood" curve as the figures on which it is based are derived from comparatively few cases. Taking the curves in order, beginning at old age, we notice that the old age sickness curve is much steeper and has its origin some nine years earlier than the death curve relating to the same period of life. The normal curves representing the sickness in "Middle Age" and "Youth" have their origins a little earlier than the corresponding death curves for the English No. IV. table, but the difference is slight. In other words *on the average*, death, if it comes, follows quickly on sickness in early life, but in old age it may follow the modal age of sickness by even nine years. A noticeable character in the analysis of the sickness curve is that the proportion of sickness ascribed to the "Youth" and "Childhood" groups is much larger than the proportion of deaths which Professor Pearson found related to the equivalent death groups.

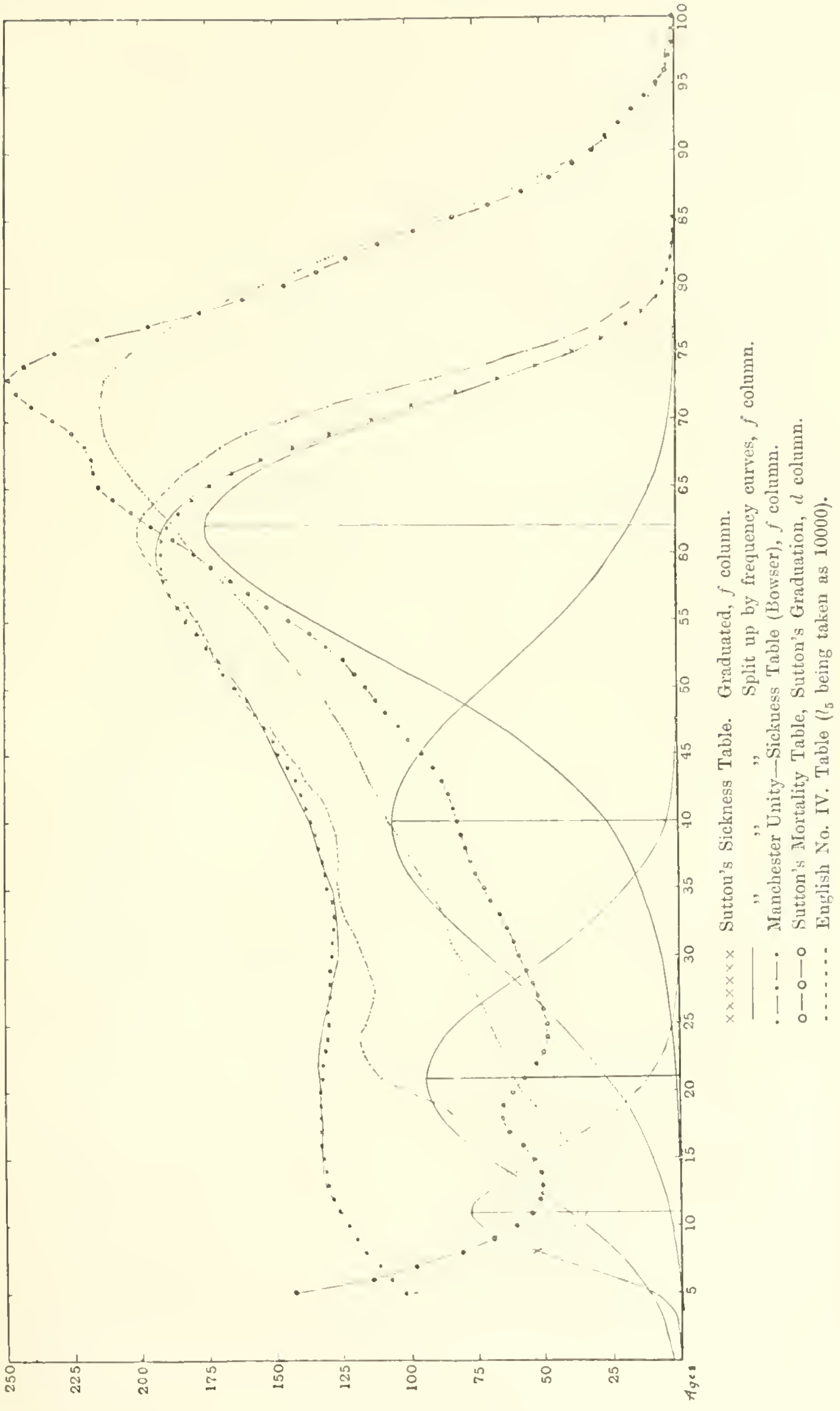
We have referred above to the fact that the old age sickness curve precedes the death curve and that the distance between them increases with the age; this is, of course, natural at the end of the table for sickness must come before death, and the curve for the latter will therefore become insignificant at a later age than the curve for the former. When we get nearer birth however it is not *necessary* for the origins of the sickness curves to be earlier than those of the deaths curves, for, if the rate of mortality is low (most obviously when it is decreasing) the fact of the sickness origin coming after the deaths origin would only indicate that the incidence of the sickness in the particular period of life was such that the sickness rate was increasing at a relatively greater rate than the death rates. The only thing that seems necessary is that the sickness curve for any group should not *start* later than the death curve.

I may here digress to refer to an auxiliary table which I found useful in splitting up the *f* column when the component part required a curve of Type III. The equation of a curve of this type may be expressed as

$$\log_{10} y_x = \log_{10} y_0 + \gamma \log_{10}(1+x) - \gamma x \log_{10} e,$$

where the range on the side of the origin where the curve does not extend to infinity, is taken as the unit for x. Now since y_0 can be taken as unity the right-hand side of this equation may be written $\gamma [\log_{10}(1+x) - x \log_{10} e]$ and a table of the function in square brackets was formed to six places of decimals for values of x from -1 to 2·5 with intervals of ·05. The use of such a table for finding γ and a is obvious and as it is also a help in calculating the actual curve when

Fig. 2.


250
225
200
175
150
125
100
75
50
25
Ages
5
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55
60
65
70
75
80
85
90
95
100
× × × × × Sutton's Sickness Table. Graduated, f column.
——— " " Split up by frequency curves, f column.
. — . — . Manchester Unity—Sickness Table (Bowser), f column.
o—o—o Sutton's Mortality Table, Sutton's Graduation, d column.
. . . . English No. IV. Table (l_5 being taken as 10000).

γ and a have been found, the table is given below. The normal curves were obtained by comparing the column of first differences in the table of the integral in Galloway's *Probability* with the data to which it was required to fit the curve. The normal curve was tried in each case and when it failed to give a satisfactory representation a Type III. curve was substituted. The work was originally done with four figures and afterwards reduced to three and I made it a rule in finding my curves not to pass a difference larger than 30 when the figure in the "f" column was more than 1000, my reason for choosing the limit being that the square of the difference divided by the frequency is a measure of the goodness of fit, and I noticed that if each of these measures was about ·8 the fit would be very good. As will be seen from the table most of the differences were considerably smaller; the two differences entered as 3 for ages 23 and 24 lie between 2·5 and 3·0. The average difference shewn by the table is ·9 and this seems good evidence that though the method adopted was tedious it had the merit of bringing out a satisfactory result.

I have drawn out the "f" column obtained from the graduated sickness table and the values of the same function for the Manchester Unity table* are also given in the diagram. In order to bring out the connection between the "f" (sickness) and "d" (mortality) columns I have also shewn the "d" column obtained from the "l" column given on p. 1176 of Sutton's work relating to 10,000 living at age 5 and the same column from the English No. IV. table altered so as to give the same number living at age 5. The analysis of the "f" curve is also given. The ages at the bottom of the diagram are at the centre of the groups to which they refer; so that f_{30} is above age 30 not 30·5.

We may however view the comparison between the rates of sickness and mortality from the point of view of our sickness graduation and find the curve that will represent the number of deaths if the exposed to risk is the same series as that assumed in the sickness graduation. Mr Sutton in his Report gave a table of the ungraduated rates of mortality (from the graduated rates the "d" column referred to above was calculated) obtained from the same experience as that from which the sickness rates were found and I applied the method of graduation to these rates and found that the number of deaths could be represented by

$$y = 174{\cdot}19\left[1+\frac{x}{68{\cdot}0265}\right]^{10{\cdot}35195}\left[1-\frac{x}{43{\cdot}7927}\right]^{6{\cdot}66415},$$

having its origin at age 61·9319. To obtain this curve the areas of the normal curve were multiplied by the corresponding values of q_x by Crelle's Tables and not more than three figures were used in the products which were grouped in fives before the moments were calculated. These abbreviations saved a great deal of labour and though, combined with the irregularity of the data before age 20 and after age 75, they render the graduated rates somewhat unsatisfactory at the ends of the experience (probably above 87 and under 12) they would only

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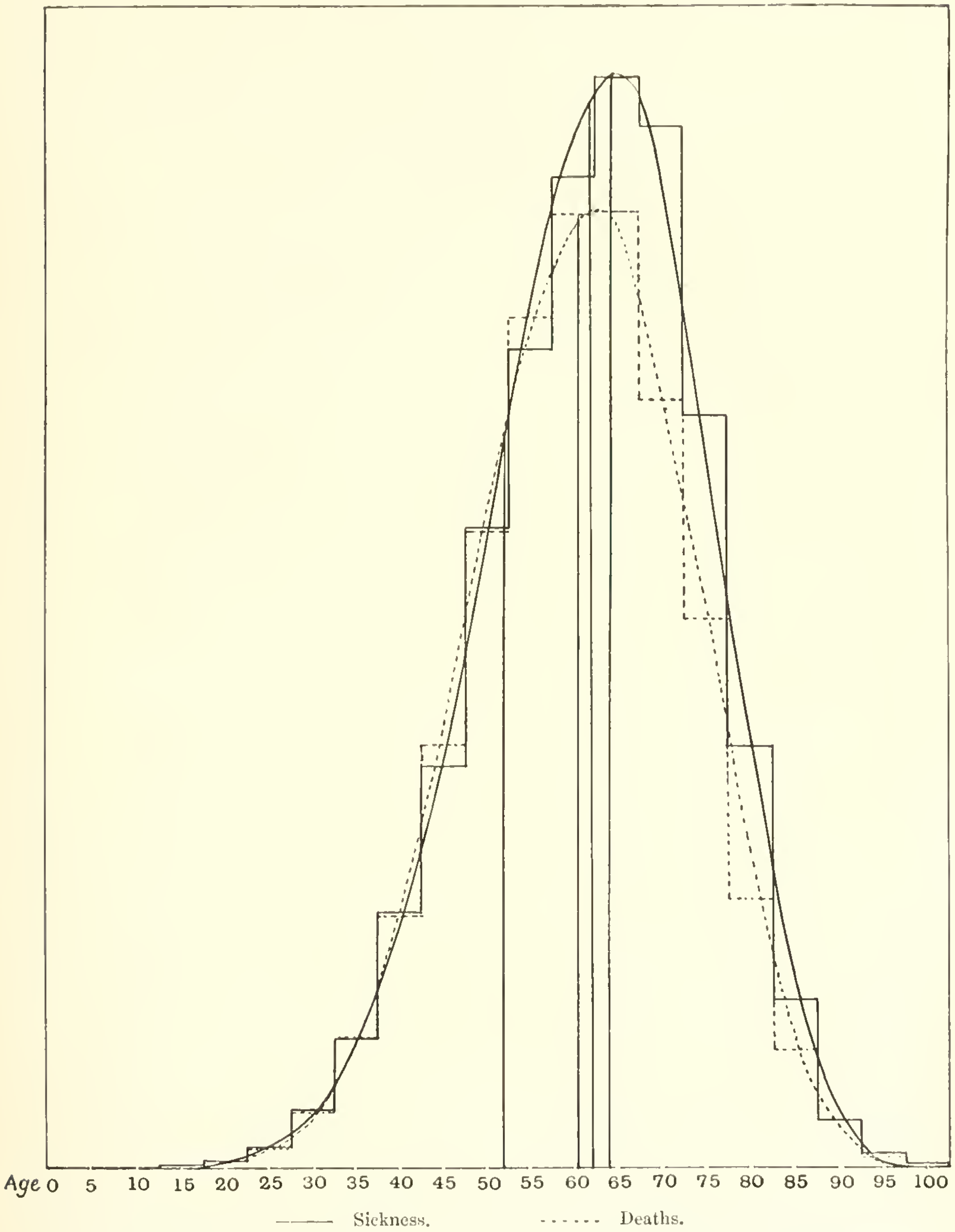
* When these values were calculated it was assumed that the value of h_{20} by the Manchester Unity table was the same as that given by Sutton's Sickness table. Some such assumption was necessary because Bowser's tables begin at age 20.

FIG. 3. *Amount of Sickness and Number of Deaths when the Exposed to Risk is represented by the same Normal Curve.*



Each square half-inch = 200 weeks' sickness = 160 deaths. The vertical lines taken in order from left to right shew (1) axis of normal curve giving exposed to risk, (2) mean of deaths, (3) mode of deaths coinciding to our scale with mean of sickness, (4) mode of sickness.

affect the main part of the table to a small extent and as the death curve was only required for comparison I considered it perfectly safe to use them.

The curve has been plotted side by side with the sickness curve in figure 3 and the following points may be noticed:—

(1) The skewness of the sickness curve is ·2 and of the death curve ·1.

(2) The mode and mean of the sickness curve are nearer old age than the mode and mean of the death curve.

(3) The sickness curve starts 18 years and ends 4 years before the death curve.

An examination of the two curves shews the connection between sickness, mortality and age and is perhaps the simplest way of doing so.

This concludes my work on Sutton's Table but the analysis of sickness will be very incomplete till other tables have been studied and infantile sickness has received a thorough examination.

TABLE I. (*for use with Type III. curves*).

X	$\log_{10}(1+X)-X\log_{10}e$[*]	X	$\log_{10}(1+X)-X\log_{10}e$[*]
− ·95	·888450	+ ·80	·092163
− ·90	·609135	+ ·85	·101979
− ·85	·454759	+ ·90	·112111
− ·80	·351534	+ ·95	·122545
− ·75	·276339	+ 1·00	·133265
− ·70	·218873	+ 1·05	·144255
− ·65	·173641	+ 1·10	·155505
− ·60	·137363	+ 1·15	·167000
− ·55	·107926	+ 1·20	·178731
− ·50	·083883	+ 1·25	·190685
− ·45	·064205	+ 1·30	·202855
− ·40	·048131	+ 1·35	·215230
− ·35	·035084	+ 1·40	·227801
− ·30	·024614	+ 1·45	·240561
− ·25	·016365	+ 1·50	·253502
− ·20	·010051	+ 1·55	·266616
− ·15	·005437	+ 1·60	·279898
− ·10	·002328	+ 1·65	·293340
− ·05	·000562	+ 1·70	·306937
·00	·000000	+ 1·75	·320683
+ ·05	·000525	+ 1·80	·334572
+ ·10	·002037	+ 1·85	·348600
+ ·15	·004446	+ 1·90	·362761
+ ·20	·007678	+ 1·95	·377052
+ ·25	·011664	+ 2·00	·391468
+ ·30	·016345	+ 2·05	·406004
+ ·35	·021669	+ 2·10	·420657
+ ·40	·027590	+ 2·15	·435422
+ ·45	·034065	+ 2·20	·450298
+ ·50	·041056	+ 2·25	·465279
+ ·55	·048530	+ 2·30	·480363
+ ·60	·056457	+ 2·35	.495547
+ ·65	·064808	+ 2·40	·510828
+ ·70	·073557	+ 2·45	·526202
+ ·75	·082683	+ 2·50	·541668

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[*] This column is negative throughout and as it will have to be multiplied it will be found more convenient to deal with the logarithms in this form than in the usual one giving a positive mantissa and negative characteristic.

TABLE II.

Ages	Descriptions of Curves					Total by Curves	"f" column	Difference	
	Old Age Origin 62·5 $175{\cdot}7\left(1-\frac{x}{25}\right)^{7\cdot 15}e^{\cdot 0286x}$	Middle Age Origin 40·5 $106{\cdot}0e^{-\cdot 003844x^2}$	Youth Origin 21·5 $91{\cdot}1e^{-\cdot 01235x^2}$	Childhood Origin 11·5 $77{\cdot}0\left(1+\frac{x}{10}\right)^{6\cdot 2}e^{-\cdot 62x}$	Infancy			+	−
85	·0	—	—	—	—	—	·1	—	—
84	·1	—	—	—	—	·1	·1	—	—
83	·3	·1	—	—	—	·4	·4	—	—
82	1	—	—	—	—	1	1	—	—
81	2	—	—	—	—	2	2	—	—
80	4	—	—	—	—	4	4	—	—
79	7	—	—	—	—	7	7	—	—
78	12	—	—	—	—	12	12	—	—
77	19	—	—	—	—	19	18	1	—
76	27	1	—	—	—	28	27	1	—
75	37	1	—	—	—	38	38	—	—
74	50	1	—	—	—	51	52	—	1
73	64	2	—	—	—	66	66	—	—
72	78	2	—	—	—	80	82	—	2
71	94	3	—	—	—	97	98	—	1
70	109	3	—	—	—	112	113	—	1
69	123	4	—	—	—	127	129	—	2
68	136	5	—	—	—	141	143	—	2
67	148	7	—	—	—	155	155	—	—
66	158	8	—	—	—	166	166	—	—
65	165	10	—	—	—	175	174	1	—
64	171	12	—	—	—	183	181	2	—
63	174	14	—	—	—	188	186	2	—
62	175	17	—	—	—	192	190	2	—
61	174	20	—	—	—	194	192	2	—
60	171	23	—	—	—	194	192	2	—
59	167	27	—	—	—	194	192	2	—
58	161	31	—	—	—	192	191	1	—
57	153	35	—	—	—	188	189	—	1
56	145	40	—	—	—	185	186	—	1
55	137	45	—	—	—	182	183	—	1
54	128	50	—	—	—	178	179	—	1
53	119	55	—	—	—	174	176	—	2
52	109	61	—	—	—	170	172	—	2
51	100	67	—	—	—	167	169	—	2
50	91	72	—	—	—	163	165	—	2
49	82	78	—	—	—	160	161	—	1
48	74	83	—	—	—	157	157	—	—
47	66	88	—	—	—	154	154	—	—
46	59	92	1	—	—	152	151	1	—
45	53	96	1	—	—	150	149	1	—
44	47	100	1	—	—	147	146	1	—
43	41	102	2	—	—	145	143	2	—
42	36	104	3	—	—	143	141	2	—
41	31	105	4	—	—	140	139	1	—

TABLE II.—(*continued*).

Ages	Descriptions of Curves					Total by Curves	"*f*" column	Difference	
	Old Age Origin 62·5 $175\cdot7\left(1-\frac{x}{25}\right)^{7\cdot15}e^{\cdot0286x}$	Middle Age Origin 40·5 $106\cdot0e^{-\cdot003844x^2}$	Youth Origin 21·5 $91\cdot1e^{-\cdot01228x^2}$	Childhood Origin 11·5 $77\cdot0\left(1+\frac{x}{10}\right)^{6\cdot2}e^{-\cdot62x}$	Infancy			+	−
40	27	106	5	—	—	138	137	1	—
39	24	105	7	—	—	136	135	1	—
38	20	104	9	—	—	133	134	—	1
37	18	102	12	—	—	132	133	—	1
36	15	100	16	—	—	131	132	—	1
35	13	96	20	—	—	129	131	—	2
34	11	92	24	—	—	127	129	—	2
33	10	88	30	—	—	128	128	—	—
32	8	83	36	—	—	127	128	—	1
31	7	78	43	—	—	128	129	—	1
30	6	72	49	—	—	127	129	—	2
29	5	67	56	1	—	129	130	—	1
28	4	61	64	1	—	130	130	—	—
27	3	55	71	1	—	130	130	—	—
26	3	50	77	2	—	132	131	1	—
25	2	45	83	3	—	133	131	2	—
24	2	40	88	4	—	134	131	3	—
23	2	35	92	6	—	135	132	3	—
22	1	31	94	8	—	134	133	1	—
21	1	27	94	11	—	133	133	—	—
20	1	23	94	16	—	134	134	—	—
19	1	20	92	21	—	134	134	—	—
18	1	17	88	27	—	133	134	—	1
17	1	14	83	35	—	133	134	—	1
16	1	12	77	43	—	133	134	—	1
15	—	10	71	52	—	133	133	—	—
14	—	8	64	61	—	133	132	1	—
13	—	7	56	69	—	132	131	1	—
12	—	5	49	75	—	129	129	—	—
11	—	4	43	76	2	125	126	—	1
10	—	3	36	74	10	123	123	38	38
9	—	3	30	67	20	120	120		
8	—	2	24	54	37	117	117		
7	—	2	20	39	51	112	112		
6	—	1	16	23	67	107	107		
5	—	1	12	11	78	102	102		
4	—	1	9	3					
3	—	—	7	1					
2	—	—	5						
1	—	—	4						
0	—	—	3						

On the Probable Errors of Frequency Constants.

(EDITORIAL.)

Introductory.

In all reasoning on statistical data we have to determine whether differences between statistical constants are significant or not. Half the blunders made in superficial statistical investigations arise from neglecting the values of the probable errors of the results obtained. But the calculation of probable errors becomes somewhat complex, when we have to deal not merely with the probable errors of means, but with those of constants depending in a much more intricate manner on the moments of the material.

The fundamental memoirs on the subject are W. F. Sheppard: "On the Application of the Theory of Error to cases of Normal Distribution and Normal Correlation," *Phil. Trans.* A., Vol. 192, pp. 101—167. L. N. G. Filon and K. Pearson: "On the Probable Errors of Frequency Constants and on the Influence of Random Selection on Variation and Correlation," *Phil. Trans.* A., Vol. 191, pp. 229—311.

The present discussion of the probable errors of frequency constants presents, perhaps, little that is novel, but it endeavours to give simple proofs of the main propositions of the subject. It is published in *Biometrika*, because several readers of that Journal have written to the Editors making enquiries on these points, recognising that the non-publication of probable errors in statistical memoirs is a serious disadvantage.

The simple idea involved in the probable error of a statistical constant is of the following kind: If the whole of a population were taken we should have certain values for its statistical constants, but in actual practice we are only able to take a sample, which should if possible be a "random sample." If a number of random samples be taken any statistical constant will vary from sample to sample, and its variation is distributed according to some law round the actual value of the constant for the total population. This variation would be very properly measured by the standard deviation of the constant for an indefinitely great series of random samples. Unfortunately custom has not taken this standard deviation as the measure of the goodness of the sample, but the whole theory having ultimately developed from the normal curve, the probable error instead of the standard deviation has been chosen, i.e. ·67449 × standard deviation. The adoption of the "probable error" of a constant as a measure of its exactness must not, however, be taken as equivalent to asserting the validity of the normal law of errors or deviations, but merely as a purely conventional reduction of the standard deviation. It would be equally valid provided it were customary to omit this reduction or indeed to multiply the standard deviation by any other conventional factor.

PROBLEM I. *Let there be n frequency groups, containing in the total population of N individuals, $y_1, y_2, y_3 \ldots y_n$ individuals respectively. To find the standard deviation of y_s, when random samples are made of m individuals from the whole population.*

The chance of an individual being drawn from the s^{th} group on one trial $= \frac{y_s}{N} = p$ say, and of its not being drawn $= 1 - \frac{y_s}{N} = q$, say. Then in m trials the distribution of frequency of this group will be given by $(p+q)^m$, with the known standard deviation $\sigma_{y_s} = \sqrt{mpq} = \sqrt{m \cdot \frac{y_s'}{m}\left(1 - \frac{y_s'}{m}\right)}$, where $y_s' = \frac{m}{N} y_s$ and is accordingly the proportion of y_s which we should expect in the typical group of m out of the N individuals. But in actual practice we have only the sample and do not know N or y_s. If y_s'' be the observed frequency of the s^{th} group y_s'' will almost certainly lie within $y_s \pm 3\sigma_{y_s}$. Hence in the above formula for σ_{y_s} we may replace, if σ_{y_s} be small compared with y_s, y_s' by y_s'', the observed frequency of the sample. Or, we read

$$\sigma^2_{y_s} = y_s\left(1 - \frac{y_s}{m}\right) \quad \text{..(i)},$$

where y_s is now taken as the frequency of the s^{th} group in the sample.

PROBLEM II. *To find the correlation between deviations in y_s and $y_{s'}$, or between deviations in the frequencies of the s^{th} and s'^{th} groups.*

Let $\delta y_s =$ deviation from y_s the most probable value in the s^{th} group, then since

$$y_1 + y_2 + y_3 + \ldots + y_s + \ldots + y_{s'} + \ldots + y_n = m,$$
$$\delta y_1 + \delta y_2 + \delta y_3 + \ldots + \delta y_s + \ldots + \delta y_{s'} + \ldots + \delta y_n = 0.$$

Now if our sample has given δy_s too many in y_s, it is proper to suppose that this error will be distributed among the other groups in the proportion of their relative frequencies*. In other words we should have

$$\delta y_s' = - \delta y_s \times \frac{y_{s'}}{m - y_s}.$$

Thus:

$$\delta y_s' \, \delta y_s = - \frac{(\delta y_s)^2}{m} \frac{y_{s'}}{1 - y_s/m}.$$

Summing for all samplings

$$\sigma_{y_{s'}} \sigma_{y_s} r_{y_s y_{s'}} = - \frac{\sigma^2_{y_s}}{m} \frac{y_{s'}}{1 - y_s/m} = - \frac{y_s y'_{s'}}{m} \text{ by (i).}$$

Hence

$$\sigma_{y_s} \sigma_{y_{s'}} r_{y_s y_{s'}} = - \frac{y_s y'_{s'}}{m} \quad \text{.. (ii).}$$

PROBLEM III. *To find the standard deviation σ_h of the mean h of a system of observations.*

Measured from a fixed point of reference, we have:

$$h = \frac{S(xy)}{S(y)} = \frac{S(xy)}{m},$$

where y is the frequency of individuals of size x.

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* This of course assumes that the error is merely due to random sampling, and not to defective measurement or classificatory judgment. In such cases the error in δy_s might, for example, be especially drawn from the "adjacent" classes y_{s-1} and y_{s+1}.

Therefore

$$m\delta h = S(x\delta y),$$

$$m^2(\delta h)^2 = S(x_s^2\,\delta y_s^2) + 2S'(x_s x_{s'}\,\delta y_s\,\delta y_{s'}),$$

where S' is a sum for all values of s and s' for which s is not equal to s'. This gives dividing by the number of random samplings after summing for all such samples:

$$m^2\sigma_h^2 = S(x_s^2\,\sigma^2_{y_s}) - 2S'(x_s\,x_{s'}\,\sigma_{y_s}\,\sigma_{y_{s'}}\,r_{y_s y_{s'}}),$$

or, using (i) and (ii)

$$m^2\sigma_h^2 = S(x_s^2 y_s) - S\left(x_s^2\frac{y_s^2}{m}\right) - 2S'\left(x_s x_{s'}\frac{y_s y_{s'}}{m}\right)$$

$$= m\mu_2' - m\,\frac{S(x_s y_s)}{m} \times \frac{S(x_{s'} y_{s'})}{m}$$

$$= m(\mu_2' - h^2),$$

where $m\mu_2'$ is the second moment about the fixed point. But $\mu_2' - h^2 = \sigma^2 =$ square of standard deviation of sample. Hence:

$$\sigma_h = \sigma/\sqrt{m} \;\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(iii)}.$$

Problem IV. *To find the standard deviation of the* q^{th} *moment* $= m\mu_q'$ *of the sample* **about a fixed point.**

$$m\mu_q' = S(x_s^q y_s).$$

Therefore

$$m\delta\mu'_q = S(x_s^q\,\delta y_s),$$

and

$$m^2\sigma^2_{\mu'_q} = S(x_s^{2q}\sigma^2_{y_s}) + 2S'(x_s^q x_{s'}^q \sigma_{y_s}\sigma_{y_{s'}} r_{y_s y_{s'}})$$

as before. Using (i), and (ii), we have

$$m^2\sigma^2_{\mu'_q} = S(x_s^{2q} y_s) - \frac{S(x_s^{2q} y_s^2)}{m} - \frac{2S'(x_s^q x_{s'}^q y_s y_{s'})}{m}$$

$$= m\mu'_{2q} - m\mu_q' \times \mu_q',$$

or

$$\sigma_{\mu'_q} = \frac{\sqrt{\mu'_{2q} - \mu'^2_q}}{\sqrt{m}} \;\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(iv)}.$$

Problem V. *To find the correlation of errors of the* q^{th} *and* q'^{th} *moments,* **both moments being taken about the same fixed point.**

As in Problem IV.:

$$m\delta\mu'_q = S(x_s^q\,\delta y_s),$$

$$m\delta\mu'_{q'} = S(x_s^{q'}\delta y_s).$$

Multiplying these together:

$$m^2\delta\mu'_q\delta\mu'_{q'} = S(x_s^{q+q'}\delta y_s^2) + S'\{(x_s^q x_{s'}^{q'} + x_{s'}^q x_s^{q'})\,\delta y_s \delta y_{s'}\},$$

or, using the definitions of correlation and standard deviation:

$$m^2\sigma_{\mu'_q}\sigma_{\mu'_{q'}} r_{\mu'_q\mu'_{q'}} = S(x_s^{q+q'}\sigma^2_{y_s}) + S'\{(x_s^q x_{s'}^{q'} + x_{s'}^q x_s^{q'})\,\sigma_{y_s}\sigma_{y_{s'}} r_{y_s y_{s'}}\}.$$

Hence by (i) and (ii)

$$m^2\sigma_{\mu'_q}\sigma_{\mu'_{q'}} r_{\mu'_q\mu'_{q'}} = m\mu'_{q+q'} - m\mu'_q\mu'_{q'}.$$

Thus

$$\sigma_{\mu'_q}\sigma_{\mu'_{q'}} r_{\mu'_q\mu'_{q'}} = \frac{\mu'_{q+q'} - \mu'_q\mu'_{q'}}{m} \;\;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(v)},$$

(iv) and (v) thus give the standard deviation of errors in any moment and the correlation in errors between two moments, when the moments are taken *about a fixed point*. We now require to find the standard deviation in errors in any moment, and the correlation of errors in moments, when these moments are taken, not about a fixed point, but about the mean, which is a point varying with the errors of random sampling.

PROBLEM VI. *To find the correlation between an error in the mean and an error in the q^{th} moment both determined from a fixed point.*

This is given at once by (v) since $h=\mu_1'$. We have only to put $q'=1$, and we have

$$\sigma_{\mu'_q}\sigma_h r_{h\mu'_q}=\frac{\mu'_{q+1}-h\mu'_q}{m} \quad \text{..(vi).}$$

Now it may be noted once and for all that *after* these expressions like (iv), (v) and (vi) have been found we can calculate the right-hand side taking for our fixed point any origin we please, i.e. we can take the point which actually coincides with the mean in the sample, i.e. write $h=0$, and $\mu'_q=\mu_q$, where μ_q is the moment about the observed mean.

PROBLEM VII. *To find the standard deviation for errors in the q^{th} moment μ_q* **taken about the mean.**

We have:

$$\begin{aligned} m\mu_q &= S\{(x_s-h)^q y_s\} \\ &= m\left(\mu'_q - qh\mu'_{q-1}+\frac{q(q-1)}{1\,.\,2}h^2\mu'_{q-2}+\ldots\right). \end{aligned}$$

Or:

$$\mu_q=\mu'_q-qh\mu'_{q-1}+h^2\chi,$$

where χ is a function of q, h and μ'_{q-2}, μ'_{q-3}, etc.

Hence:

$$\delta\mu_q=\delta\mu'_q-qh\,\delta\mu'_{q-1}-\delta h\,(q\mu'_{q-1}+2h\chi)+\text{terms in } h^2 \text{ and higher powers.}$$

Thus if we are going to put $h=0$ finally or measure our moments from a fixed point, which coincides with the actual position of the mean in the sample, we may write:

$$\delta\mu_q=\delta\mu'_q-q\,\delta h\,\mu'_{q-1}+\text{terms which vanish with } h.$$

Thus:

$$\delta\mu_q^2=\delta\mu_q'^2+q^2\delta h^2\,\mu'^2_{q-1}-2q\,\mu'_{q-1}\,\delta h\,\delta\mu'_q+\text{terms vanishing with } h.$$

Or:

$$\sigma^2_{\mu_q}=\sigma^2_{\mu'_q}+q^2\sigma_h^2\mu'^2_{q-1}-2q\mu'_{q-1}\,\sigma_h\sigma_{\mu'_q}r_{h\mu'_q}+\text{terms in } h,$$

and using (iv), (iii) and (vi):

$$\sigma^2_{\mu_q}=\frac{\mu'_{2q}-\mu'^2_q+q^2\sigma^2\mu'^2_{q-1}-2q\mu'_{q-1}\mu'_{q+1}+\text{terms in } h}{m}.$$

Put $h=0$, and $\mu'_i=\mu_i$, we have:

$$\sigma_{\mu_q}=\sqrt{\frac{\mu_{2q}-\mu^2_q-2q\mu_{q+1}\mu_{q-1}+q^2\sigma^2\mu^2_{q-1}}{m}} \quad \text{.................................... (vii).}$$

Illustrations.

(i) Put $q=2$, and remember that $\mu_1=0$.

$$\text{Probable error of } \mu_2=\cdot 67449\sqrt{\frac{\mu_4-\mu_2^2}{m}}.$$

For example, in the case of a normal curve this $=\cdot 67449\sqrt{\frac{2}{m}}\,\mu_2$, since $\mu_4=3\mu_2^2$.

Further we have, since $\sigma=\sqrt{\mu_2}$, for the normal curve

$$\text{Probable error of } \sigma=\cdot 67449\sqrt{\frac{1}{2m}}\,\sigma.$$

(ii) Put $q=3$. $\quad$ Probable error of $\mu_3=\cdot 67449\sqrt{\frac{\mu_6-\mu_3^2-6\mu_4\mu_2+9\mu_2^3}{m}}$.

For example, in the case of a normal curve $\mu_4=3\mu_2^2$, $\mu_6=15\mu_2^3$, and $\mu_3=0$, therefore

$$\text{Probable error of } \mu_3=\cdot 67449\sqrt{\frac{15\sigma^6-18\sigma^6+9\sigma^6}{m}}=\cdot 67449\sqrt{\frac{6}{m}}\,\sigma^3,$$

and a curve should not by this criterion alone be assumed not to be normal unless the observed μ_3 is at least two or three times as large as

$$\cdot 67449\sqrt{\frac{6}{m}}\,\sigma^3.$$

(iii) Put $q=4$.

$$\text{Probable error of } \mu_4=\cdot 67449\sqrt{\frac{\mu_8-\mu_4^2-8\mu_5\mu_3+16\mu_2\mu_3^2}{m}}.$$

For the normal curve since $\mu_3=0$, this reduces to

$$\cdot 67449\sqrt{\frac{\mu_8-\mu_4^2}{m}}=\cdot 67449\sqrt{\frac{96}{m}}\,\sigma^4,$$

the former value being exactly the same expression as holds for moments about a fixed point.

PROBLEM VIII. *To find the correlation in errors in two moments* **both measured from the mean.**

As in the last problem:

$$\delta\mu_q=\delta\mu'_q-q\delta h\mu'_{q-1}+\text{terms in } h,$$
$$\delta\mu_{q'}=\delta\mu'_{q'}-q'\delta h\mu'_{q'-1}+\text{terms in } h.$$

Hence:

$$\delta\mu_q\,\delta\mu_{q'}=\delta\mu'_q\delta\mu'_{q'}+qq'(\delta h)^2\mu'_{q-1}\mu'_{q'-1}-q'\delta h\,\delta\mu'_q\mu'_{q'-1}-q\delta h\,\delta\mu'_{q'}\mu'_{q-1}+\text{terms in } h.$$

Thus:

$$\sigma_{\mu_q}\sigma_{\mu_{q'}}r_{\mu_q\mu_{q'}}=\sigma_{\mu'_q}\sigma_{\mu'_{q'}}r_{\mu'_q\mu'_{q'}}+qq'\sigma_h^2\mu'_{q-1}\mu'_{q'-1}-q'\sigma_h\sigma_{\mu'_q}r_{h\mu'_q}\mu'_{q'-1}-q\sigma_h\sigma_{\mu'_{q'}}r_{h\mu'_{q'}}\mu'_{q'-1}$$
$$+\text{terms in } h.$$

Using (v) and (vi) we find:

$$\sigma_{\mu_q}\sigma_{\mu_{q'}}r_{\mu_q\mu_{q'}}=\frac{\mu'_{q+q'}-\mu'_q\mu'_{q'}+qq'\sigma^2\mu'_{q-1}\mu'_{q'-1}-q'\mu'_{q+1}\mu'_{q'-1}-q\mu'_{q-1}\mu'_{q'+1}+\text{terms in } h}{m}.$$

Putting $h=0$, $\mu_i'=\mu_i$, we find:

$$\sigma_{\mu_q}\sigma_{\mu_{q'}}r_{\mu_q\mu_{q'}}=\frac{\mu_{q+q'}-\mu_q\mu_{q'}+qq'\sigma^2\mu_{q-1}\mu_{q'-1}-q'\mu_{q+1}\mu_{q'-1}-q\mu_{q-1}\mu_{q'+1}}{m}\ \ldots\ldots\ldots\ \text{(viii)}.$$

Illustrations.

(i) Suppose $q=2$, $q'=3$, then remembering $\mu_1=0$,

$$\sigma_{\mu_2}\sigma_{\mu_3}r_{\mu_2\mu_3}=\frac{\mu_5-\mu_2\mu_3-3\mu_3\mu_2}{m}=\frac{\mu_5-4\mu_2\mu_3}{m}.$$

(ii) Suppose $q=2$, $q'=4$,

$$\sigma_{\mu_2}\sigma_{\mu_4}r_{\mu_2\mu_4}=\frac{\mu_6-\mu_2\mu_4-4\mu_3^2}{m}.$$

(iii) Suppose $q=3$, $q'=4$,

$$\sigma_{\mu_3}\sigma_{\mu_4}r_{\mu_3\mu_4}=\frac{\mu_7-\mu_3\mu_4+12\mu_2^2\mu_3-4\mu_4\mu_3-3\mu_2\mu_5}{m}$$

$$=\frac{\mu_7-5\mu_3\mu_4+12\mu_2^2\mu_3-3\mu_2\mu_5}{m}.$$

Lastly, to find certain probable errors we require:

PROBLEM IX. *To find the correlation between errors in the q^{th} moment about the mean and the q'^{th} moment about a fixed point, in terms of these moments.*

In order to work out this problem we take a preliminary proposition:

To find the correlation between an error in the group y_s and an error in the moment $\mu'_{q'}$ about a fixed point.

We have:

$$m\mu'_{q'} = S(x_s^{q'} y_s),$$

therefore:

$$m\delta\mu'_{q'}\delta y_s = x_s^q (\delta y_s)^2 + S'(x_{s'}^{q'} \delta y_s \delta y_{s'}),$$

$$m\sigma_{\mu'_{q'}}\sigma_{y_s} r_{\mu'_{q'} y_s} = x_s^{q'} y_s \left(1 - \frac{y_s}{m}\right) - \frac{S'(x_{s'}^{q'} y_s y_{s'})}{m}$$

$$= y_s \left(x_s^{q'} - \frac{S(x_s^{q'} y_s)}{m}\right),$$

or:

$$\sigma_{\mu'_{q'}}\sigma_{y_s} r_{\mu'_{q'} y_s} = \frac{y_s (x_s^{q'} - \mu'_{q'})}{m} \quad \ldots\ldots\ldots\ldots \quad \text{(ix)}.$$

This is the correlation between an error in y_s and one in the q'^{th} moment about a fixed point.

To find the correlation between an error in y_s and one in the q^{th} moment about the mean, we have as before:

$$\delta\mu_q = \delta\mu'_q - q\delta h \mu'_{q-1} + \text{terms in } h,$$

$$\delta\mu_q \delta y_s = \delta\mu'_q d y_s - q\,\delta h\,\delta y_s \mu'_{q-1} + \text{terms in } h,$$

$$\sigma_{\mu_q}\sigma_{y_s} r_{\mu_q y_s} = \sigma_{\mu'_q}\sigma_{y_s} r_{\mu'_q y_s} - q\sigma_h \sigma_{y_s} r_{h y_s} \mu'_{q-1} + \text{terms in } h,$$

$$= \frac{y_s (x_s^q - \mu'_q)}{m} - \frac{q y_s (x_s - h)}{m} \mu'_{q-1} + \text{ terms in } h,$$

since $h = \mu_1'$ and we can use (ix). Hence putting $h = 0$, we have

$$\sigma_{\mu_q}\sigma_{y_s} r_{\mu_q y_s} = \frac{y_s (x_s^q - \mu_q - q x_s \mu_{q-1})}{m} \quad \ldots\ldots\ldots\ldots \quad \text{(x)},$$

x being here measured from the mean.

Illustrations.

(i) To find the correlation between an error in any single group and an error in the mean.

We have from (ix) when $q' = 1$, $h = \mu_1'$,

$$\sigma_h \sigma_{y_s} r_{h y_s} = \frac{y_s (x_s - h)}{m}$$

$$= \frac{y_s x_s}{m}, \text{ if } x_s \text{ be measured from mean.}$$

(ii) To find the correlation between the error in the standard deviation and an error in any single group.

We have from (x) if $q = 2$,

$$\sigma_{\mu_2}\sigma_{y_s} r_{\mu_2 y_s} = \frac{y_s (x_s^2 - \mu_2)}{m}.$$

But $\mu_2 = \sigma^2$

$$\delta\mu_2 = 2\sigma\delta\sigma,$$

therefore:

$$\delta\mu_2 \delta y_s = 2\sigma\delta y_s \delta\sigma,$$

and

$$\sigma_{\mu_2}\sigma_{y_s} r_{\mu_2 y_s} = 2\sigma\,\sigma_{y_s}\sigma_\sigma r_{y_s \sigma}.$$

Further:
$$\delta\mu_2{}^2 = 4\sigma^2\delta\sigma^2,$$

$$\sigma^2_{\mu_2} = 4\sigma^2\sigma^2_\sigma,$$

whence we see:
$$r_{\mu_2 y_s} = r_{y_s\sigma}.$$

Thus we find:
$$\sigma_{\mu_2}\sigma_{y_s}r_{\sigma y_s} = -\frac{y_s(\sigma^2 - x_s{}^2)}{m}.$$

Thus if an error be made in the frequency of a group with character less than σ, the error in the standard deviation will be negatively correlated with it. If the error be in a frequency group with character greater than σ, the error in σ will be positively correlated with the error in the group.

We can now return to our original problem. We have:
$$m\delta\mu'_{q'} = S(x_s{}^{q'}\delta y_s),$$

$$\delta\mu_q = \delta\mu'_q - q\,\delta h\,\mu'_{q-1} + \text{terms in } h,$$

$$m\delta\mu'_{q'}\delta\mu_q = S(x_s{}^{q'}\delta y_s\delta\mu'_q) - q\mu'_{q-1}S(x_s{}^{q'}\delta y_s\delta h) + \text{terms in } h,$$

$$\sigma_{\mu'_{q'}}\sigma_{\mu_q}r_{\mu'_{q'}\mu_q} = \frac{S\{x_s{}^{q'}y_s(x_s{}^q - \mu'_q)\}}{m^2} - q\mu'_{q-1}\frac{S\{x_s{}^{q'}y_s(x_s - h)\}}{m^2} + \text{terms in } h,$$

$$= \frac{\mu'_{q+q'} - \mu'_{q'}\mu'_q}{m} - \frac{q\mu'_{q-1}\mu'_{q'+1}}{m} + \text{terms in } h.$$

Whence putting $h=0$, we have for the correlation of errors in the two kinds of moments, i.e. those measured from a fixed point and those measured from the mean:

$$\sigma_{\mu'_{q'}}\sigma_{\mu_q}r_{\mu'_{q'}\mu_q} = \frac{\mu_{q+q'} - \mu_{q'}\mu_q - q\mu_{q-1}\mu_{q'+1}}{m} \quad \ldots\ldots\ldots\ldots \text{(xi)}.$$

The chief use of this formula is to find the correlation of errors in the mean and in the moments about the mean. For this purpose put $q'=1$, i.e. $\mu'_{q'}=h$, and we have since $\mu_1=0$, on right-hand side:

$$\sigma_h\sigma_{\mu_q}r_{h\mu_q} = \frac{\mu_{q+1} - q\mu_{q-1}\mu_2}{m} \quad \ldots\ldots\ldots\ldots \text{(xii)}.$$

Illustrations.

(i) To find the correlation in error between the mean and the standard deviation.

All we need to do is to put $q=2$, then:
$$\sigma_h\sigma_{\mu_2}r_{h\mu_2} = \mu_3/m.$$

But $r_{h\mu_q} = r_{h\sigma}$, therefore
$$\sigma_h\sigma_{\mu_2}r_{h\sigma} = \mu_3/m \quad \ldots\ldots\ldots\ldots \text{(xiii)}.$$

Hence if a random sample be taken which is more variable than the general population, the probability is that the mean of the sample will be higher than the mean of the general population if the third moment be positive, and lower if it be negative. Thus we cannot by random sampling change the variability without changing the type or the type without changing the variability. The only exception occurs when $\mu_3=0$, for example in the normal curve. In that case errors in the mean are independent of errors in the variability.

(ii) To find the correlation in error between the mean and the third moment.

Put $q=3$ in (xii), we have
$$\sigma_h\sigma_{\mu_3}r_{h\mu_3} = \frac{\mu_4 - 3\mu_2{}^2}{m}.$$

This vanishes again in the case of the normal curve. It may be shewn that the correlation between the mean and all the moments vanishes for normal systems and normal systems only.

GENERAL PROBLEM. *To find the probable error of any constant c_i of a frequency distribution and the correlation of any two constants c_i and $c_{i'}$.*

Any constant will be a function of the mean h and the moments $\mu_2, \mu_3, \mu_4, \ldots \mu_q \ldots$ about the mean. Hence:

$$c_i = \phi(h, \ \mu_2, \ \mu_3, \ \mu_4 \ldots \mu_q \ldots),$$

and if the errors be small as compared with the quantities in which they occur we can write

$$\delta c_i = \phi_h \delta h + \phi_{\mu_2} \delta\mu_2 + \phi_{\mu_3} \delta\mu_3 + \ldots\ldots$$

where $\phi_h, \ \phi_{\mu_2}, \ldots$ are the differentials of the known function ϕ with regard to the subscript quantities. Hence their numerical values are known and we may write, if λ denotes a known numerical quantity:

$$\delta c_i = \lambda_1 \delta h + \lambda_2 \delta\mu_2 + \lambda_3 \delta\mu_3 + \ldots\ldots .$$

Square, sum and divide by the number of cases and we have:

$$\begin{aligned}\sigma^2{}_{c_i} &= \lambda_1{}^2\sigma_h{}^2 + \lambda_2{}^2\sigma^2{}_{\mu_2} + \lambda_3{}^2\mu_3{}^2 + \ldots\ldots \\ &\quad + 2\lambda_1\lambda_2\sigma_h\sigma_{\mu_2}r_{h\mu_2} + 2\lambda_1\lambda_3\sigma_h\sigma_{\mu_2}r_{h\mu_3} + \ldots\ldots \\ &\quad + 2\lambda_2\lambda_3\sigma_{\mu_2}\sigma_{\mu_3}r_{\mu_2\mu_3} + \ldots\ldots \\ &= \lambda_1{}^2\sigma_h{}^2 + S(\lambda_q{}^2\sigma^2{}_{\mu_q}) \\ &\quad + 2\lambda_1\sigma_h S(\lambda_q\sigma_{\mu_q}r_{h\mu_q}) \\ &\quad + 2S'(\lambda_q\lambda_{q'}\sigma_{\mu_q}\sigma_{\mu_{q'}}r_{\mu_q\mu_{q'}}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(xiv)}.\end{aligned}$$

Our formulæ (vii), (viii) and (xii) give all the standard deviations and correlations required on the right-hand side. Similarly

$$\delta c_{i'} = \lambda_1{}'\delta h + \lambda_2{}'\delta\mu_2 + \lambda_3{}'\delta\mu_3 + \ldots\ldots .$$

Hence:

$$\begin{aligned}\delta c_i \delta c_{i'} &= \lambda_1\lambda_1{}'\delta h^2 + S\{(\lambda_q\lambda_1{}' + \lambda'_q\lambda_1)\,\delta h\,\delta\mu_q\} \\ &\quad + S(\lambda_q\lambda'_q\,\delta\mu_q{}^2) \\ &\quad + S\{(\lambda_q\lambda'_{q'} + \lambda'_q\lambda_{q'})\,\delta\mu_q\,\delta\mu_{q'}\}.\end{aligned}$$

Or:

$$\begin{aligned}\sigma_{c_i}\sigma_{c_{i'}}r_{c_ic_{i'}} &= \lambda_1\lambda_1{}'\sigma_h{}^2 + S(\lambda_q\lambda'_q\sigma^2{}_{\mu_q}) \\ &\quad + S\{(\lambda_q\lambda_1{}' + \lambda'_q\lambda_1)\,\sigma_h\sigma_{\mu_q}r_{h\mu_q}\} \\ &\quad + 2S'\{(\lambda_q\lambda'_{q'} + \lambda'_q\lambda_{q'})\,\sigma_{\mu_q}\sigma_{\mu_{q'}}r_{\mu_q}r_{\mu_{q'}}\} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(xv)}.\end{aligned}$$

Equations (xiv) and (xv) give one the means of determining the probable errors and the correlations of the constants of any frequency distribution whatever.

Theoretically the investigation is quite straightforward. Practically it is often very laborious. Thus if our constants only go as far as the fourth moment, we shall still want all the moments up to the eighth to determine their probable errors (see equations (vii) and (viii)). But to calculate the first *eight* moments is a laborious bit of arithmetic. It is often convenient and sufficient not to go further than the third or fourth moment, and to express by some assumed form of frequency curve, the remaining moments in terms of the earlier. This can always be done* if the frequency curves satisfy the relation $\dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{a_0 + a_1x}{c_0 + c_1x + c_2x^2}$, for in these cases there is a reduction formula giving the n^{th} moment about the mean in terms of the $n-1^{\text{th}}$ and $n-2^{\text{th}}$ moments. For example we have:

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* *Phil. Trans.* A., Vol. 186, p. 381.

(*a*) Normal Curve:

$$y = y_0 e^{-\frac{x^2}{2\sigma^2}},$$

$$\mu_{2n} = (2n-1)\,\sigma^2 \mu_{2n-2}.$$

(*b*) Curve of Type :

$$y = y_0 \left(1 + \frac{x}{a}\right)^p e^{-\gamma x},$$

Origin at mode and $p = a\gamma$,

$$\mu_{n+1} = \frac{n}{\gamma}\mu_n + \frac{n(p+1)}{\gamma^2}\mu_{n-1}.$$

(*c*) Curve of Type :

$$y = y_0 \left(1 + \frac{x}{a_1}\right)^{m_1} \left(1 - \frac{x}{a_2}\right)^{m_2},$$

Origin at mode and $b = a_1 + a_2$, $r = m_1 + m_2 + 2$,

$$\mu_{n+1} = \frac{nb\,(m_2 - m_1)}{r\,(r+n)}\mu_n + \frac{nb^2\,(m_1+1)\,(m_2+1)}{r^2\,(r+n)}\mu_{n-1}.$$

(*d*) Curve of Type :

$$y = y_0 \left\{1 + \frac{x^2}{a^2}\right\}^{-\frac{r+2}{2}} e^{-\nu \tan^{-1}\frac{x}{a}},$$

Origin at $-a\nu/r$ from mean and $\tan\phi = \nu/r$,

$$\mu_{n+1} = \frac{n}{r-n}\left(\frac{a^2\mu_{n-1}}{\cos^2\phi} - 2a\tan\phi\,\mu_n\right).$$

(*e*) The general formula of reduction for

$$\frac{1}{y}\frac{dy}{dx} = \frac{a_0 + a_1 x}{c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + \dots},$$

Origin at the mean is,

$$nc_0\mu_{n-1} + \{(n+1)\,c_1 + a_0\}\,\mu_n + \{(n+2)\,c_2 + a_1\}\,\mu_{n+1} + (n+3)\,c_3\mu_{n+2} + (n+4)\,c_4\mu_{n+3} + \dots = 0.$$

As a practical illustration of the use of these formulae the reader is referred to a criticism of Professor von Török's recent craniological work appearing in this part of *Biometrika*.

THIRD REPORT ON HYBRIDS BETWEEN WALTZING MICE AND ALBINO RACES.

ON THE RESULT OF CROSSING JAPANESE WALTZING MICE WITH "EXTRACTED" RECESSIVE ALBINOS.

By A. D. DARBISHIRE, Balliol College, Oxford.

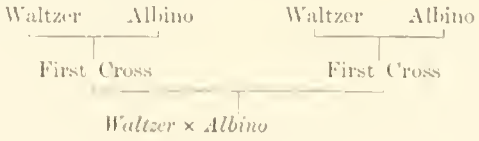
As is well known to those familiar with this question, it is an essential part of the Mendelian hypothesis that the (so-called "extracted") recessive individual which is produced by pairing two first crosses, is in every respect similar to the original pure recessive. It forms, in fact, the foundation on which the doctrine of the purity of the germ-cells rests. To put the matter more explicitly, the extracted recessive is identical with the original pure recessive and is not affected by its unusual ancestry in any way.

It is the object of the present report to describe three different kinds of crosses which show that the character of the offspring of extracted albinos is determined by (or at any rate correlated with) the ancestry of the albino parent.

I have crossed pure-bred waltzing mice, not with pure-bred albinos, but with extracted recessive albinos: now, the offspring thus obtained are either correlated with their grandparents (i.e. the hybrid parents of the albino), or, they are not. That is to say the character of the grandparent either has some effect on the character of the grandchildren or it has not.

I hope the following cases will show that it has.

1. Pure-bred waltzing mice were crossed with albinos produced by pairing two first crosses together. Thus:

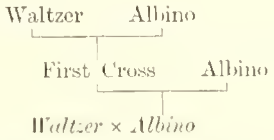


Three families have been produced from such a cross. The parents of the albinos used had always wild-colour in the coat, and the offspring of these unions had invariably the same colour. The results are given in Table I., below.

TABLE I*.

Catalogue Number of Cross	Character of Offspring	Character of Grandparents on albino side
130	2c, 2c	♀ 2c × ♂ 3c
148	3c, 3c, 3c, 3c, 3c, 3c	♀ 3c × ♂ 3c
150	2c, 2c, 2c, 2c, 3c	♀ 3c × ♂ 3c

2. Pure-bred waltzing mice have also been crossed with albinos produced by crossing a hybrid with an albino. Thus:



And the measure of the correlation which exists with regard to colour between offspring thus produced and the hybrid grandparent is shown in Table II.

TABLE II.

Colour of Grandchild.

Hybrid Grandparent.		Yellow (*a*)	Wild (*c* or *d*)	Totals
	(*a*)	8	4	12
	(*c* or *d*)	1	67	68
	Totals	9	71	80

Working out this table by the method for dealing with quantitatively indeterminate characters, we find, using the notation of *Phil. Trans.* A., Vol. 195, p. 6,

$$h = 1{\cdot}21344, \quad k = 1{\cdot}03643,$$

with an equation of many terms having the solution

$$r = {\cdot}951.$$

This correlation between grandparent and grandchild with regard to a "dominant character" transmitted through the recessive parent seems to dispose entirely of any theory which involves the gametic purity of the recessive parent.

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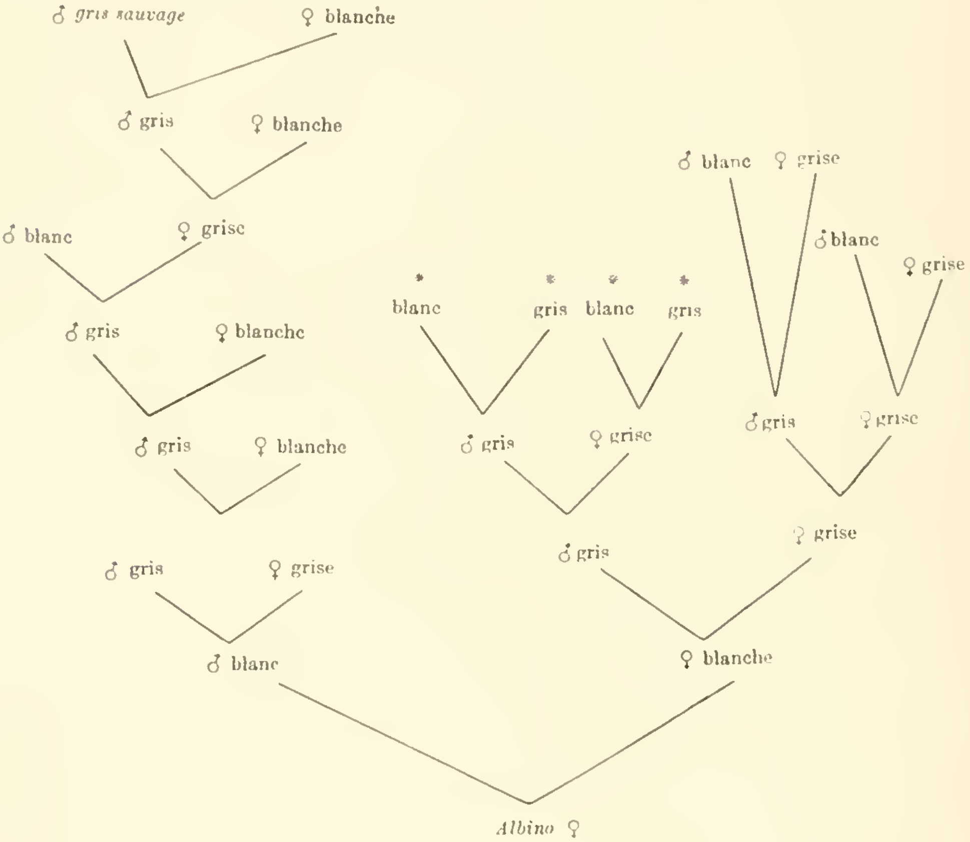
* The notation being the same as in the last Report. *Biometrika*, Vol. II. p. 165.

3. I have made one cross between a pure-bred waltzing mouse ♂ and an albino ♀ extracted from crosses between albinos and house-mice. The albino was given me by Professor Cuénot: and I wish to take this opportunity for thanking him for his courtesy and kindness to me. The ancestry of the albino is, in Professor Cuénot's words, as follows:


♂ gris sauvage
♀ blanche
♂ gris
♀ blanche
♂ blanc
♀ grise
♂ gris
♀ blanche
♂ gris
♀ blanche
♂ gris
♀ grise
♂ blanc
♂ blanc
♀ grise
♂ blanc
♀ grise
* blanc
* gris
* blanc
* gris
♂ gris
♀ grise
♂ gris
♀ grise
♂ gris
♀ grise
♀ blanche
Albino ♀

That is to say the amount of wild-colour in the ancestry is very great: the seven offspring produced by this mouse crossed with a waltzer were all completely wild-coloured.

4. So far we have been considering the colour of those portions of the coat which were not white.

The correlation between the amount of white in the coats of hybrid grandparent, parent of albino, and its grandchildren is given in Table III.

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* Which parent was white and which grey is not known.

TABLE III.

*Amount of Whiteness**.

Grandchild.

Grandparent.	*2—3*	*4—6*	Totals
2—3	23	15	38
4—6	13	29	42
Totals	36	44	80

5. A last important difference, upon which I will not at present comment, lies in this. The cross between the ordinary albino and the pure waltzer has so far invariably given dark-eyed young, but out of the eighty young already produced by crossing extracted albinos with pure waltzers five have pink eyes.

Doubtless different people will interpret these phenomena in different ways: but the subject of the present paper is not a question of interpretation but of fact: and I have confined myself to the mere statement of the fact until a larger number of cases justify an attempt at interpretation.

Postscript. Added June 4th. The foregoing account treats only of the offspring of extracted recessives. A mouse of the same generation as the latter, a waltzer with pink eyes and a yellow and white coat ($G'G'$ of Mr Bateson's notation,—the form which Castle asserts to be the extracted dominant) has served four pure-bred albino does; three of which have families old enough for description. In the notation of my previous reports they may be described as follows:

No. of Cross	Offspring
145	5*b*, 5*b*, 6*b*, 5*b*, 6*b*, 5*c*, and 1 albino
146	5*a*, 5*c*, 5*c*, 4*c*, and 4 albinos
156	2*c*, 5*c*, and 3 albinos

This mouse is clearly not a pure dominant, because it produces albinos; it is not a dominant hybrid because it has pink eyes; and it cannot be a recessive because when paired with an albino it produces some black-eyed forms.

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* Scale of whiteness is that of the Second Report. I find $h = \cdot 1257$, $k = \cdot 0627$, and the equation for r

$$\cdot 04204 r^7 + \cdot 00243 r^6 + \cdot 07207 r^5 + \cdot 00294 r^4 + \cdot 16339 r^3 + \cdot 00394 r^2 + r - \cdot 46798 = 0.$$

The solution of which is $r = \cdot 451$.

Thus in this case as in the previous one we see the ancestry of recessive individuals is of importance, for a very sensible correlation exists between the offspring and the grandparent with which the offspring are connected through their recessive parent.

MR BATESON'S REVISIONS OF MENDEL'S THEORY OF HEREDITY.

BY W. F. R. WELDON, F.R.S.

THE results which Mr Darbishire has so far obtained by crossing pink-eyed piebald waltzing mice with normal pink-eyed albinos have been recorded in the last two numbers of *Biometrika*. In *Nature* of March 19 and April 23 Mr Bateson has stated his belief that the inheritance of eye-colour in these mice is "strictly Mendelian," and he has published a Mendelian formula which implies his belief that the inheritance of coat colour is Mendelian also. In order to fully understand Mr Bateson's treatment it is necessary to realise not only Mendel's own doctrines, but the various modifications of these which Mr Bateson has from time to time propounded. Only after such preliminary survey can we discuss the bearing of the facts established by Mr Darbishire in his work on mice.

Mendel's Theory of Hybrids.

Mendel made two general statements, describing the results which he says he invariably obtained in his experiments with cross-bred peas; the first is what is often called his *Law of Dominance*, and describes the manner in which characters are transmitted from the pure-bred parents to the hybrid resulting from their union; the second is his *Law of Segregation* (Spaltungsgesetz, Loi de Disjonction) and describes the characters of the subsequent generations, descended from the hybrids originally produced; to account for the phenomena described by his Law of Segregation, he puts forward a theory of the constitution of the germ-cells, both in pure-bred individuals and in hybrids.

Mendel was the first to systematically analyse the differential characters of a race or species into a series of unit-characters, each of which might, under suitable conditions, be inherited independently of the others; in this he anticipated the essential features of the view afterwards developed (without knowledge of Mendel's work) by de Vries. Mendel further anticipated speculations of de Vries and Weismann by attributing the inheritance of each unit-character to the presence of a particular unit element in the germ-cell; and this conception of the "mechanism" of inheritance enabled him to formulate his hypothesis of the constitution of the germ-cells in the class of hybrids which he studied. He believed that if two races differ in a particular unit-character, which had the condition *A* in one race, the condition *a* in another, the gametes of the two races will differ: in the one race, each gamete produced will contain an element *A*, each gamete of the other race containing an element *a*. In the normal process of reproduction by individuals of like race, male and female gametes of like constitution meet, and their corresponding elements unite; so that the complete zygote from which a pure-bred individual of our first race arises, contains an element *AA*, derived in part from each parent; when an individual of one race is paired with an individual of the other race, the hybrid zygote contains an element *Aa*. Now in cases such as those Mendel observed, he supposes that when the hybrid, resulting from a zygote *Aa* forms germ-cells for itself, each of its germ-cells contains *either* an element *A* only, *or* an element *a* only; and germ-cells of each kind are formed in equal numbers, half of each kind being male, and half female. When such hybrids are paired together, three kinds of union are possible; gametes of like constitution may unite, giving rise to zygotes of constitution *AA* or *aa*, or gametes of unlike kind may unite, and give rise to zygotes of constitution *Aa*. As the union is supposed to occur by chance, zygotes of constitution *Aa* will occur, in any large series of cases, twice as frequently as zygotes *AA* or *aa*. In all his discussions of such hybrids Mendel assumes that the Law of Dominance will hold, as he tells us it always held in the cases he examined; he therefore assumes that the first formed hybrid, though containing elements *Aa*, will present the appearance of an individual belonging to the race in which the unit character has the "dominant" condition, say *A*; the offspring of such hybrids will externally be of two kinds; one quarter resembling the race *a* and capable of transmitting only the characters proper to that race, three quarters resembling the race *A*; but of these three quarters with dominant appearance, one will contain only elements *A* and will, therefore, be capable of transmitting only the characters of race *A*, while the other two are of composition *Aa*, and behave like their hybrid parents if allowed to breed.

If the two pure-bred parents differ in more than one unit character, all the elements which represent the various differential characters are separated in the hybrid body during gamete-formation, and each behaves independently of the others; thus an individual of pure race distinguished by two unit characters *A* and *B*, when paired with an individual of another pure race with characters *a* and *b*, will give rise to a hybrid of constitution $Aa + Bb$; and when this hybrid forms its gametes, separate elements *A*, *a*, *B*, *b*, will be formed in equal numbers and united in the gametes in each of the four possible ways with equal frequency; so that there will be four equally numerous sets of gametes, of constitution $(A + B)$, $(A + b)$, $(a + B)$, and $(a + b)$. A similar process of resolution and reconstruction is supposed to occur in more complex cases.

Mendel considered that the unit character could only be determined by experiment; on the one hand groups of apparently distinct characters might be inherited as one unit (for example seed-coat colour and flower colour in certain peas) while on the other hand an apparently single character, such as the colour of some flowers, might be due to the simultaneous presence of several unit characters, each capable of being inherited separately from the others.

*Mr Bateson's Revisions of Mendel**.

1. *The nature of the gametes in hybrids.*—Mendel says that the gametes, produced by a hybrid zygote of constitution Aa, are not themselves hybrid, but are of two kinds, both equally numerous; one kind contains only elements A, the other containing only elements a. Now when similar gametes, produced in this way, unite to form a zygote, what will happen? The individuals will behave like pure-bred individuals of the race represented by their gametes. This is all that Mendel ever asserts of them; when he says that their characters remain constant, it is clear that he means a constancy of the same order as that observed within the limits of a pure-bred species because he treats the species as themselves constant, and so neglects individual variation within the limits of a single race. Mr Bateson however, alone among modern naturalists, roundly declares that "the pure dominant and pure recessive members of each generation are not merely like but identical with the pure parents" (II. p. 12); so that one of the effects attributed by him to cross-breeding is the disappearance of those differences which normally exist between members of the same species. He asserts that by cross-breeding a group of individuals may be produced, including one ancestor and some of its grandchildren, great grandchildren and more remote posterity ("members *of each generation*") which are "not merely like but identical with" each other. This statement is supported by no scrap of evidence; it is flatly contradicted by experiments which Mr Bateson himself adduces as "following Mendel's law with considerable accuracy" (e.g., the experiments with *Datura*, (II. pp. 21—32)); nevertheless, it is made the basis of the imposing dogma that any attempt to express the characters of the hybrids described in terms of all their various ancestors is henceforth futile; the suggestion that the discrepancies between Mendel's own results and those of subsequent observers who have endeavoured to repeat his observations may be accounted for by differences in the ancestry of the peas used is regarded by Mr Bateson as an attempt to mask a difficulty one dare not face (I. p. 200).

2. *The essentials of Mendel's theory.*—Having made the striking addition to Mendel's statements, described in the last paragraph, Mr Bateson proceeds to

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* The statements here discussed are chiefly contained in the following works, which will be referred to as I. and II.:

I.—*Mendel's Principles of Heredity:* a Defence, by W. Bateson. 8vo. Cambridge, 1902.

II.—*Reports to the Evolution Committee of the Royal Society:* I. Experiments undertaken by W. Bateson and Miss E. R. Saunders. Roy. Soc., 1902.

separate what he considers essential in them from what is not. The phenomenon of dominance, though always observed by Mendel, is not essential; and in asserting this proposition Mr Bateson does not hesitate to accuse his master of something very like a gross breach of good faith. Mendel's description of the result obtained by crossing peas is translated by Mr Bateson in these words: "In the case of each "of the seven crosses the hybrid character resembles that of one of the parental "forms so closely that the other either escapes observation completely or cannot "be detected with certainty" (I. p. 49). This is a clear and unequivocal statement of an observed experimental result, that in each of seven crosses dominance was observed. No honest observer could have formulated this statement if he had observed any large number of exceptions, and it seems to me that no one can read Mendel's paper without believing him to be absolutely honest; yet Mr Bateson thinks it fair to suppose that the behaviour of 12 out of 34 varieties of peas, cultivated by Mendel, was not recorded in his final statement because amongst other drawbacks they exhibited "defective and irregular dominance" (I. p. 137).

By an analysis of this remarkable kind we are led to the statement that what is essential in Mendel's work is no ascertained fact at all, but Mr Bateson's peculiar interpretation of the hypothesis concerning the nature of germ-cells, which he calls "the great fact of gametic purity" (I. p. 117, II. p. 12).

3. *Atavism in hybrids.*—By rejecting dominance as "unessential," Mr Bateson is able to apply the theory of "pure" gamete formation to a number of cases in which the offspring of hybrids exhibit real or apparent reversion to the characters of the pure parental races, although nothing like dominance is seen in the hybrids themselves. But here he meets with a difficulty. In many of the newly-included cases the hybrids exhibit characters which suggest reversion to *remote* ancestors; for example, when von Guaita crossed pink-eyed albino mice with dark-eyed, black and white waltzing mice, he says that the hybrid young always resembled a wild mouse in size, colour, and wildness; here we have at least an apparent reversion to some characters of the common ancestors of both races. Again, Mr Bateson finds that there are several distinct races of Sweet Pea, which give rise to similar hybrids when crossed; and of these hybrids he says "it is characteristic of such "forms that they may reproduce *in appearance* some putative ancestor" (II. p. 144). It is not thought necessary to include facts like these, which surely depend upon the constitution of the germ-cells as much as any other phenomena of inheritance, in a theory of the gametes concerned. The apparent atavism is said to be peculiar to heterozygotes, and is so dismissed; its bearing on the phenomena of cross-breeding as a whole, and especially on "the great fact of gametic purity," being neglected.

We have here a second addition to Mendel's statements—the manifestation of a spurious atavism as a property of hybrids, and the doctrine that several different gametic combinations may produce identical individuals.

So far as I can discover, from an attempt to collate Mr Bateson's various statements, the only thing which can be predicated of the result of crossing pure races is that the resulting hybrids, though their "character may be anything," will at least be uniform; this uniformity is indeed considered to differentiate Mendelian hybrids from those in which the ancestral characters are blended (II. p. 153).

4. *The similarity of reciprocal crosses.*—Mendel says that in all his experiments "it is perfectly immaterial whether the dominant character belong to the seed-"bearer or to the pollen-parent; the form of the hybrid remains identical in both "cases" (I. p. 50); and this, at least, seems "essential," because the law of segregation which he formulates to describe his results depends upon it. Mr Bateson however propounds a theory by which "the great fact of gametic purity" may apply to cases in which reciprocal crosses are unlike. He conceives a species in which the male gametes contain only one element A of a Mendelian pair, the female gametes containing only the complementary member a of the pair. When a male and female of such a species breed together, the fertilised zygote is of composition Aa; but the gametes produced by such a zygote will contain only A elements if the zygote body be a male, only a elements if it be a female. Each individual will therefore completely suppress half the heritage it receives from its parents during the course of its development. This novel and striking conception is not only completely foreign to Mendel but to all other naturalists, except Mr Bateson and his colleague Miss Saunders (II. p. 132).

The Horse and the Ass among animals, and certain species of *Digitalis* among plants, are adduced as examples of species between which the reciprocal crosses are unlike. If Mr Bateson's remarkable hypothesis be applied, it follows that in either the Horse or the Ass, and in some species of *Digitalis*, there are characters which cannot be transmitted from the female through a son to grand-daughters, others which cannot be inherited by grandsons from the maternal grandfather. With so rich a store of new and startling theories to propound, Mr Bateson naturally has no time to illustrate each of them in detail; but it would be interesting if he would spare a moment to tell us what these characters are in one of the cases he mentions.

5. *The behaviour of "compound characters" on crossing.*—The facts on which Mendel bases his theory of compound characters are these: A purple-flowered bean (*Phaseolus multiflorus*) was crossed with a white-flowered bean (*Ph. nanus*); from the account given we may assume that the hybrids produced were uniform in character, and we are told that whiteness of flower was completely recessive*. These hybrids, when self-fertilised, gave a whole series of flower colours; among 31 plants obtained, one had white flowers, the others had flowers of various shades from purple-red to pale violet. The experiment was not continued. The explanation offered is simple, and exhibits the phenomena observed as exactly parallel with those seen when Peas, of races which differ in two pairs of Mendelian characters, are crossed. Taking the well-known statements concerning the result

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* I consider only Mendel's statements of flower-colour; the races differed, of course, in other characters, but the discussion of these is not immediately relevant.

of crossing a Pea of green (recessive) cotyledon colour and wrinkled (recessive) shape with one of yellow (dominant) colour and round (dominant) shape, and denoting the gametic elements by the initial letters of the qualities they represent, the gametes of one pea will on Mendel's view contain the elements $G + W$, those of the other $Y + R$. On cross fertilisation, each element of one gamete will unite with the complementary element of the other, to form a fertilised zygote element; the zygote will therefore be of constitution $(YG + RW)$. Now the plant resulting from this zygote will produce each of the elements Y, G, R, and W in equal numbers; and one member of each pair will be included in each gamete formed; a member of each pair will enter a given gamete independently of the member of the other pair; and there will result four kinds of gametes, each kind equally numerous, of composition $(Y + R)$, $(Y + W)$, $(G + R)$, and $(G + W)$; half the gametes of each kind will be of one sex, half of the other sex, and when self-fertilisation occurs, the gametes of opposite sexes will be mated at random. It is clear that the composition of the zygotes formed, and their relative frequency, will be represented by

$$\begin{array}{lll}
(WW + YY), & 4\,(WR + YG), & (WW + GG), \\
2\,(WW + YG), & 2\,(WR + GG), & (RR\ + YY), \\
2\,(WR + YY), & 2\,(RR\ + YG), & (RR\ + GG).
\end{array}$$

It will be noticed that only one individual dominant in both characters, and one recessive in both characters, occur in every 16 individuals.

Now Mendel assumes that the flower-colour of his purple bean is a combination of two or more entirely independent colour elements "which individually act like any other constant character of the plant," and he proceeds to treat each of these as if it were a separate unit character, like the shape or cotyledon colour of a pea. Supposing two such colour elements, A_1 and A_2, in the purple flower, he regarded the gamete of his *Ph. multiflorus* as containing these two elements $(A_1 + A_2)$, while the gamete of the white-flowered *Ph. nanus* contained two identical recessive white elements $(a + a)$. The constitution of the hybrid zygote he writes $(A_1a + A_2a)$ and as in the case of the peas referred to, he deduces nine possible forms among the descendants of such a zygote, occurring with the relative frequency

$$\begin{array}{lll}
(A_1A_1 + A_2A_2), & 4\,(A_1a + A_2a), & (A_1A_1 + aa), \\
2\,(A_1A_1 + A_2a), & 2\,(A_1a + aa), & (aa + A_2A_2), \\
2\,(A_1a + A_2A_2), & 2\,(aa + A_2a), & (aa + aa).
\end{array}$$

This treatment is quite similar to that of the simple unit characters of peas, in every respect but one. Every unit character in a pea is treated as if it were represented in the gamete by only a single element; but the simple unit character, whiteness, in *Ph. nanus*, is represented in the gamete by a number of similar unit elements equal to the whole number of dissimilar unit elements present in the compound character, purple, of the *multiflorus* flower. Admitting this, for the moment, as a possibility, what happens when the same white-flowered *Ph. nanus* is crossed in one case with a coloured flower containing two unit colours, in another with a flower containing four or five?

Mr Bateson seems to have perceived this difficulty, and he has conceived what he calls a "compound allelomorph," the source of further complications. He conceives, if I understand him, that the colour-characters, A_1 and A_2 of a purple bean flower are not, as Mendel says, independent, but that they are combined, while in the pure-bred plant or its gametes, into a system which behaves like a single unit, so that he would, I think, agree to write the zygote, produced by crossing the purple-flowered and the white-flowered bean together, as $(A_1 + A_2)\,a$, where the portion enclosed in a bracket behaves like one unit character. Concerning the behaviour of a "compound allelomorph" when the hybrid forms its gametes, Mr Bateson has suggested a series of possibilities, each involving a process not included in Mendel's theory, and each incompatible with the view that a simple unit of colour character is comparable with "any other constant character of the plant." The principal suggestions so far put forward by Mr Bateson may be classified as follows:

A. *Suggestions in harmony with Mendel's view that elementary unit-characters are separated during gamete formation:*

(*a*) *The number of elements contained in the gametes produced by a hybrid need not be the same as the number contained in the pure parental gametes, and each gamete of hybrid origin may contain more than one element of the same kind.*

In this case Mr Bateson supposes that a hybrid zygote, of constitution $(A_1 + A_2 + A_3 + \ldots \text{etc.})\,a$ will, during gamete formation, give rise to a number of independent elements, A_1, A_2, etc., ... and a, and these will be distributed among the gametes; but he supposes that a gamete may be produced of constitution A_1A_1, A_2A_2, etc. (I. p. 80, II. p. 145). This is clearly not a phenomenon comparable with anything which can be supposed to occur when a zygote cross-bred from a green-wrinkled and a yellow-round pea produces gametes; for a gamete of constitution YY would have two elements determining cotyledon colour, and none whatever determining shape. The union of two such gametes would produce an unthinkable result, a pea with yellow colour and without shape; to aid our conception of such a pea we can only appeal to that "residue," the plant with all its unit characters removed, which Mr Bateson considers to be the foundation on which the idea of "Species" must ultimately rest! (I. p. 28, II. p. 148).

(*b*) *Each gamete produced by a hybrid contains one, and one only of the unit characters transmitted to it from its parents.* (*Proc. Cambridge Phil. Soc.* XII.)

In this case the heterozygote $(A_1 + A_2 + \ldots)\,a$ produces gametes of simple constitution A_1, A_2, ... or a, and here again we have a conception which cannot be applied to anything but colour without raising the difficulties pointed out in the case of the previous hypothesis. This second non-Mendelian hypothesis has so far been suggested in connection with two subsidiary suggestions:

(α) *The number of gametes of each kind is the same*; this is of course a Mendelian hypothesis.

(β) *The number of gametes, containing elements derived from the parent with simple character, is equal to the sum of all the other gametes, each of which has a single element of the compound parental character.* Here the view that gametes of all kinds are produced in equal numbers is given up, in order to fit a particular result.

It is important to notice that neither of the two hypotheses above described can be applied to any system of separable organs whose modifications are united to form a "compound allelomorph," without involving the production of an individual in which some of the characters are determined solely by that "residue" from which all unit characters are abstracted, pictured by Mr Bateson as the basis of our conception of Species. Yet he tells us that he has taken colour-characters as examples of the way in which other cases may be treated, because "the facts in those cases are clearer, but their nature is probably not different" (II. p. 146).

B. *The Mendelian theory, that all inherited characters are reduced to their elementary units and redistributed during gamete formation, is given up.*

Under this head we have two possibilities; for some gametes of the hybrid may contain the parental "compound allelomorph" *either* entire, *or* partly resolved into its constituents.

The doctrines put forward by Mr Bateson in connection with "compound allelomorphism" have been considered at some length, because of their great interest to anyone who wishes to appreciate his attitude. It must be remembered that no character can, on Mendel's view, be asserted *a priori* to be simple or compound. If a simple result, such as that obtained by Mendel when he crossed yellow and green peas, be obtained on crossing two animals or plants which differ in respect of any character, that charactér is simple, and there is no need to go further. Should the hybrids exhibit any unexpected character, a reference to the properties of "heterozygotes" will remove all difficulties; should the offspring of the hybrids be of more than two kinds, one or other of the foregoing hypotheses will generally be found to evade the difficulty; if not, "our conceptions of allelomorphic characters, simple and compound, are in process of formation*," and a new theory will quickly be provided.

We need not discuss the attempt to regard the sex of offspring as a phenomenon of gametic differentiation, a doctrine more difficult than even those we have described, excepting only that of the marvellous residue without unit characters, on which the future definition of species must be based. We may now proceed to examine the way in which Mr Bateson, after formulating all these hypotheses, treats Mr Darbishire's new facts.

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* Bateson: "Note on the Resolution of Compound Characters by Cross-breeding." *Proc. Cambridge Phil. Soc.* XII. p. 53.

Mr Darbishire's Results and Mr Bateson's First Explanation.

The mice used by Mr Darbishire are normal pink-eyed albinos, of various ancestry, and a race of "Japanese" waltzing mice, having pink eyes and a piebald (fawn-coloured and white) coat. The normal albino mouse is said by Crampe, and by many mouse-breeders, to breed perfectly true, whatever its ancestry, and Mr Darbishire's albinos certainly so far do so. The waltzing mice also breed true in the sense that their offspring always waltz, and always have pink eyes with a fawn-and-white coat; but there are of course slight variations in the size and shape of the coloured patches. There is no proof that the results of reciprocal crosses between these two races are different. Therefore, since these two races each breed true, and give similar results in reciprocal crosses, they satisfy the conditions of "gametic purity" laid down by Mr Bateson, and there is no reason to assume that Mr Bateson's remarkable hypothesis of a difference in constitution between the male and female gametes applies to either.

The first hybrid generation, directly resulting from the cross, is so far constant in two respects; none of the individuals "waltz," and all have pigmented (dark) eyes. In coat-colour the hybrids are very variable; none have the fur entirely white, but the colour, and the extent of the coloured patches, vary widely. As to the amount of *white* fur; out of 203 mice available when Mr Darbishire's last Report was published, he tells me that one had *no* white fur; 69 had white on the belly only; 51 had some white on the back and sides, but less than in pure-bred waltzing mice; 30 had about the same amount of white as a waltzing mouse, and 3 had more. The colour of the coloured fur was *yellow* in 7 cases, a pale or dark "*wild-colour,*" like that of the wild mouse, in 138 cases, and *black* in 9 cases.

The uniformly dark colour of the eyes is difficult to explain on any Mendelian hypothesis: for since pink eyes may coexist with a coloured coat, and dark eyes with a white coat, it is clear that in many cases at least eye-colour and coat-colour can be separately inherited, and should therefore on Mendel's view be represented by separate gametic elements. But we are told by Mr Bateson, and on better evidence by von Guaita, that albinism is a recessive character; and it seems natural to suggest that "albinism" is a "compound" character, composed of two or more elements, among which are whiteness of coat and pinkness of eye. On this view, the two purely recessive eye-colours of Mr Darbishire's mice should give pink eyes when crossed; and we see that they do not. No Mendelian hypothesis will account for the production of a new or atavistic eye-colour by the union of similar elements; and therefore, although eye-colour is certainly separable from whiteness of coat, because dark-eyed mice with white fur are well known; although it is similarly separable from a fawn-coloured-and-white coat, because among the offspring of Mr Darbishire's hybrids fawn-and-white individuals occur, some dark-eyed and some pink-eyed, yet a method must be devised by which this darkness of the hybrid eye may be exhibited as a "property of heterozygotes," and so neglected, with the other inconvenient properties of first crosses, in applying the "great fact of gametic purity." Mr Bateson accordingly writes (*Nature*, No. 1742, March 19, 1903) "The first cross shows that when *in this case* an albino "(pink-eye) gamete, *G*, meets a colour-bearing (pink-eye) gamete, *G'*, in fertilisation "we must expect the resulting heterozygote, *GG'*, to be coloured in coat, with a "*dark* eye." Now if this means anything more than a mere re-statement of Mr Darbishire's result, it means that so far as coat-colour and eye-colour are concerned the elements in each gamete are to be treated as *single units*; they are either actually single, or they are "compound allelomorphs." The attempted explanation, even on this view, only succeeds in placing the phenomena in the same category with many others avowedly inexplicable, but it does avoid the difficulty of the origin of a new or atavistic character from the union of two similar eye-colour elements.

The coat-colour of the hybrids is still unexplained by Mr Bateson's statement; for since every hybrid by hypothesis results from the union of similar pairs of gametes, the hybrids themselves should be similar. Mr Bateson has hitherto assumed in all his discussions that whatever the characters of similar heterozygotes may be, at least they are similar! In this case, he passes over the difficulty in silence. To me (and I am glad to know that Mr Darbishire agrees with me) this variability of the hybrids is highly significant, when these results are compared with those obtained by others. Using a race of albinos, known to be absolutely pure because they had been inbred for 29 generations, and crossing these with pure-bred Japanese waltzing mice with dark eyes and black-and-white fur, von Guaita tells us that he obtained hybrids which uniformly resembled wild mice; making a similar cross, but using albinos of which no definite history is recorded, Haacke obtained hybrids sometimes like wild mice, and sometimes black. Mr Darbishire has deliberately used albinos which were sometimes pure-bred, sometimes known to have near ancestors of various colours, and he obtains a highly variable group of hybrids. Now the gametic purity of an extracted recessive form which breeds true cannot be questioned without abandoning the whole Mendelian position, or any modification of it, and therefore all these albino mice must be treated as equally pure on Mr Bateson's view; but I submit that the three sets of facts already published by Mr Darbishire, by von Guaita, and by Haacke, show a correlation between the ancestry of the pure recessive albino and the character of the hybrids, sufficiently strong to disprove the "great fact of gametic purity" in this case, at least. The correlation between the colour of the coat in dominant hybrids, and in their hybrid grandchildren, which Mr Darbishire has shown to be established by transmission through a pure recessive albino, is still more conclusive evidence against the gametic purity of recessives, (*supra*, pp. 282–5).

So much for the result of the first cross. What happens when the hybrids of this generation are paired together? Mr Bateson represents the result as a division of the second generation into three groups, of constitution and relative frequency indicated by the formula

$$GG + 2GG' + G'G'$$

where the homozygotes GG are pink-eyed albinos, the homozygotes $G'G'$ are pink-eyed "with some colour in the coat," and the heterozygotes GG' have "dark eyes "and some colour in the coat."

In this formula either coat-colour and eye-colour together are treated as a single unit character, or they are treated as a series of unit elements combined in a "compound allelomorph" which is transmitted to half the germ-cells of either sex completely unchanged, and the Mendelian principle that compound characters are reduced to their elements and redistributed in germ-cell formation is abandoned. The formula gives 25 per cent. pink-eyed albinos, 50 per cent. dark-eyed mice "with some colour in the coat," and 25 per cent. pink-eyed mice with some colour in the coat; the numbers given in Mr Darbishire's last Report, from which Mr Bateson deduced his formula, are

13 pink-eyed albinos,
36 dark-eyed with colour in the coat,
17 pink-eyed with colour in the coat,

and considering the small number of individuals, this record fits Mr Bateson's non-Mendelian formula very well. Any Mendelian hypothesis, which should attempt to account for the great variability in coat-colour by postulating a resolution of the "compound allelomorph" would lead to one of two results, either of which would be inconsistent with the experiments.

For if we suppose the elements which represent coat-colour and eye-colour to break up into only two components during the formation of gametes by the hybrid, then if both of these be supposed dominant over albinism, the number of pure albinos, or of pure dominants in the second generation is reduced to one in sixteen, a proportion which is not in accord with observation: if on the other hand one of the two colour elements be supposed recessive to albinism, we may get $\frac{3}{16}$ of the second generation showing albinism: such a proportion would fit the facts so far obtained, but two out of every three such albinos should be dominant hybrids, producing some coloured young, and no such albinos have been observed by Mr Darbishire or by any one else.

Mr Bateson's is therefore the only pseudo-Mendelian formula which will account for the observed distribution of eye-colour and albinism. But it is fatal to the "great fact of gametic purity," either in Mr Bateson's sense or in Mendel's. For this "great fact" involves much more than is implied by the statement that the $G'G'$ individuals are "pink-eyed, with some colour in the coat"; these individuals are said to be formed of gametes identical in constitution with those of their pure-bred waltzing grandparents, at least in respect of eye-colour and coat-colour; the individuals produced from the union of such gametes should on Mendel's view be like individuals of the pure waltzing race, on Mr Bateson's view they should be "not like but identical with" their pure-bred waltzing grandparent, both in eye-colour and in coat-colour (II. p. 12). Now Mr Darbishire has pointed out (*Biometrika*, Vol. II, Part 2, pp. 167 and 168) that the mice, which Mr Bateson calls of constitution $G'G'$ may exhibit a blue-grey colour which breeders call "lilac," unknown in the pure-bred waltzing race (at least in the recent ancestry of the mice used) and equally unknown in hybrids of the first generation, which Mr Bateson calls GG'. At the time of writing, there are 18 mice of the group $G'G'$, all "pink-eyed, with some colour in the coat," and of these no less than six are either wholly or in part lilac.

The coat-colour of the 36 mice GG' is also variable, as is that of those mice (which Mr Bateson regards as having the same constitution) produced by crossing hybrids of the first generation with albinos. The variability of coat-colour, unaccompanied by a resolution of the allelomorph, is as great an obstacle to Mr Bateson's interpretation in this case as in the case of the first cross.

Here, as in so many other cases, a result in superficial agreement with those observed by Mendel can only be obtained by devising a series of categories so vaguely defined that they convey wholly inadequate information; and with a little skill such categories may be found to fit almost any series of results. Just as on a former occasion Mr Bateson has used the category "hairy" to imply identity of character between leaves in which the number of hairs per square centimetre may be anything from a dozen to over a thousand, so he here assumes that the conditions of equality, suggested when he asserts that 18 mice are of identical constitution $G'G'$, are satisfied when he has shown that they all have "pink eyes "and some colour in the coat." Such a category would include a pink-eyed mouse with green fur and a purple tail: and he makes it in fact include "lilac" and yellow.

Surely we want some better demonstration of the "identity" between pure dominants and their parents than this sort of category can give, before the "great "fact of gametic purity" can serve as a foundation for the huge superstructure of hypothesis built upon it!

When it is pointed out to Mr Bateson that he cannot legitimately class yellow, lilac, and black in a single colour category, he abandons his formula of March 19 and puts forward a new set of hypotheses (*Nature*, No. 1747, April 23, p. 585). He now explains the variability among Mr Darbishire's first crosses as due to gametic impurity in one or both of the races crossed. Now, apart from the difficulty of regarding a "pure recessive" albino as impure in the Mendelian sense, let us see whether the new statement fits the observed behaviour of the "pure" waltzing mice. The waltzing mice used by Mr Darbishire are chiefly descended from about 80 mice, 40 of each sex, bought by me some time ago, and paired at random under my direction. Now if we suppose these mice to have been impure in Mr Bateson's sense, that is, if we suppose them to have produced gametes of more than one kind, this may have happened in either of two ways: there may have been two kinds of gametes, one kind produced by each sex, as supposed by Mr Bateson in the case of the Horse or the Ass, in which case the race would breed true, every individual being produced by a heterozygote which lost half its heritage during development. Such a form of gametic heterogeneity would be consistent with the observed fact that the mice bred true, but it would involve difference between the crosses Albino ♀ × waltzer ♂ and waltzer ♀ × Albino ♂ which has not been observed, and it would fail to account for the observed differences between individual hybrids produced by the same parents. The hypothesis that each mouse of either sex produces gametes of more than one kind involves *either* the hypothesis that these gametes do not obey Mendel's law at all, *or* that a constant percentage of "rogues," dominant or recessive in one or more elementary characters, is produced in each generation. No such rogues have been observed either in a very considerable number (many hundreds) of young examined before Mr Darbishire's experiments were begun or in pure waltzing families raised during the course of the experiments: so that this hypothesis also fails to fit the facts.

The second new assumption made by Mr Bateson, that coat-colour is a "compound" character, is probably true; but it completely fails to account for the observed variability among the offspring of the first cross, since no "resolution of compound characters" can occur during the formation of the first hybrid generation, and it involves a reduction of the number of pure recessive albinos, among the offspring of hybrids, to a *maximum* far below the observed proportion. Mr Bateson's new hypothesis therefore fails to fit the facts any better than his first.

I am as anxious as Mr Bateson can be to test the possible ways of bringing the behaviour of these mice into accord with Mendel's "principles" or indeed with any other theory of heredity. The two hypotheses already put forward fail, as we have seen. I earnestly appeal to Mr Bateson, now that the facts are fully before him, to produce some final formula, expressed in terms of categories so definite that they really describe the mice included, which seems to him capable of bringing the behaviour of these hybrid mice and their offspring into harmony with the "principles" he so strongly supports.

[A letter to *Nature*, May 14, containing a new hypothesis, has appeared while this article is going through the press. This letter and a paper before the Zoological Society of London, of which Mr Bateson has given notice, could not be discussed in the present number of *Biometrika* without unduly delaying its publication.]

MENDEL'S LAWS AND SOME RECORDS IN RABBIT BREEDING.

By FREDERICK ADAMS WOODS, M.D.

OWING to the current interest in Mendel's Laws of Heredity and the heated discussion going on in some quarters as to their validity, the following records from rabbit breeding may be of value, in showing what practical application the principles find when applied to a certain species of mammals not heretofore studied.

Within the last year many of the records formerly obtained by breeding mice have been gone over again in light of the Mendelian principles, and in some instances there is a clear uniformity with what might be expected, notably the point that albinism is pure recessive and always when mated with itself gives rise to albinos and nothing else. (Crampe (5) and von Guaita (8).) The waltzing character is also recessive according to Bateson, who in his recent book, *Mendel's Principles of Heredity* (p. 174), discusses this point. It is, however, not clear that all the results of different observers can be harmonized with the Mendelian principles. (Conf. Weldon (10) p. 244.)

In view of this condition of the subject further studies of the old material, as well as new records among other groups of mammals, must be made before we shall know what practical application the experiments of the botanists, Mendel, de Vries, Correns, Spillman, and others will have when applied in the breeding of animals, or what will be the importance of these remarkable laws to the student of heredity in its widest scope.

Rats and mice are excellent material for use in this connection as they breed true to certain definite types, and do not easily blend. The same is true of rabbits. The progeny of crosses illustrate "alternative" inheritance, each offspring resembling in colour some one parent or other ancestor, rather than a mixture of many. As this is equally true of common pet rabbits, these animals should be good material to furnish evidence for the Mendelian principles.

The following experiments on the colour inheritance of rabbits were originally undertaken in order to get exact material for the study of Galton's Law of Ancestral Heredity (9); and were begun in the spring of 1899. Up to the present time I have bred some 350 young, most of whom have ancestry now kept to the third or fourth generation, yet there has always been such a marked deviation from the law as announced by Galton that I have wished to withhold the results until a larger number of figures might be at hand. Thinking, however, that if Mendel's principles are to be of practical application to the breeder, they should show their force in a group of at least 300 individuals, I determined to see what help might be derived by using this old and forgotten work now so fortunately rediscovered.

My original purpose was to obtain by means of artificial selection a race of pure blacks and then cross these with albinos. Although I have selected the blacks with this end in view for several generations a pure black race has not yet been formed. This part of the work then is very similar to the efforts of a breeder of fancy stock, trying to improve his strain. Most of the other experiments are the crossing of nearly pure black with albinos, or the mating of albinos which have black in the ancestry. I have always felt that it was very strange that two albino parents which are themselves the descendants of an ancestry almost entirely black on all sides, should never be able to throw a trace of black in any of their progeny. Such a fact being absolute in every instance cannot be passed over by calling it "a prepotency," since if there were a prepotency of white over black, the black ought, nevertheless, once in a while to show itself, which under these circumstances it never does, according to both the reports of breeders and the records of investigators.

We may now turn to the analysis of the figures, taking the three main groups according to colour of parents: 1st, black × black; 2nd, black × white; 3rd, white × white.

The colours met with are indicated as follows: B = black; W = albino; G = grey; Y = yellow. A few spotted ones which could not be placed in any one of these categories were occasionally met with; these will be described in detail.

The number of each experiment (or litter) is given in Roman numerals. The numbers in brackets, e.g. (1) (2) etc. are used to distinguish the individual rabbits.

The 150 offspring when both parents were black or nearly so, were 105 of similar colour; 28 albino; 9 grey; and 8 yellow. Intermediate or spotted forms did not occur from these pairings.

We analyse the results of mating Black with Black in Table I.

TABLE I.

Black × *Black*.

	Parents		Offspring			
	♂	♀	*B*	*W*	*G*	*Y*
i	(1)	(2)	4	—	1	2
ii	(7)	(3)	5	1	—	—
iii	(1)	(14)	3	—	—	—
iv	(1)	(2)	5	—	—	1
vii	(1)	(14)	3	1	—	—
viii	(7)	(28)	2	1	—	—
xi	(7)	(6)	2	—	—	—
xiii	(7)	(28)	2	1	1	—
xiv	(7)	(5)	3	—	—	—
xv	(7)	(6)	1	—	—	—
xviii	(32)	(2)	4	1	—	—
xix	(21)	(34)	2	2	1	—
xx	(21)	(37)	5	1	—	—
xxi	(33)	(6)	4	1	—	—
xxiv	(32)	(20)	4	—	—	—
xxv	(32)	(2)	2	—	—	2
xxvi	(21)	(37)	3	1	—	—
xxvii	(21)	(34)	2	—	—	—
xxviii	(32)	(20)	3	3	—	—
xxix	(54)	(51)	4	—	—	—
xxx	(50)	(5)	4	—	2	—
xxxi	(21)	(37)	2	3	2	—
xxxiii	(21)	(37)	4	2	1	—
xxxvi	(115)	(51)	4	—	—	—
xxxvii	(32)	(37)	3	1	1	1
xliii	(32)	(37)	3	1	—	1
xliv	(32)	(116)	1	1	—	—
xlviii	(121)	(132)	2	1	—	1
xlix	(32)	(37)	4	—	—	—
li	(32)	(116)	3	1	—	—
lvi	(32)	(116)	3	3	—	—
lxiv	(131)	(116)	6	1	—	—
lxviii	(131)	(116)	3	1	—	—
			105	28	9	8
			Total	150		

It can be seen after a careful survey of the parents that there were four rabbits used who gave no white when mated with any other. These were (5), (50), (51), and (54). The others, since they at some time were parents of one or more albinos, must have had albinism recessive, if the Mendelian principle be correct.

There were 17 offspring born from (5), (50), (51) and (54). (5) × (7) = 3; (5) × (50) = 6; (54) × (51) = 4; (115) × (51) = 4. Subtracting these 17 we then have 133 young, the descendants of the mating of black parents, who in the Mendelian sense are known to have albinism recessive.

Of these 133 young 28 only were albinos. 25 $^{\circ}/_{\circ}$ should have been if Mendel's law is correct, for the product of $DR \times DR$ should be $D + 2DR + R$. Or in other words, three-fourths should appear with the dominant colour, in two-thirds of which the white is recessive but not manifest.

$$\tfrac{28}{133} = 21{\cdot}05\,^{\circ}/_{\circ} \text{ instead of } 25\,^{\circ}/_{\circ}.$$

Results of Black crossed with White or *vice versa* are shown in Table II.

TABLE II.

Black × *White.*

	Parents		Offspring				
	♂	♀	*B*	*W*	*G*	*Y*	
xii	(7) *B*	(40) *W*	1	2	—	—	—
xxiii	(21) *B*	(38) *W*	2	2	—	—	—
xxxv	(112) *W*	(85) *B*	4	2	—	—	1*
xxxviii	(112) *W*	(85) *B*	2	3	—	—	—
xli	(112) *W*	(85) *B*	1	5	—	—	—
xlii	(112) *W*	(51) *B*	3	—	—	—	—
xlv	(112) *W*	(85) *B*	2	1	3	—	3†
liv	(112) *W*	(85) *B*	5	1	—	—	1‡
lvii	(124) *B*	(139) *W*	1	—	—	—	—
lviii	(124) *B*	(139) *W*	3	—	—	—	—
lxix	(112) *W*	(85) *B*	3	3	—	—	—
lxvi	(112) *W*	(85) *B*	1	1	—	—	—
lxxi	(112) *W*	(85) *B*	1	2	—	—	2‡
lxxiv	(112) *W*	(85) *B*	5	3	—	—	—
Totals	—	—	34	25	3	—	7

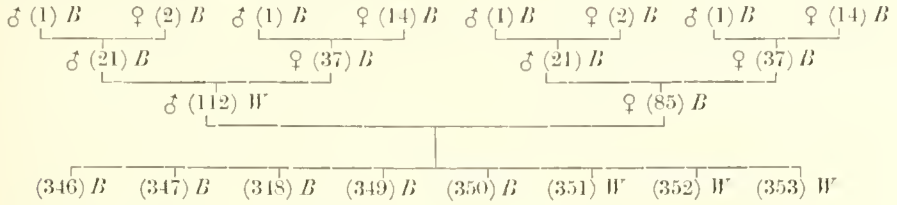
* 3/4 *W*, 1/4 *B*. † 4/5 *W*, 1/5 *B*. ‡ Spotted.

Rabbit (51), suspected of being a pure dominant in the $B \times B$ matings, occurs here once, therefore her litter, as well as the four from (124) may be taken out, since we cannot be sure that these, as parents, contained any white recessive.

If these seven young are taken out of the consideration it will be seen that 27 *B*; 25 *W*; 3 *G*, and 7 spotted were born from $DR \times R$.

The product of $DR \times R$ according to Mendel should give equal numbers of *D*s and *R*s (conf. Bateson (2) p. 158). The figures are 25 albinos in 62 or 40·3 $^{\circ}/_{\circ}$ instead of 50 $^{\circ}/_{\circ}$. Here as before the error is on the side of too little white in the offspring.

In most of these the back ancestry was black. A characteristic pedigree lxxiv is shown below, where it will be seen that one white in every two, which is demanded by Mendel's Law, would not be expected from Galton's. A less amount of white would be called for by the latter law on account of the influence of grandparents and great-grandparents. Hence the error from Mendel's Law may be due to ancestral influence.



Results of White crossed with White are shown in Table III.

TABLE III.

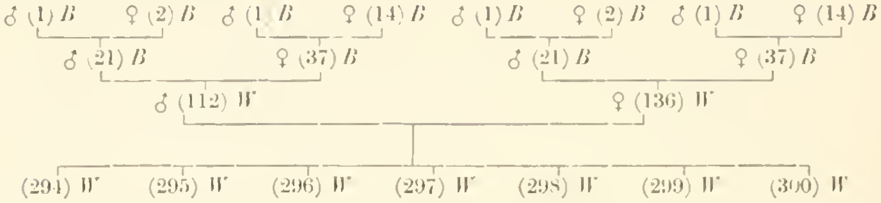
White × White.

	♂ and ♀ Parents both albinos	*B*	*W*	*G*	*Y*
ix	(38) × (39)	—	6	—	—
x	(38) × (39)	—	4	—	—
xvi	(46) × (40)	—	4	—	—
xxii	(46) × (40)	—	3	—	—
xxxii	(46) × (40)	—	7	—	—
xxxiv	(112) × (38a)	—	6	—	—
xxxix	(112) × (120)	—	5	—	—
xl	(112) × (120)	—	5	—	—
lii	(112) × (136)	—	4	—	—
lv	(112) × (36)	—	5	—	—
lx	(112) × (136)	—	6	—	—
lxii	(112) × (120)	—	6	—	—
lxiii	(112) × (136)	—	7	—	—
lxv	(112) × (136)	—	1	—	—
lxvii	(112) × (120)	—	4	—	—
lxix	(112) × (136)	—	3	—	—
lxx	(112) × (136)	—	5	—	—
lxxiii	(112) × (136)	—	5	—	—
Total	—	—	86	—	—

Here we have 86 offspring, all albinos. This is of course in keeping with Mendel's principles, since *R* × *R* can give only *R*. It is not in harmony with Galton's Law, and especially so in this instance, since all the latter two-thirds of the experiments were made with the three rabbits (112), (120), and (136), who had two black parents, and four black grandparents as well. These animals have been intentionally used to see if some black reversions might not be obtained from such a curious mating.

The pedigree of experiment lxiii is given below, and is characteristic of the rest, and can hardly be explained in any other way than recourse to the

Mendelian principles. The buck and doe albino, brother and sister, were closely inbred from black stock.



In addition to the foregoing a few crosses were made between the white buck (112) and a brownish yellow (108), herself the descendant of black ancestry.

Table IV brings out the curious fact that among 19 of her young, her own colour does not appear at all.

TABLE IV.

	Parents	Offspring			
	♂ *B* ♀ *Y*	*B*	*W*	*G*	*Y*
liii	(112) × (108)	1	3	1	—
lxi	(112) × (108)	4	1	1	—
lxxii	(112) × (108)	3	5	—	—
Totals	—	8	9	2	—

Such a result continually obtained would not be in keeping with Galton's Law, since at least 25 $^{\circ}/_{\circ}$ of the offspring ought to be yellow as a result of the influence of one of two parents. It can perhaps be explained by Mendel's principles when a further knowledge of coloration enables us to say how yellow is broken up when combined with pure gametes like those produced from albinos. This may be similar to the "mosaic inheritance" described by Castle (3) and (4).

I have made as yet no accurate correlations to test the Galton-Pearson Law of Ancestral Heredity, as I wish to have later generations giving a more complete back pedigree and larger figures, before such is attempted. Rough draughts, however, show that there is a wide discrepancy of from 15 $^{\circ}/_{\circ}$ to 25 $^{\circ}/_{\circ}$ from the calculated.

In the case of *W* × *W* this is even worse, since the amount of black in the grandparents and great-grandparents calls for 30 $^{\circ}/_{\circ}$ to 45 $^{\circ}/_{\circ}$ black in offspring, whereas we find none at all. The experiments, therefore, confirm the idea that the Law of Ancestral Heredity will not hold so well in cases of alternative inheritance as it does when applied to characteristics which freely blend.

The results set forth in Davenport's review of von Guaita's experiments (8) in breeding mice are also confirmed.

It is however to be noticed that the deviations from Mendel's Law in Tables I and II, namely too little white, are on the side of suggesting an ancestral influence which Mendel's Law does not recognize. This neglect of ancestry Weldon considers to be the fallacy of the Mendelian principles, and writes as follows (10), p. 252.

"The fundamental mistake which vitiates all work based upon Mendel's method is the neglect of ancestry and the attempt to regard the whole effect upon offspring, produced by a particular parent, as due to the existence in the parent of particular structural characters; while the contradictory results obtained by those who have observed the offspring of parents apparently identical in character show clearly enough that not only the parents themselves, but their race, that is their ancestry, must be taken into account before the results of pairing them can be predicted."

It certainly would seem that as regards the breeding of mammals we cannot with our present knowledge give up the idea that ancestry has some influence.

This influence of ancestry is contended for by Darbishire (7)* in a short article in *Biometrika* in which he clearly shows the influence of ancestry in a number of crosses between the Japanese waltzing and albino mice. It seems in every one of a few instances given to make a considerable difference in the results whether his albinos are from pure-bred stock or not. Perhaps further experiments may show what characteristics are pure *unit-characters*†, as albinism appears to be, and then all results will agree closely with the expected.

Or, on the other hand, further experiments may overthrow the reliance to be placed on the few figures already collected. At any rate, the future only can decide. The figures given in this article show more than anything else, that any law yet formulated, when applied to an instance similar to practical stock improvement, gives results not remarkably in keeping with the observed. In this case the truth seems to lie between the expected as announced by Mendel and that formulated by Galton.

Before closing it might be interesting to compare the figures, 21 °/o, obtained from rabbits by $DR \times DR$ with those of other observers for mice. Crampe (5), p. 542, crossed grey wild mice with albinos and obtained 15 young, all resembling the wild form. The next generation gave 79 young, 22 of whom were albino. $\frac{22}{79} = 27\cdot8$ °/o. Von Guaita (8), (p. 122 Davenport's review), shows under similar conditions 14 albinos out of 44, or 31·8 °/o. Cuénot (6), out of 270 young, 72 albino, or 26·6 °/o; and again 162 grey to 57 albino, or 26 °/o. Thus all the figures range between 21 and 31·8 °/o, and seem pretty close considering the comparatively small numbers used.

As was stated above, my result for $DR \times R$ is not so close as that for $DR \times DR$; and instead of 50 °/o R we find 40·3 °/o.

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* Since writing a second article by Darbishire has appeared.

† Bateson (1), p. 27, discusses "unit-characters."

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UEBER ASYMMETRIE BEI "GELASIMUS PUGILATOR" LATR.

Von GEORG DUNCKER, Ph.D.

Die vorliegende Arbeit soll einen Beitrag zur Kenntniss der Abhängigkeit der individuellen Symmetrieverhältnisse verschiedener Paare bilateral-homologer Merkmale von einander liefern. Bei symmetrischen sowohl wie bei asymmetrischen derartigen Merkmalpaaren bilateral gebauter Organismen sind die Unterschiede ihrer Einzelmerkmale mit Nothwendigkeit variabel, sofern diese Einzelmerkmale selbst variabel sind und zu einander in unvollständiger Korrelation stehen. Man erhält also bei der statistischen Untersuchung numerisch ausdrückbarer, bilateral-homologer Merkmale ungleiche individuelle Differenzen derselben, deren Zusammenstellung eine den Variationsreihen ähnliche Reihe, die Differenzreihe des Merkmalpaares, ergiebt. Die Differenzen verschiedener Grösse entsprechen den verschiedenen Graden individueller Symmetrie resp. Asymmetrie des Merkmalpaares; seine Kollektiv-Symmetrie, resp. Asymmetrie bei der untersuchten Formengemeinschaft wird nach dem durchschnittlichen Verhalten der Differenzreihe beurtheilt. Dadurch dass man bisher ausschliesslich das letztere, nicht aber die thatsächlich existirenden Einzeldifferenzen zu berücksichtigen pflegte, entstand z. B. die irrige Auffassung von der spiegelbildlichen Aehnlichkeit bilateral-homologer Organe bei symmetrischen Thierarten.

Wie ich bereits an einzelnen Beispielen nachgewiesen habe [1.] und demnächst an einem umfangreicheren Vergleichsmaterial darzuthun beabsichtige, sind in der Regel die individuellen Symmetrieverhältnisse verschiedener Merkmalpaare von einander mehr oder weniger unabhängig; es besteht keine nennenswerthe Korrelation zwischen ihren Differenzreihen. Das im Folgenden zu besprechende Material bietet gewisse Abweichungen von dieser Regel und ist überdies dadurch besonders instruktiv, dass bei ihm Dimorphismus der Kollektiv-Asymmetrie vorliegt; es kommen entweder rechts- oder linksseitig asymmetrische, dagegen keine symmetrischen Individuen vor.

1901 veröffentlichte Robert M. Yerkes eine Abhandlung [2.]: "A study of Variation in the Fiddler-Crab (*Gelasimus pugilator* Latr.)." Die Männchen der

Gelasimus-Arten sind durch den Besitz zweier hochgradig differenzierter Scheeren ausgezeichnet. Eine von ihnen ist verhältnissmässig sehr gross, auffällig gefärbt, mit Zähnen bewehrt und dient nach Yerkes (p. 440) sowohl zur Vertheidigung als auch zum Graben, in vielen Fällen wohl auch zum Anlocken der Weibchen. Die andere Scheere ist zwerghaft, unscheinbar gefärbt, unbezahnt und dient zum Zuführen der Nahrung in den Mund. Wir wollen die beiden im Folgenden als Kampf- und als Fressscheere unterscheiden. Die Weibchen weisen stets nur zwei kleine, übereinstimmende Fressscheeren auf.

Die Stellung der Kampf- zu der Fressscheere bei den Männchen wechselt derart, dass jene bald der rechten, bald der linken Körperseite angehört; wir theilen dann nach der Stellung der Kampfscheere die Gesammtheit der Männchen in zwei Gruppen, die Rechts- (*R*) und die Linkshänder (*L*) ein. Beide sind ungefähr gleich häufig, denn Yerkes (p. 424, Tab. III, IV) fand in West Falmouth Harbor:

	bei *Gelasimus pugilator*		*Gelasimus pugnax*	
am	*R*	*L*	*R*	*L*
18—22. VIII. 99	501 = 51·9 %	465 = 48·1 %	—	—
16. VII. 00	431 = 50·0 %	431 = 50·0 %	552 = 48·8 %	578 = 51·1 %

Aus Yerkes' Untersuchungen ergeben sich neben den sehr ausgeprägten Ungleichheiten der bilateral-homologen Scheerendimensionen auch noch schwächere der Längen der beiden Lateralränder (*M*) des Cephalothorax-Panzers und der beiderseitigen Meropoditen (*G*) des zweiten Beinpaares (= 1. Gangbeinpaares). Die an dem Scheerenpaare gemessenen Dimensionen sind Längen der Meropoditen (*Ch*), der Carpopoditen (*C*) und der Propoditen (*P*) von ihrer Basis bis zum Gelenk des Daktylopoditen. Endlich waren an jedem der je 400 untersuchten rechts- und linkshändigen Thiere die Medianlänge des Panzers (*T*) und seine Orbitofrontalbreite (*F*) zwischen den vorderen Seitenzähnen auf 0·1 mm. gemessen worden; cf. Yerkes' Figg. 1 u. 2 (pp. 419—420), sowie seine Beschreibung des Messungsverfahrens (p. 420).

Es lag nun der Gedanke nahe, dass die Ungleichheiten der einzelnen Dimensionen Folgen der Scheerendifferenzirung seien, da ja bereits die Gewichtsverschiedenheit der Scheeren sehr wohl zu Ungleichheiten der einzelnen Merkmale auf den beiden Körperseiten Anlass geben könnte. War diese Annahme richtig, so stand zu erwarten, dass auch ihre individuellen Unterschiede, im Gegensatz zu anderweitigen Befunden, korrelativ von einander abhängig seien. Auf diesen Punkt also richteten sich die Untersuchungen, deren Resultate im Folgenden mitgetheilt werden sollen.

Herr Yerkes hatte die grosse Liebenswürdigkeit, mir auf meine Bitte seine handschriftlichen Messungstafeln, welche 400 *R*- und 400 *L*-Thiere enthalten, leihweise zu überlassen. Die auf 0·1 mm. ausgeführten Einzelmessungen derselben fasste ich zu solchen von 0·5 mm. Varianteneinheit derart zusammen, dass z. B. 7·3—7·7 mm. = 7·5 mm. = $15\,\frac{\text{mm.}}{2}$, 7·8—8·2 mm. = 8·0 mm. = $16\,\frac{\text{mm.}}{2}$

gesetzt wurden, und berechnete aus den so entstandenen reducierten Variationsreihen deren Bestimmungswerte sowie die Korrelationskoeffizienten der paarigen Merkmale. Aus den für die Berechnung der letzteren nothwendigen Kombinationsschemata ergaben sich durch einfache Auszählung die absoluten Differenzreihen der bilateral-homologen Merkmalpaare (s. Anhangstab.), deren Bestimmungswerthe zu denen der beiden Variationsreihen der Einzelmerkmale I und II in folgenden einfachen Beziehungen stehen:

$$A_D = A_I - A_{II}, \quad \sigma_D = \sqrt{\sigma_I^2 + \sigma_{II}^2 - 2\rho\, \sigma_I\, \sigma_{II}},$$

$$\beta_{3D} = \frac{\Sigma\,(x_I^3) - \Sigma\,(x_{II}^3) - 3\,\Sigma\,(x_I^2\, x_{II} - x_I\, x_{II}^2)}{n\,(\sigma_I^2 + \sigma_{II}^2 - 2\rho\, \sigma_I\, \sigma_{II})^{\frac{3}{2}}},$$

$$\beta_{4D} = \frac{\Sigma\,(x_I^4) + \Sigma\,(x_{II}^4) + 6\,\Sigma\,(x_I^2 x_{II}^2) - 4\,\Sigma\,(x_I^3 x_{II} + x_I x_{II}^3)}{n\,(\sigma_I^2 + \sigma_{II}^2 - 2\rho\, \sigma_I \sigma_{II})^2}.$$

Die benannten Bestimmungswerthe (A und σ) der Variations- und der Differenzreihen sind, wie die Varianten derselben, im Folgenden stets in der Varianteneinheit $u = 0{\cdot}5$ mm. ausgedrückt.

Die Berechnung der Korrelationskoeffizienten erfolgte nach Bravais' Formel in Pearson's Vereinfachung des Rechenprozesses

$$\rho = \frac{\Sigma\,(x_I\, x_{II})}{n\, \sigma_I \sigma_{II}} = \frac{\frac{1}{n}\Sigma\,([V - V_0]_I \cdot [V - V_0]_{II}) - V_I\, V_{II}}{\sigma_I \sigma_{II}},$$

wo $$x_I = (V - A)_I \text{ und } V_I = \frac{1}{n}\Sigma\,(V - V_0)_I.$$

Die Beziehung zwischen ρ, σ_D, σ_I und σ_{II} diente zur Kontrole der vorherigen Rechnung nach der Formel

$$\rho' = \frac{\sigma_I^2 + \sigma_{II}^2 - \sigma_D^2}{2\sigma_I \sigma_{II}}.$$

Häufig stimmten die für ρ und ρ' gefundenen Werthe noch auf fünf Dezimalen überein und niemals erreichte die Differenz derselben eine Einheit der vierten Dezimalstelle. Ihr wahrscheinlicher Fehler nach der Formel

$$p\,.\,\Sigma_\rho = \frac{(1 - \rho^2)\,.\,0{\cdot}67449}{\sqrt{n}}$$

ermittelt, wurde den einzelnen Korrelationskoeffizienten hinzugefügt. Sämmtliche für die vorliegende Arbeit in Betracht kommende Kombinationsschemata (30) sind bei dem Herausgeber dieser Zeitschrift hinterlegt.

Da zwischen der Totallänge und der Orbitofrontalbreite sehr hohe Korrelation besteht, nämlich für

	Rechtshändige	Linkshändige
ρ	0·91711 ± 0·00536	0·90085 ± 0·00636

so benutzte ich diese beiden Dimensionen gleichzeitig, um das Material in drei Grössengruppen zu zerlegen, für deren mittlere T 23 bis 26 $\frac{\text{mm.}}{2}$, F 30 bis 32 $\frac{\text{mm.}}{2}$ inclusive beträgt. Ein Uebergreifen der kleinsten Gruppe mit einer dieser Dimensionen in das Gebiet der grössten oder umgekehrt fand in Folge der bedeutenden Korrelation nicht statt.

Soweit bisher statistische Untersuchungen bilateral-homologer Merkmalpaare vorliegen, ergeben dieselben ausnahmslos unvollständige Korrelation variabler Merkmale. Hieraus und aus den Beziehungen der Bestimmungswerthe der Differenzreihe zu denen der Variationsreihen der Einzelmerkmale folgt, dass der Grad der individuellen Asymmetrie variabel ist, dass also neben symmetrischen stets auch asymmetrische Individuen hinsichtlich des Merkmalpaares auftreten müssen. Die Kollektiv-Symmetrie eines Merkmalpaares kann daher nicht wohl auf dem "spiegelbildlich gleichen Verhalten der beiden Körperseiten zu einander" beruhen, sondern sie ist, wie in [1.] gezeigt, definiert durch eine symmetrische Differenzreihe mit dem Mittel Null. Ist nun eine Differenzreihe um Null als Mittel symmetrisch, so ist auch ihr Centralwerth gleich Null, folglich sowohl die Anzahl als auch die Summe der positiven gleich der der negativen Differenzen, wenn letztere ihrem absoluten Zahlenwerth nach gerechnet werden. Für dies Verhalten aber haben wir als einfachen Ausdruck den *Asymmetrieindex* der Differenzreihe

$$\alpha = \frac{\Sigma(f').\Sigma(D') - \Sigma(f'').\Sigma(D'')}{n[\Sigma(D') + \Sigma(D'')]} = 0,$$

wo mit f' und D' Frequenz- resp. Differenzwerthe des positiven, mit f'' und D'' die entsprechenden Werthe des negativen Abschnittes der Differenzreihe bezeichnet sind. Existieren dagegen nur positiv oder nur negativ asymmetrische, aber keine symmetrischen Individuen, so wird $\alpha = \pm 1$, d. h., es liegt vollkommene positive oder negative Asymmetrie des Merkmalpaares bei der untersuchten Individuengemeinschaft vor. Das Vorzeichen des Asymmetrieindex entspricht den bei bilateralen Organismen denkbaren einander entgegengesetzten Möglichkeiten der Kollektiv-Asymmetrie, z. B. rechts- oder linksseitig, oder, in unserem Falle, auf der Kampf- oder der Fressscheerenseite überwiegende Entwicklung der homologen Merkmale. Der Asymmetrieindex* stellt somit den kürzesten Ausdruck der Kollektiv-Asymmetrie eines Merkmalpaares bei einer Individuengemeinschaft dar.

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* Rechnungsbeispiel: Absolute Differenzen der Lateralränder bei den Rechtshändern. (Cf. Anhangstab.)

$D = \text{Md} - \text{Ms}$:	-1	0	1	2	3
f:	1	63	310	23	3

$$\Sigma(D') = 310 \times 1 + 23 \times 2 + 3 \times 3 = 365, \quad \Sigma(f') = 336,$$

$$\Sigma(D'') = 1, \quad \Sigma(f'') = 1, \quad n = 400,$$

$$\alpha = \frac{336 \times 365 - 1 \times 1}{400 \times 366} = \frac{122639}{146400} \quad 0{\cdot}83770.$$

Da die Variationsreihen der untersuchten Merkmale gegenüber den in Yerkes' Arbeit angeführten ([2.], p. 42 Tab. I u. Figg. 1—12) durch die Reduktion der Messungen auf 0·5 mm. etwas verändert erscheinen, gebe ich zunächst eine Zusammenstellung ihrer Bestimmungswerthe, nämlich ihr arithmetisches Mittel (A), ihren Variabilitätsindex (σ), beides in halben Millimetern als der Varianteneinheit ausgedrückte Zahlen, ihren dritten* (β_3) und ihren vierten Momentquotienten (β_4), welche unbenannte Zahlen sind. Ferner habe ich die Quartilwerthe Q_1 und Q_3, deren arithmetisches Mittel q, und den Medianwerth M für Totallänge und Orbitofrontalbreite aufgeführt.

TABELLE I.

R, *L*, = rechtshändige resp. linkshändige Thiere. *s* (sinister), *d* (dexter) = linke oder rechte Körperhälfte.

				A	σ	β_3	β_4	δ
Dimensionen des Panzers	Medianlänge (T)		R	24·48750	1·76489	0·57873	3·46339	—
			L	24·45000	1·82414	0·47809	3·36645	—
	Orbitofrontalbreite (F) ...		R	31·44250	2·24537	0·73608	3·42996	—
			L	31·26750	2·44457	0·58306	3·32595	—
	Lateralränder (M)...	R	s	16·76750	1·46234	0·54573	3·21308	25·11°/₀
			d	17·67750	1·54386	0·60112	3·45931	
		L	s	17·37750	1·57321	0·49973	3·38461	25·18°/₀
			d	16·54250	1·47587	0·64010	3·37943	
Meropoditenlänge des 1. Gangbeinpaares (G) ...		R	s	14·22750	1·25329	0·63178	3·29739	29·17°/₀
			d	15·14500	1·39247	0·57075	3·39429	
		L	s	15·11250	1·49159	0·52587	3·25918	30·84°/₀
			d	13·95750	1·32125	0·40979	3·11063	
Dimensionen des Scheerenpaares	Meropoditenlänge (Ch)	R	s	11·08750	1·02462	0·45204	2·96605	97·88°/₀
			d	17·50500	1·59540	0·39896	3·09451	
		L	s	17·27000	1·68288	0·34230	3·25285	98·03°/₀
			d	10·59500	1·02761	0·42801	3·23408	
	Carpopoditenlänge (C)	R	s	6·09750	0·63482	0·32710	3·48979	100·00°/₀
			d	14·23750	1·50036	0·60851	4·09883	
		L	s	13·72000	1·38802	0·35723	3·29010	100·00°/₀
			d	5·94000	0·66813	0·32044	3·07763	
	Propoditenlänge (P)	R	s	5·43750	0·54872	0·29404	2·26207	100·00°/₀
			d	18·45500	1·89023	−0·06681	3·72480	
		L	s	18·05750	1·82049	−0·04050	3·22911	100·00°/₀
			d	5·50750	0·59156	−0·01074	2·53760	

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* $\beta_3 = \frac{\Sigma (x^3)}{n\sigma^3} = \pm \sqrt{\beta_1}$ (Pearson).

ANHANG ZUR TABELLE I.

Erste und dritte Quartilwerthe (Q_1 u. Q_3), sowie deren Mittelwerth (q), und Medianwerthe (M) der Panzerlänge und Orbitofrontalbreite.

		Q_1	M	Q_3	q
Panzerlänge (T) …	R	23·26623	24·24545	25·57547	1·15462
	L	23·21429	24·24074	25·59434	1·19003
Orbitofrontalbreite (F)	R	29·92169	31·09770	32·45588	1·26710
	L	29·72989	30·94000	32·46667	1·36839

Die Durchschnittswerthe fast sämmtlicher untersuchten Merkmale sind bei den rechtshändigen Thieren grösser, als bei den linkshändigen; die einzige Ausnahme bildet der Propodit der Fressscheere. Die Durchschnittswerthe aller paarigen Merkmale sind auf der Seite der Kampfscheere grösser, als auf der der Fressscheere.

Die Variabilität der Merkmale ist meistens bei den linkshändigen Thieren bedeutender; Ausnahmen bilden der Carpopodit der Kampfscheere und vielleicht auch ihr Propodit. Wie die Durchschnittswerthe der paarigen Merkmale, so sind auch ihre Variabilitätsindizes stets grösser auf der Seite der Kampfscheere.

Die dritten Momentquotienten sind meistens positiv; negativ nur für die Propoditenlänge der Kampfscheere bei Rechts- und Linkshändern, sowie vielleicht bei letzteren für die der Fressscheere, wo er jedoch kaum von Null verschieden ist. Ihrem absoluten Werth nach bleiben sie sämmtlich unter 0·65, sind also nur klein. Bestimmte Grössenbeziehungen hinsichtlich ihrer bei Rechts- und Linkshändern, sowie auf den homodynamen Körperseiten bestehen nicht, obwohl sie im allgemeinen bei rechtshändigen etwas höher sind als bei linkshändigen Thieren.

Die vierten Momentquotienten sind meistens bei den Rechtshändern grösser, als bei den Linkshändern; ausgenommen sind hiervon diejenigen des Lateralrandes auf der Seite der Fressscheere und der beiderseitigen Scheerenmeropoditen. Die der paarigen Merkmale sind stets auf der Seite der Kampfscheere grösser, stimmen also hierin mit dem Verhalten von A und σ überein.

Die Variationsgebiete der Lateralränder und die der Meropoditenlängen des zweiten Beinpaares fallen weitgehend zusammen, wie aus der verhältnissmässig kleinen prozentualen Differenzfläche ihrer Variationspolygone hervorgeht. Die der chelaren Meropoditenlängen greifen noch ein wenig ineinander über, während jene der übrigen bilateral-homologen Merkmale vollständig getrennt sind. Demnach erscheint bei dem vorliegenden Material das gemeinsame Variationsgebiet zweier bilateral-homologer Merkmale um so grösser, je näher dieselben einander am Körper der Thiere liegen.

Parallel zu dem letzterwähnten Verhalten geht das der Korrelationsintensität zwischen den paarigen Merkmalen. Ihre nachstehend aufgeführten stets positiven

Korrelationskoeffizienten schwanken zwischen 0·95 (Lateralränder) und 0·47 (chelare Propoditen), also innerhalb eines weiten Spielraumes, bald bei Rechts-, bald bei Linkshändern etwas überwiegend. Die Beziehung der Korrelationsintensität zur räumlichen Anordnung der bilateral-homologen Merkmale erscheint mir besonders beachtenswerth.

TABELLE II.

Korrelationskoeffizienten zwischen rechter und linker Körperseite.

R = rechtshändige, *L* = linkshändige Thiere.

		R	*L*
Laterale Panzerränder		0·94678 ± 0·00349	0·94976 ± 0·00330
Meropoditenlänge des I. Gangbeinpaares		0·91796 ± 0·00531	0·87901 ± 0·00767
Scheeren	Meropoditenlänge ...	0·75447 ± 0·01453	0·78895 ± 0·01276
	Carpopoditenlänge ...	0·69750 ± 0·01732	0·69895 ± 0·01725
	Propoditenlänge ...	0·47333 ± 0·02617	0·54861 ± 0·02357

Beim Vergleich der Bestimmungswerthe der Differenzreihen ergiebt sich Folgendes:

TABELLE III.

Differenzreihen.

R, *L* = rechtshändige resp. linkshändige Thiere. *D* = Differenzen.

	Laterale Panzerränder (*M*)		Meropoditenlänge d. 1. Gangbeins (*G*)		Meropoditenlänge d. Scheeren (*Ch*)		Carpopoditenlänge d. Scheeren (*C*)		Propoditenlänge der Scheeren (*P*)	
	D		*D*		*D*		*D*		*D*	
	R	*L*	*R*	*L*	*R*	*L*	*R*	*L*	*R*	*L*
A	0·91000	0·83500	0·91750	1·15500	6·41750	6·67500	8·14000	7·78000	13·01750	12·55000
σ	0·49689	0·49272	0·55290	0·71132	1·06216	1·07674	1·15126	1·03759	1·70065	1·57560
β_3	0·06026	0·03896	−0·02236	−0·23337	−0·34074	−0·16845	0·32505	0·03241	− 0·60675	− 0·48127
β_4	5·99735	7·51097	4·48052	5·26774	3·71957	3·57525	3·88292	3·75459	4·52343	4·31115
Asymmetrie-index	0·83770	0·79263	0·81320	0·87238	1·00000	1·00000	1·00000	1·00000	1·00000	1·00000

Die Durchschnittswerthe der Differenzreihen überwiegen bei den Rechtshändern in allen Fällen mit Ausnahme der Meropoditenlängen des Scheeren- und des zweiten Beinpaares. Da alle gemessenen Dimensionen auf der Seite der Kampfscheere die grösseren Durchschnittswerthe ergeben, so sind sie stets positiv.

Ihre Variabilitätsindizes verhalten sich bei links- und rechtshändigen Thieren wie die Durchschnittswerthe.

Die dritten Momentquotienten sind positiv für M und C, negativ für G, Ch und P. Ihrem absoluten Zahlenwerth nach sind sie überall bei den rechtshändigen Thieren grösser, ausser für G. Sie sind meistens kleiner, als die korrespondierenden Werthe der Einzelmerkmale.

Die vierten Momentquotienten sind für die drei Paare der Scheerendimensionen grösser bei den rechts-, für die beiden anderen Merkmalpaare bei den linkshändigen Thieren, und stets grösser als die korrespondierenden Werthe der Einzelmerkmale.

Die Asymmetrieindizes betragen für die drei paarigen Scheerendimensionen in Folge ihrer nahezu oder vollkommen auseinander fallenden Variationsgebiete stets Eins; die Kollektiv-Asymmetrie dieser Merkmalpaare ist demnach vollständig. Für die Lateralränder und die Meropoditen des zweiten Beinpaares ergeben sich echte Bruchwerthe, die bei Rechts- und Linkshändern etwas verschieden ausfallen.

Die Symmetrieverhältnisse der einzelnen Merkmalpaare werden durch das Wachsthum der Thiere derart beeinflusst, dass die Differenzen der bilateralhomologen Merkmale mit zunehmendem Alter an Grösse zunehmen. Die paarigen Merkmale wachsen also ungleich rasch, je nachdem sie sich auf der Seite der Fress- oder der Kampfscheere befinden, und zwar schneller auf der Seite der letzteren. Es muss daher zwischen den absoluten Differenzreihen positive Alterskorrelation bestehen.

Untersucht man die *Abhängigkeit der Kollektiv-Asymmetrie* der unvollständig asymmetrischen Merkmalpaare M und G von der hier als primär betrachteten Asymmetrie der Scheerenentwicklung, nämlich der Rechts- und Linkshändigkeit, nach dem von Pearson ([3.] pp. 435—436) für nicht numerische Merkmale angegebenen Verfahren*, so erhält man

$$\text{für } M \qquad \rho = 0{\cdot}95933 \pm 0{\cdot}00269$$
$$\text{,, } G \qquad \rho = 0{\cdot}97302 \pm 0{\cdot}00180$$

also nahezu vollkommene Abhängigkeit.

Für die Merkmalpaare mit vollständiger Asymmetrie ist natürlich auch diese Abhängigkeit eine vollkommene; man findet für sie stets den Werth $\rho = 1$.

Zum Vergleich füge ich die bisher unveröffentlichten korrespondierenden Werthe für paarige Merkmale zweier anderer asymmetrisch differenzierter Thierarten hier bei:

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* Rechnungsbeispiel: $D(M)$. Cf. Anhangstabelle.

D:	−4	−3	−2	−1	0	1	2	3	Σ
f L	1	0	14	303	81	1			400
f R				1	63	310	23	3	400
Σ $(L+R)$	4	0	14	304	144	311	23	3	800

	−4 bis C	C bis 3	Σ
	363·56	36·44	400
	36·44	363·56	400
Σ	400	400	800

$$C = -0{\cdot}5 + \frac{81}{144} = 0{\cdot}06250 \qquad \rho = \sin 360 \,.\, \frac{163{\cdot}56}{800} = 0{\cdot}95933.$$

a. Pleuronectes flesus L. 1060 rechts- und 60 linksaugige (♂ + ♀) Individuen.

		α	
	ρ	R	L
Gesammtzahl der Brustflossenstrahlen	0·80530	0·60443	− 0·58333
Zahl der Brustflossen-Theilstrahlen	0·99782	0·97778	− 0·98305
Gesammtzahl der Bauchflossenstrahlen	0·07811	0·01957	− 0·08333
Zahl der Bauchflossen-Theilstrahlen	0·54902	0·32600	− 0·37424
Endigungsstelle der Supraoccipitaläste der Seitenlinien	0·34170	0·12403	− 0·32305

b. Eriphia spinifrons Herbst. 694 rechts- und 255 linkshändige ♂.
544 „ „ 206 „ ♀.

	ρ	
	♂	♀
Zahl der Anterolateraldornen	0·01257	0·01885
„ „ Infraorbitaldornen	0·07533	0·10661
„ „ Facialdornen	− 0·01571	− 0·01571
„ „ Frontaldornen	0·05338	0·09411
„ „ Suprafrontaldornen	0·04711	0·00940

Die Asymmetrieindizes dieser Merkmalpaare liegen sämmtlich zwischen + 0·1 und − 0·1.

Aus den unter *a* und *b* aufgeführten Werthen ergiebt sich, dass die Kollektiv-Asymmetrie verschiedener bilateral-homologer Merkmalpaare von der jeweils als primär betrachteten bilateralen Differenzierung in ausserordentlich ungleichem Maasse abhängig ist, ferner, dass der Grad dieser Abhängigkeit bei unvollständiger Kollektiv-Asymmetrie eines Merkmalpaares zwar im allgemeinen konform, jedoch nicht proportional dem Grade der letzteren (dem Asymmetrieindex) zunimmt.

Die gegenseitige Abhängigkeit der individuellen Symmetrieverhältnisse der verschiedenen Paare bilateral-homologer Merkmale von einander ergiebt sich aus den korrelativen Beziehungen ihrer Differenzreihen. Hinsichtlich letzterer sind drei einander ausschliessende Möglichkeiten denkbar:

1. Mit überdurchschnittlicher Asymmetrie eines Merkmalpaares ist überdurchschnittliche eines anderen individuell verbunden und vice versa. Dies Verhalten würde seinen Ausdruck in positiver Korrelation der Differenzreihen finden.

2. Mit überdurchschnittlicher Asymmetrie eines Merkmalpaares ist unterdurchschnittliche eines anderen verbunden und umgekehrt. Individuelle Asymmetrien verschiedener Merkmalpaare gleichen sich gegenseitig aus. Hierbei würde negative Korrelation der Differenzreihen bestehen.

3. Der individuelle Grad der Asymmetrie eines Merkmalpaares ist unabhängig von demjenigen des anderen. Es besteht keine Korrelation zwischen ihren Differenzreihen.

Die wenigen, ausschliesslich von mir angestellten Untersuchungen an verschiedenartigem Material ergaben bisher übereinstimmend auffällig niedrige Korrelationskoeffizienten der Differenzreihen bilateral-homologer Merkmalpaare.

1. *Acerina cernua* L. (692 ♀).

	ρ_D	
Seitliche Kopf- und Mandibellängen in Procenten der Totallänge	0·10775	± 0·02536

2. *Pleuronectes flesus* L. (1060 rechtsäugige ♂ + ♀).

Gesammt- und Theilstrahlzahlen der Brustflossen	0·11104	± 0·02092
„ „ „ „ Bauchflossen	0·01330	± 0·02077
Gesammtstrahlzahlen der Brust- und der Bauchflossen	0·01359	± 0·02076
Theilstrahlzahlen der Brust- und der Bauchflossen	− 0·04125	± 0·02121
Gesammtstrahlzahlen der Brust- und Theilstrahlzahlen der Bauchflossen	0·07154	± 0·02068
Theilstrahlzahlen der Brust- und Gesammtstrahlzahlen der Bauchflossen	− 0·03568	± 0·02120

3. *Eriphia spinifrons* Herbst (Rechtshänder; 694 ♂ und 544 ♀).

Differenzreihen der beiderseitigen Dornzahlen der Anterolateral- und der Infraorbitalränder	♂:	− 0·00024	± 0·02583
	♀:	0·00345	± 0·02911
der Infraorbital- und der Frontalränder	♂:	0·04880	± 0·02584
	♀:	0·08088	± 0·02908
der Frontalränder und der Suprafrontalleisten	♂:	0·00966	± 0·02583
	♀:	− 0·06765	± 0·02897

Somit besteht bezüglich dieser Merkmalpaare, die mit Ausnahme der beiden ersten (*Acerina*) keinen wesentlichen Altersveränderungen ausgesetzt sind, nur sehr geringe oder—in den weitaus meisten Fällen—keine mit Sicherheit nachweisbare Korrelation ihrer Differenzreihen, d. h. die individuellen Symmetrieverhältnisse der verschiedenen bilateral-homologen Merkmalpaare sind von einander unabhängig*.

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* Durch Entwicklung des Ausdrucks

$$(a)\quad \rho_D = \frac{\Sigma\,(\chi_{DI}\,\chi_{DII})}{n\sigma_{DI}\,\sigma_{DII}} = \frac{\Sigma\,(\chi_d - \chi_s)_I\,(\chi_d - \chi_s)_{II}}{n\sigma_{DI}\,\sigma_{DII}}$$

erhält man

$$(b)\quad \rho_D = \frac{\rho_{dIdII}\,\sigma_{dI}\sigma_{dII} - \rho_{dIsII}\,\sigma_{dI}\sigma_{sII} - \rho_{sIdII}\,\sigma_{sI}\sigma_{dII} + \rho_{sIsII}\,\sigma_{sI}\sigma_{sII}}{\sqrt{(\sigma_d^2 + \sigma_s^2 - 2\rho\sigma_d\sigma_s)_I \cdot (\sigma_d^2 + \sigma_s^2 - 2\rho\sigma_d\sigma_s)_{II}}},$$

wobei *I* und *II* die Merkmalpaare, deren Differenzreihen kombiniert sind, bezeichnen. Dieser Ausdruck ergiebt zunächst dann von Null abweichende (positive oder negative) Werthe, wenn die Variabilität der Merkmale beider Paare beträchtlich auf den beiden Körperseiten differiert. Ist die Variabilität der homologen Merkmale auf beiden Körperseiten gleich, so reduziert sich obiger Ausdruck auf die Form

$$(c)\quad \rho_D = \frac{\rho_{dIdII} - \rho_{dIsII} - \rho_{sIdII} + \rho_{sIsII}}{\sqrt{(1 - \rho_I)\,(1 - \rho_{II})}}$$

Bei dem vorliegenden Material ist nun von vorn herein stärkere Korrelation zwischen den Differenzreihen deshalb zu erwarten, weil die individuellen Differenzen der paarigen Merkmale mit der Grössenzunahme der Thiere wachsen. Ich habe diesen Umstand einstweilen nur an zwei Kombinationen solcher Merkmalpaare geprüft, deren Beziehungen ein besonderes Interesse boten. Die eine derselben enthält die metamer-homologen Meropoditenlängen der Scheeren und der vorderen Schreitbeine (*Ch* und *G*), die andere die Merkmalpaare stärkster Asymmetrie (*C* und *P*). Ausser den Korrelationskoeffizienten der Differenzreihen (ρ_D) wurden auch diejenigen der Einzelmerkmale ermittelt.

TABELLE IV.

	R		*L*	
	Chs	*Chd*	*Chs*	*Chd*
Gs	0·80217 ±0·01202	0·82526 ±0·01076	0·84841 ±0·00948	0·79955 ±0·01217
Gd	0·79660 ±0·01182	0·84257 ±0·00978	0·83269 ±0·01037	0·80305 ±0·01198
ρ_D	0·17358 ± 0·03721		0·18654 ± 0·03255	
	Cs	*Cd*	*Cs*	*Cd*
Ps	0·61676 ±0·02090	0·56311 ±0·02303	0·83941 ±0·00985	0·63589 ±0·02009
Pd	0·59847 ±0·02162	0·83549 ±0·01018	0·61757 ±0·02080	0·64632 ±0·01964
ρ_D	0·71636 ± 0·01715		0·67041 ± 0·01857	

Die Kollektiv-Asymmetrie des untersuchten Materials findet auch in dem Verhalten der zusammengehörigen Tetraden von Korrelationskoeffizienten der nicht homologen paarigen Einzelmerkmale ihren Ausdruck. Bei Symmetrie sind in der Regel die Korrelationskoeffizienten von derartigen Merkmalen derselben Körperseite einander gleich und meistens höher als die von solchen entgegengesetzter Körperseiten, welche ihrerseits wiederum gleich sind. Im vorliegenden

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und ergiebt merkliche Werthe nur dann, wenn ein Unterschied der korrelativen Beziehungen zwischen den nicht homologen Merkmalen derselben und den nicht homologen Merkmalen entgegengesetzter Körperseiten besteht.

Nach Formel (*b*) finde ich für ρ_D unserer Beispiele

	G : *Ch*	*C* : *P*
R	0·17360	0·71636
L	0·18656	0·67041.

Die Korrelation des ersten Falles wird wesentlich durch den Unterschied der Variabilität der homologen Einzelmerkmale auf den beiden Körperseiten, die des zweiten sowohl hierdurch, als auch durch die Verschiedenheit der korrelativen Beziehungen der nicht homologen Einzelmerkmale bedingt.

Falle finden wir zwar, dass die höchste Korrelation zwischen den Merkmalen der Kampfscheerenseite besteht, dass aber diejenige zwischen den Merkmalen der Seite der Fressscheere nicht oder kaum höher ist, als die zwischen Merkmalen entgegengesetzter Körperseiten. Die beiden Paare metamer-homologer Merkmale stehen in engerer Korrelation, als die einander unmittelbar benachbarten Paare nicht-homologer Dimensionen der Scheerenglieder.

Das Gesammtresultat unserer Untersuchungen der Symmetrieverhältnisse der verschiedenen bilateral-homologen Merkmalpaare von *Gelasimus pugilator* lässt sich etwa folgendermaassen ausdrücken:

Die Kollektiv-Asymmetrie der einzelnen bilateral-homologen Merkmalpaare, welche ihren Ausdruck in den Asymmetrieindizes findet, hängt de facto gänzlich von der als primär betrachteten, als "Rechts-" oder "Linkshändigkeit" bezeichneten Differenzierung der durch die Medianebene getrennten Körperhälften ab. Trotzdem stehen die individuellen Unterschiede der einzelnen Merkmalpaare in nur schwacher oder in mittlerer Korrelation zu einander.

Demnach entwickeln sich die untersuchten bilateral-homologen Merkmale auf den beiden seitlichen Körperhälften in hohem Grade verschieden. Dies ist jedoch nicht die Folge eines die beiden individuellen Körperhälften gemeinschaftlich in dem Sinne beeinflussenden Prozesses, dass die Grösse der Unterschiede eines Merkmalpaares diejenige von solchen anderer reguliert, denn die individuellen Symmetrieverhältnisse der einzelnen Merkmalpaare sind ja von einander entweder unabhängig oder in nur mässigem Grade abhängig. Kollektiv-Symmetrie oder -Asymmetrie ist die Folge grösserer oder geringerer Aehnlichkeit des Entwicklungsganges der beiden homologen Körperhälften, deren einzelne Merkmale individuell variabel sind. Je ähnlicher die Variationsreihen der homologen Merkmale, je höher die gewöhnlich zwischen ihnen bestehende, positive Korrelation, desto grösser allerdings die Wahrscheinlichkeit, hinsichtlich ihrer vollkommen symmetrische Individuen anzutreffen. Doch steht die Grösse des individuellen Unterschiedes eines Merkmalpaares für gewöhnlich überhaupt nicht im Zusammenhang mit der eines anderen, und auch bei dem vorliegenden Material nur mittelbar, auf Grund besonderer mechanischer und trophischer Bedingungen.

Zum Schlusse möchte ich nicht verfehlen, Herrn Robert M. Yerkes auch an dieser Stelle meinen verbindlichen Dank für die liebenswürdige Bereitwilligkeit auszusprechen, mit der er mir sein interessantes Material zur Benutzung überliess.

LITTERATUR.

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Zusammenfassung.

1. Bei den Männchen der *Gelasimus*-Arten besteht eine ausgeprägte Differenzierung der beiden Scheeren nach Form und Grösse. Die grössere (Kampf-) Scheere kann der rechten oder der linken Körperseite angehören.

2. Die Untersuchungen von Yerkes an männlichen *Gelasimus pugilator* Latr. ergaben ungleiche Grössenverhältnisse nicht nur der korrespondierenden Scheerenglieder beiderseits, sondern auch der Lateralränder des Cephalothorax und der Meropoditen des zweiten Beinpaares, derart, dass sämmtliche bilateral-homologen Dimensionen im Durchschnitt auf der Seite der Kampfscheere stets grösser als auf der Gegenseite sind.

3. Die Variabilität bilateral-homologer Dimensionen überwiegt auf der Seite der Kampfscheere.

4. Die Differenz zweier bilateral-homologer Dimensionen nimmt mit steigender Totallänge zu; es findet rascheres Wachsthum der auf der Seite der Kampfscheere befindlichen Organe statt.

5. Der wie bei bilateral-homologen Merkmalen überhaupt stets positive Korrelationskoeffizient der untersuchten Paare homologer Dimensionen ist um so grösser, je näher die gemessenen Organe einander liegen.

6. Die Scheerendimensionen verhalten sich vollständig, die übrigen zwar stark, aber unvollständig asymmetrisch. Erstere sind also ausnahmslos auf der Seite der Kampfscheere grösser als auf der der Fressscheere, während sich bezüglich letzterer vereinzelt auch symmetrische oder gegensätzlich asymmetrische Individuen finden.

7. Die Abhängigkeit der geschilderten Ungleichheiten der bilateral-homologen Merkmale von der Stellung der beiden Scheeren zu den Körperseiten (Rechts- und Linkshändigkeit) entspricht ihren Asymmetrieindizes. Sie ist nahezu vollständig bezüglich der Lateralränder und der Meropoditenlängen des zweiten Beinpaares, absolut bezüglich der bilateral-homologen Scheerendimensionen. Die Kollektiv-Asymmetrie der Formengemeinschaft ist demnach hinsichtlich der untersuchten Merkmalpaare so gut wie vollkommen.

8. Die Differenzreihen ergeben, da sie nach (4) in Wachsthumskorrelation stehen, bei dem vorliegenden Material höhere Korrelationskoeffizienten, als bei früheren Beobachtungen. Doch ist an den beiden einstweilen untersuchten Kombinationen derselben die Zunahme der Korrelationskoeffizienten nicht sehr bedeutend; insbesondere stehen trotz ihrer die individuellen Unterschiede der Meropoditenlängen des ersten und des zweiten Beinpaares in bemerkenswerth niedriger Korrelation.

9. Das Bestehen von Korrelation in den sub (8) genannten Fällen bildet eine Ausnahme von der sonst geltenden Regel, dass die individuellen Verschiedenheiten eines bilateral-homologen Merkmalpaares ihrer Grösse nach gänzlich

unabhängig von denen eines anderen erscheinen. Sie erklärt sich zum Theil durch Wachsthumskorrelation, zum Theil ist sie wahrscheinlich die Folge der ungleichen mechanischen und trophischen Bedingungen, welchen die paarigen Organe auf Grund der ausserordentlichen Grössendifferenzierung der beiden Scheerenbeine unterliegen.

10. Sind nun im vorliegenden Falle die Differenzen der verschiedenen bilateralhomologen Merkmalpaare auch nicht gänzlich unabhängig von einander, so genügt die Intensität ihrer korrelativen Beziehungen doch bei weitem nicht zur Erklärung ihrer nahezu resp. durchaus vollkommenen Kollektiv-Asymmetrie. In dieser Beziehung schliessen sich also die hier erhaltenen Befunde denen an, in welchen die individuellen Differenzen verschiedener paariger Merkmale überhaupt keine Abhängigkeit von einander aufweisen.

ANHANGSTABELLE.

Differenzreihen ($u = 0{\cdot}5$ mm.).

Differenzen		*−2*	*−1*	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	*18*
Lateralränder des Cephalothorax ...	*R*	—	1	63	310	23	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	L	—	1	81	303	14	0	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Meropoditenlängen d. 1. Gangbeinpaares	*R*	—	2	71	287	38	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	L	2	4	39	252	91	12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Chelare Meropoditenlängen	*R*	—	—	—	—	1	2	12	52	140	140	46	7	—	—	—	—	—	—	—	—	—
	L	—	—	—	—	—	2	8	38	119	150	72	8	3	—	—	—	—	—	—	—	—
Chelare Carpopoditenlängen	*R*	—	—	—	—	—	—	—	4	20	82	162	87	35	8	1	1	—	—	—	—	—
	L	—	—	—	—	—	—	1	4	33	113	161	72	14	1	1	—	—	—	—	—	—
Chelare Propoditenlängen	*R*	—	—	—	—	—	—	—	—	2	0	3	11	13	31	65	118	100	31	21	4	1
	L	—	—	—	—	—	—	—	—	2	1	3	10	27	44	88	133	53	29	9	1	—

VARIATION AND CORRELATION IN ARCELLA*.

By RAYMOND PEARL and FRANCES J. DUNBAR.

I. Introduction.

The purpose of this paper is to present the results of a quantitative study of the variability of the common shelled rhizopod, *Arcella vulgaris*. Comparatively little is known regarding the amount of the individual variation which exists in the different groups of the Protozoa, and it seems highly desirable that our knowledge in this field should be extended. Almost the only piece of quantitative work on the variability of these forms which has appeared is that of Simpson† on *Paramœcium*. It was with the idea of determining the variation in a still lower form than *Paramœcium* that the present work was undertaken. The plan followed in this work was, however, somewhat different from that of Simpson. Since his primary purpose was the determination of the amount of variation following binary fission, only organisms of known immediate ancestry were measured. The labour involved in isolating a particular *Paramœcium* undergoing division, and in measuring the daughter individuals after fission has been completed, is very considerable, and consequently it is not surprising that measurements for but 100 pairs were collected.

Our plan was to determine the amount of variation in a representative sample of a homogeneous population of *Arcellae*, without reference to the immediate ancestry of the individuals of the sample. This is the course which not only usually is, but in most cases must be, followed in studying the variation in higher forms, since the difficulties of getting a sufficiently large number for statistical purposes of individuals of known ancestry are very great. By working in this way it would seem that we might get values of the variation coefficients which would be better for the purposes of comparing protozoan with metazoan variability than those of Simpson. The practical difficulties in handling the material in this way are of course very greatly reduced, and consequently it becomes possible to measure a large series of individuals.

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* Contributions from the Zoological Laboratory of the University of Michigan.

† Simpson, J. Y., "The Relation of Binary Fission to Variation." *Biometrika*, Vol. I., pp. 400—404, 1902.

Besides the determination of the constants of the simple variation in single dimensions we also wished to ascertain the amount of the correlation in the variation of different parts. As in the case of simple variation Simpson's is the only work which we have on correlation in any protozoan. His data enabled him to determine, in addition to the correlation between the two individuals of the divided pair, the correlation between length and breadth of the same individual. His value for the length-breadth correlation in the same individual was ·421. It is very important to determine whether there exists in general a higher, or lower, or equal amount of correlation between the parts of lower organisms as compared with higher. This question of the relative amount of correlation in different forms becomes very significant when any attempt is made to solve the more general problem of the physiological basis of the phenomenon of correlation in variation[*]. Although the fundamental character of this general problem is apparent, yet comparatively little has been done towards solving it. It seems not unlikely that the ultimate method of attacking this problem will be by means of experiment, but some light can certainly be thrown on the question by determining the amount of correlation in variation in different groups of organisms. Such determinations can then be used as a definite quantitative basis, from which to proceed by experimental methods. It has seemed to one of the writers that one good starting point for the investigation of this problem would be a thorough study of variation and correlation in the cell, including both free-living and tissue cells. The present paper forms our first contribution to this study.

II. Material and Methods.

The form used in this work was the common *Arcella, A. vulgaris.* A very complete description of this organism, illustrated with coloured figures, is given by Leidy[†]. It is also fully described by Bütschli[‡].

Arcella is so generally distributed and well known an organism as to make only a very brief account of its taxonomic position, structure, etc., necessary here. *Arcella* is a shelled rhizopod belonging to the group *Imperforata*, family *Arcellina*, as defined by Bütschli. The shell is composed of chitinous material, and varies in colour from almost purely colourless transparency to a deep brown. In surface view the shell is seen to be composed of very small hexagonal elements packed closely together. The normal *Arcella* shell is circular in outline, with an approximately plane ventral surface, containing in its centre the circular "mouth" opening. The form of the shell is shown in side view in Fig. 1.

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[*] For an historical account of the discussion of the phenomenon of correlation by earlier workers cf. Radl, E., "Ueber die Bedeutung des Prinzips von der Korrelation in der Biologie." *Biol. Centralbl.*, Bd. xxi. pp. 401–416, 491–497, 550–560, 585–591, 605–621, 1901.

[†] Leidy, J., "Freshwater Rhizopods of North America." *Rep. U. S. Geol. Surv. of the Territories* (Hayden). Vol. xii. 1879, pp. 166–173. Plate XXVII.

[‡] Bütschli, O., "Protozoa," in Bronn's *Klassen und Ordnungen des Thierreiches.* Bd. i. 1 Abth. 1880–82.

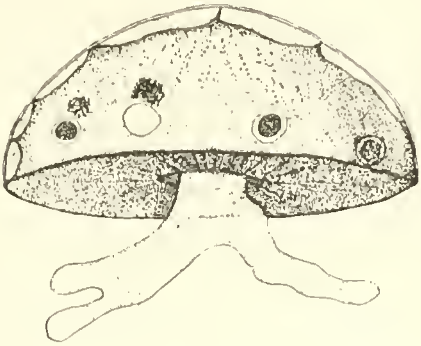

FIG. 1.

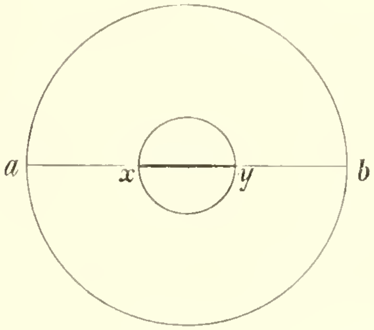

FIG. 2.

The points studied in the work were: (i) the variation of the diameter of the shell (Fig. 2, *ab*), (ii) the variation in the diameter of the "mouth" opening (Fig. 2, *xy*), (iii) the variation in colour, and (iv) the correlation between (i) and (ii), and (i) and (iii). For verbal convenience we shall refer throughout the paper to character (i) (the total diameter of the shell) as the *outer* diameter, and to character (ii) (the diameter of the "mouth" opening) as the *inner* diameter.

The measurements of the outer and inner diameters were made in the following way. At the beginning of the taking of the series of measurements a compound microscope with a camera lucida attached was set up in a certain place in the laboratory and adjusted so that both microscope and camera gave optically their best results. The arrangement was then left undisturbed*. The amount of magnification obtained by the use of microscope and camera with the paper at the level of the table was then very carefully measured several times and the mean of all these observations used as the true value of the magnification.

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* Except once during the course of the measuring, when it became necessary to use the microscope elsewhere. When replaced it was adjusted so as to be in the same condition as before the removal.

The *Arcellae* were then taken with a pipette from the culture dish and placed a few at a time on a slide in water, and covered with a cover-slip. Then with a sharp needle the points marking the limits of the maximum diameter of the shell and of the "mouth" opening as seen in the camera image were pricked on a card of smooth drawing-paper laid on the table under the mirror of the camera. After the size of all the *Arcellae* on a slide had been pricked in this way, the slide was cleaned off and a fresh lot of individuals taken from the culture.

In this way cards were obtained bearing the dimensions magnified approximately 400 times, and marked by needle pricks. These dimensions on the cards were then measured by means of a sliding arm calliper, bearing a vernier reading to fiftieths of a millimeter. The measurements so obtained were divided by the magnification of the microscope-camera combination and reduced to mikrons. The measurements in mikrons were then permanently recorded.

From a series of test measurements (50 in number) of distances of known size (spaces of a net micrometer) made by this method we were led to the conclusion that microscopical measurements made in this way were very accurate. In fact we are inclined to believe that microscopical measurements may be made more accurately and, considering the accuracy, more expeditiously, than in any other way except by the use of a filar ocular micrometer, which unfortunately was not available for this work.

A point which needs consideration in connection with the method of making measurements is the question of symmetry of the shells. While it is known as a matter of common observation that *Arcella* shells are in general circular in outline, it is not so certain whether they are all exactly circular. We found in examining a large number of shells that there are a considerable number which are not exactly circular but are slightly ovate in outline. With practice an observer becomes very skilful in detecting any deviation from the circular outline, and in taking our measurements whenever it was suspected that any such deviation was present in a shell two pairs of diameters (inner and outer) at right angles to one another were measured instead of a single pair. These diameters were so taken as to be, as nearly as one could judge, the maximum and minimum diameters. Then the average of the two was taken as representing what the diameter of this shell would be were it symmetrical. All these deviations observed were small—in most cases extremely small. No strikingly asymmetrical shells were found in the course of this work.

In taking the individuals for measurement there was no selective choice exercised. Some water was drawn up from the culture in a pipette and dropped on a slide. If there were any *Arcellae* in the water they were all measured. If none were found a new sample of water was taken. The sample of the population measured was a "random sample."

In order to test quantitatively the generally accepted view that there is a considerable degree of correlation between darkness of colour and size in *Arcella* shells it was desired to collect statistical data on the colour of the shells. The problem of recording definitely the colour of such an organism as *Arcella* is a very difficult one. After trying and abandoning several methods of recording colour we finally decided upon the following as sufficiently accurate for our purpose. In the work of Leidy already referred to, Plate XXVII contains a number of figures of colour varieties of *Arcella*. These figures are extremely well chosen to typify the different colour classes, ranging all the way from colourless transparency to a very deep brown. For our colour determinations we chose four of these figures on this Plate as types with which to compare the colour of the *Arcellae* observed. As typical of the lightest shells we took Fig. 14, and called this our colour type *A*. The colour of this figure on the Plate is a very light yellow-brown. It very well represents the colour of the lightest shells we found. For the next darker type we chose Fig. 7, and designated this as colour type *B*. Shells falling within this type have a distinct yellow-brown colour, and yet are still fairly transparent. Fig. 19 served as our colour type *C*. Shells falling in this colour class may be roughly described as of medium-brown colour, with a sensible though small admixture of yellow. As representative of the darkest individuals, our colour type *D*, we chose Leidy's Fig. 2. This is the colour most frequently observed in *Arcella* shells, and is a dark, even brown.

As the *Arcella* shells were measured their colour was compared with these figures and each shell was recorded as *A*, *B*, *C* or *D*, according to the figure which it most nearly matched. It was found in practice that the colour of the shells could be more accurately classified in this way than by comparing with sheets of coloured paper for example, because the thing with which the colour was compared in this method was an actual representation of an *Arcella*. While the method gives us no quantitative determination of the amount of pigmentation it does furnish a reasonably good classification of *Arcella* shells by colour, which can be used for purposes of correlation. This was all we hoped or desired to get in the present work.

The 504 individuals of which measurements were made were all taken from a single small laboratory culture dish. The material was collected from a small lake in the vicinity of Ann Arbor, and the few *Arcellae* originally in the collection multiplied rapidly under the conditions of the laboratory culture*. At the time when the measurements were begun the *Arcellae* were very abundant. Since the individuals were all taken from one small culture dish they may be considered a very homogeneous sample of this *Arcellae* population, as far as environmental conditions are concerned. The sample is not, however, quite homogeneous with respect to age, for the following reasons. Pressure of other work made it necessary for us to extend the time of making measurements over a period of about three weeks. It is likely that some of the last individuals to be measured represented later generations than those to which those measured earlier belonged. That this was the case is indicated by the form of the frequency polygons obtained. It will be shown farther on in the paper, however, that there was on the whole no sensible increase in size of the shells during the time which the making of the measurements covered.

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* [Since the original *Arcellae* were *few* in number, it should be noted that the authors are dealing rather with a few large "fraternities" than a general population of *Arcellae*; such, however, is probably the constitution of most "cultures." ED.]

What we have then as material for this biometrical study is a random sample of the individuals of a population of *Arcella,* covering a period of about three weeks in the life of the culture. It may be considered that we have a section three weeks long in the cultural history, and have investigated the conditions of the individual *Arcellae* falling within that section.

III. RESULTS.

A. *Variation in Outer and Inner Diameters.*

The results of the measurements of the outer and inner diameters of 504 individuals are shown in Tables I and II.

TABLE I.

Outer Diameter. Class unit = 2 mikrons.

Diameter of shell in mikrons	Frequency	Diameter of shell in mikrons	Frequency
42—44	1	*70—72*	2
44—46	2	*72—74*	2
46—48	—	*74—76*	3
48—50	5	*76—78*	3
50—52	22	*78—80*	4
52—54	147	*80—82*	2
54—56	210	*82—84*	—
56—58	70	*84—86*	1
58—60	13	*86—88*	1
60—62	1	*88—90*	1
62—64	5	*90—92*	—
64—66	1	*92—94*	1
66—68	3	*94—96*	1
68—70	3		
		Total ...	504

TABLE II.

Inner Diameter. Class unit = 1 mikron.

Diameter of shell in mikrons	Frequency	Diameter of shell in mikrons	Frequency
11—12	4	*23—24*	3
12—13	5	*24—25*	2
13—14	25	*25—26*	2
14—15	101	*26—27*	1
15—16	189	*27—28*	—
16—17	120	*28—29*	—
17—18	29	*29—30*	1
18—19	4	*30—39*	—
19—20	7		
20—21	3	*39—40*	1
21—22	4		
22—23	3	Total ...	504

For these frequency distributions the following constants have been determined: mean, median, mode, standard deviation, coefficient of variation, together with the probable errors of the three most important constants, the mean, the standard deviation and the coefficient of variation. Only an approximate value of the mode was obtained, the method used being that described by Yule*, wherein an approximation to the value of the mode is determined from the relation: Mode = Mean − 3 × (Mean − Median)†. In determining the value of the standard deviation from the equation, $\sigma = \sqrt{\mu_2}$, Sheppard's correction was used in obtaining μ_2. The formula used in determining the probable error of the coefficient of variation was that given by Macdonell‡ after Pearson, viz., P. E. of $v = \pm \cdot 6745 \frac{v}{\sqrt{2n}} \left[1 + 2\left(\frac{v}{100}\right)^2\right]^{\frac{1}{2}}$ where v is the coefficient of variation, and n is the number of individuals.

The values of these constants for the frequency distributions of variation in the outer and inner diameters are given in Table III.

TABLE III.

Constants for Size Variations in 504 Individuals of Arcella vulgaris.

	Outer diameter	Inner diameter
Mean	55·7897 ± ·1721 mikrons	15·9107 ± ·0653 mikrons
Mode	52·5635 ,,	15·0357 ,,
Median	54·7143 ,,	15·6190 ,,
Standard Deviation ...	5·7283 ± ·1217 ,,	2·1731 ± ·0462 ,,
Coefficient of Variation	10·2676 ± ·2204 °/₀	13·6578 ± ·2955 °/₀

Discussion of Results.

There are several points brought out by these constants regarding variation in *Arcella*, which seem to need some further discussion.

Taking first the constants defining the type of the species with respect to the characters under consideration, it is seen that the mean and the mode are separated by an appreciable distance, or in other words the distribution is skew. In the case of the outer diameter this distance amounts to 3·226 mikrons, or 1·61 abscissal units. In the case of the inner diameter the distance between mean and mode is ·875 mikrons, or approximately ·88 of the abscissal unit for this distribution. From these absolute values of the differences it would appear that the distribution for the outer diameter was markedly more skew than the distribution for the inner diameter. It is to be remembered however that the means and class units in the two cases are different. If we compare, instead of the absolute, the relative differences, by taking the percentages of the differences between mean and mode in the respective means, we reach the following results: the percentage difference between the mean outer diameter and the modal outer diameter in the mean outer diameter is 5·78 %. The percentage difference between the mean inner diameter and the modal inner diameter in the mean inner diameter is 5·49 %. These relative values are in close accord and lead to the same conclusion regarding the relative skewness of the two curves as that reached by mere inspection, namely, that the two distributions are very closely alike in their form. We would suggest this method just described of taking the percentage difference between the mean and the mode as a useful rough method of comparing the relative skewness of frequency distributions, in cases where it is not feasible to compute the skewness exactly from the moments*.

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* Yule, G. U., "Supplementary Note on the Determination of the Mode." *Jour. Roy. Stat. Soc.* Vol. LIX. p. 398.

† *Phil. Trans.* Vol. 186, A, p. 375.

‡ *Biometrika*, Vol. I. p. 189.

From the mean values for the two diameters it is seen that the whole diameter of the shell (outer) is on the average approximately 3·5 times the diameter of the "mouth" opening (inner). More exact relations between these two dimensions will be given later in the paper in the form of regression equations.

The coefficients of variation show clearly that the inner diameter is distinctly more variable than the outer. This relation is shown in Table IV.

TABLE IV.

Coefficients of Variation. Outer and Inner Diameters.

	Coefficient	Probable error of coefficient
Inner diameter	13·6578	·2955
Outer "	10·2676	·2204
Difference	3·3902	—
Probable error of difference	—	·3686

The difference is roughly ten times its probable error and hence may be considered significant.

These two frequency distributions furnish a good example of a case where the absolute concentration of variates given by the standard deviation cannot be used for the comparison of the relative variability of two characters in which the means are widely different. According to the standard deviations in the case under consideration the outer diameter would appear to be twice as variable as

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[* Skewness has been defined as the ratio of the distance between mean and mode to the standard-deviation; we find for it in the above cases: ·5632 for outer, ·4027 for inner diameter distributions. These are considerable amounts and readily comparable with the values found in other cases. Ed.]

the inner, and in fact is, so far as absolute size of deviation is concerned. Taking the size of the deviations in relation to the respective means, however, as has been seen, we find the inner diameter to be significantly more variable than the outer.

It is of interest to compare the variability of *Arcella* with that of *Paramœcium* as determined by Simpson*. In his series of 100 *Paramœcia* the coefficient of variation for length was 8·361, and for breadth 13·439. The value of this coefficient for length is less than either of the coefficients for the characters of *Arcella* studied. The value of the coefficient of variation for the breadth of *Paramœcium* (13·439) is very nearly equal to that for the inner diameter of *Arcella* (13·658). Both the coefficients for *Paramœcium* are values of the same general order of magnitude as the coefficients for *Arcella*. From a comparative standpoint this result is of considerable interest. It is highly desirable that quantitative studies of the variation in other *Protozoa* reproducing primarily by the method of fission be made, in order to determine whether a value of approximately 10 °/₀ for the coefficient of variation is characteristic of variation in individuals produced by binary fission. The finding of such closely accordant values for the variation in two such widely different protozoan groups as the rhizopods and the holotrichous ciliates would seem to be of some significance. The senior author of this paper hopes to carry on some further investigations bearing on this problem, using other species of *Protozoa*.

It may again be noted in passing that in the sample of an *Arcella* population here studied we have a random general sample of a population produced by fission, the individuals not being chosen with reference to their immediate ancestry, as was the case in Simpson's work. It is interesting to find such close agreement in the values for the variation in samples collected in these different ways.

It is of some interest to compare the size of the *Arcellae* found in this study of a single culture with the values given by Leidy from the results of his studies of *Arcellae* from all parts of the country. Leidy says†: "The shell of *Arcella vulgaris* ranges from $\frac{1}{520}$ to $\frac{1}{165}$ of an inch in breadth, $\frac{1}{690}$ to $\frac{1}{350}$ of an inch in height, with the mouth $\frac{1}{2000}$ to $\frac{1}{520}$ of an inch in breadth, and elevated $\frac{1}{4100}$ to $\frac{1}{1500}$ of an inch." Reducing the values for "breadth of shell" (outer diameter), and "breadth of mouth" (inner diameter) to mikrons and comparing with our data, we get the following results.

Leidy's data { Range of breadth of shell: ca. 49—152 mikrons.
„ „ opening: ca. 12·7—49 mikrons.

Our data { Range of outer diameter: ca. 42—96 mikrons.
Range of inner „ : ca. 11—40 „ .

(With only one individual exceeding 30 mikrons in inner diameter.)

It is seen from this that the range observed by Leidy extends beyond ours only at the upper end.

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* *Biometrika*, Vol. I. p. 405.

† *Loc. cit.* p. 173.

B. *Variation in Colour.*

The colour determinations were made in the way already described (p. 325) for 371 individuals. The frequency distribution for colour is given in Table V.

TABLE V.

Frequency Distribution of Colour.

Colour class	Frequency
A	24
B	75
C	75
D	197
Total ...	371

Class *A* includes individuals of the lightest colour; class *D* the darkest individuals, and classes *B* and *C* intermediate gradations. As was expected the largest number of individuals fell in class *D*. A majority of all the individuals fall within this darkest colour class. The number of lighter coloured individuals is, however, larger than probably would have been predicted from general observation.

The correlation between colour and size will be discussed in a later section.

C. *Correlation.*

The correlations principally studied were those existing between the outer and inner diameters, and between the outer diameter and colour.

1. *Correlation between outer and inner diameters.*

The correlation table showing the relation between the total diameter of the shell and the diameter of the opening is exhibited in Table VI.

The value of the coefficient of correlation deduced from this table by the usual formula

$$r = \frac{S(xy)}{n\sigma_1\sigma_2}$$

is

$$r = \cdot 836 \pm \cdot 007.$$

A correlation coefficient of this magnitude of course indicates a high degree of correlation between the outer and inner diameters in *Arcella*. The value is considerably greater than that obtained by Simpson* for the correlation between the length and breadth of individual *Paramœcia*. His value for r in this case is

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* *Loc. cit.* p. 403.

TABLE VI.

Correlation between Outer and Inner Diameters in Arcella.

Outer. Measurements in Mikrons.

Inner.	42—44	44—46	46—48	48—50	50—52	52—54	54—56	56—58	58—60	60—62	62—64	64—66	66—68	68—70	70—72	72—74	74—76	76—78	78—80	80—82	82—84	84—86	86—88	88—90	90—92	92—94	94—96	Totals
11—12	1	2	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
12—13	—	—	—	1	2	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
13—14	—	—	—	2	5	13	4	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	25
14—15	—	—	—	1	8	38	42	12	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	101
15—16	—	—	—	1	5	68	92	20	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	189
16—17	—	—	—	—	1	23	60	30	3	—	1	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	120
17—18	—	—	—	—	1	2	12	7	4	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1	—	29
18—19	—	—	—	—	—	—	—	—	2	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	4
19—20	—	—	—	—	—	—	—	—	1	—	3	—	—	1	1	1	—	—	—	—	—	—	—	—	—	—	—	7
20—21	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	1	—	—	—	—	—	—	—	—	3
21—22	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1	—	2	—	—	—	—	—	—	—	—	—	—	4
22—23	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	2	—	—	—	—	—	—	—	—	3
23—24	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	3
24—25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	2
25—26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	—	—	2
26—27	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
27—28	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
28—29	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
29—30	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
30—39	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•
39—40	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	1	2	—	5	22	147	210	70	13	1	5	1	3	3	2	2	3	3	4	2	—	1	1	1	—	1	1	504

·421 ± ·055. The higher value for the correlation coefficient of the characters studied in the *Arcella* shell over the value for length and breadth of *Paramœcium* is what would perhaps be expected from general considerations regarding the two cases. The shell of *Arcella* is a very definitely formed, firm structure, which after its production is not easily changed in its shape by environmental changes. On the other hand the body of a *Paramœcium* is composed of soft, yielding protoplasm, bounded externally only by a thin, flexible pellicle. It is well known to every biologist that changes of form in the body of *Paramœcium* are very easily and quickly produced by changes in the surrounding conditions. In this event it would not be expected that there would be so close and definite a relationship between the dimensions of length and breadth in this form as exists between the two diameters of the firm, unyielding, and definitely formed shell of *Arcella**.

Passing now to the regression equations, we get from the usual equations for the regressions†

$$b_1 = r\frac{\sigma_1}{\sigma_2}$$

and

$$b_2 = r\frac{\sigma_2}{\sigma_1},$$

by substituting the values already obtained for r, σ_1, and σ_2, the following values for the regression coefficients between outer and inner diameters of *Arcella* shells,

$b_1 = 0{\cdot}317$ (regression of inner on outer diameter),

$b_2 = 2{\cdot}204$ („ „ outer „ inner „).

To express in words the significance of these equations we may say that: (1) the mean increase in the diameter of the opening in *Arcella* shells for every increase of 1 mikron in the whole diameter of the shell is ·317 mikrons; and (2) the mean increase in the total diameter of *Arcella* shells for every increase of 1 mikron in the opening is 2·204 mikrons.

The regression lines referred to the means as origin have the form:

$$x = {\cdot}317\,y,$$

$$y = 2{\cdot}204\,x.$$

In these equations x stands for deviations from the mean inner diameter, and y for deviations from the mean outer diameter. Reducing these to actual values instead of deviations by the method described by Yule‡, and letting D_i denote the probable size in mikrons of the *inner diameter* and D_o the probable size in mikrons of the *outer diameter*, we get for the regression equations:

---

* We may also expect a difference from Simpson's value, because in the first place our correlation has not been corrected for growth, and in the second place two breadths expressing generally *size* are *a priori* more likely to be highly correlated than a length and breadth which also involve the factor of *shape*. Thus the length and breadth of the human skull for most races are less highly correlated than two breadths such as the frontal and bizygomatic breadths.

† Yule, G. U., "On the Theory of Correlation." *Jour. Roy. Stat. Soc.* Vol. LX. 1897, p. 818.

‡ *Loc. cit.* p. 827.

$$D_i - 15{\cdot}9107 = {\cdot}317\,(O - 55{\cdot}7897), \quad \text{or } D_i = {\cdot}317\,O - 1{\cdot}775 \;\ldots\ldots\ldots\text{(i)},$$

$$D_o - 55{\cdot}7897 = 2{\cdot}204\,(I - 15{\cdot}9107), \quad \text{or } D_o = 20{\cdot}723 + 2{\cdot}204\,I \;\ldots\ldots\text{(ii)},$$

where O stands for the observed outer, and I for the observed inner diameter.

The probable error in determining the value of the inner diameter from that of the outer by equation (i) is given by the expression $\pm {\cdot}6745\,\sigma_1\sqrt{1-r^2}$. In the present case this expression equals ± ·804 mikrons.

The probable error in determining the outer diameter from the inner by equation (ii) is 2·120 mikrons. The expression for this probable error is the same as that for equation (i) except that σ_2 is written instead of σ_1.

2. *Correlation between outer diameter and colour.*

The methods used in measuring and classifying the colour of *Arcella* shells have been fully described in another section. It will be recalled that the shells were divided into four colour classes, designated *A*, *B*, *C*, and *D*. Class *A* includes the shells lightest in colour and *D* those darkest.

In Table VII. is shown the frequency of the occurrence of the different combinations of colour and size of shells.

TABLE VII.

Correlation between Colour and Outer Diameter in Arcella.

Outer diameter in mikrons.

Colour Class	42—44	44—46	46—48	48—50	50—52	52—54	54—56	56—58	58—60	60—62	62—64	64—66	66—68	68—70	70—72	72—74	74—76	76—78	78—80	80—82	82—84	84—86	86—88	88—90	90—92	92—94	94—96	Totals
A	1	—	—	1	2	6	7	5	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	24
B	—	—	—	2	6	18	33	8	3	—	—	1	—	2	—	—	—	—	—	—	—	—	1	—	—	1	—	75
C	—	1	—	—	4	22	34	9	1	—	3	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	75
D	—	—	—	2	8	62	82	22	3	1	2	—	1	—	1	2	2	2	4	—	—	1	—	1	—	—	1	197
Totals	1	1	—	5	20	108	156	44	7	1	5	1	3	2	2	2	2	2	4	—	—	1	1	1	—	1	1	371

The colour classes are not quantitatively defined in this table, but they may be considered each as including a definite though unknown portion of the colour scale of some one pigment in the *Arcella* shell. The intensity of this colour may also be considered to increase regularly and gradually from the lower limit of class *A* to the upper limit of class *D*.

The method we have used in obtaining the coefficient of correlation between colour and diameter of shell is that described by Pearson*.

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* Pearson, K., *Phil. Trans.*, Vol. 195, A, pp. 1–47.

A fourfold table was formed from Table VII. and is exhibited in Table VIII. The divisions were taken between the outer diameter classes centering at 55 mikrons and 57 mikrons, and between the colour classes C and D. These seemed on the whole the best divisions to choose. They lie near the medians and satisfy the requirements of making $a+c>b+d$ and of also making $ad>bc$, although they do not make $a+b>c+d$. It is impossible to fulfil this condition without making our horizontal division somewhere within colour class D. As a consequence of $c+d$ being greater than $a+b$, k is, of course, negative.

TABLE VIII.

Fourfold Correlation between Colour and Outer Diameter.

Diameter:	Under 56	56 and over	Totals
Colour: Under Class D	(a) 137	(b) 37	$(a+b)$ 174
Class D	(c) 154	(d) 43	$(c+d)$ 197
Totals	$(a+c)$ 291	$(b+d)$ 80	(N) 371

$$h = \cdot 78704,$$
$$k = -\cdot 07778.$$

The equation in r is

$$\cdot 018179\, r^4 - \cdot 063044\, r^3 + \cdot 030608\, r^2 - r + \cdot 012046 = 0.$$

Solving for r by Horner's method, we get

$$r = \cdot 01205.$$

The probable error of this very low coefficient with such a comparatively small number of observations is of course large as compared with the coefficient itself. The value of the probable error in case this r had been computed by the product-moment method would be approximately ± ·035. The value of the probable error for the method here used has not been computed, but it is of course still larger.

Making all due allowance for the size of the probable error it is clearly seen that there is an extremely small degree of correlation between colour and size in the shell of *Arcella* within the limits of size in the series here discussed. It may even be said that practically there is no correlation between these two characters.

To find that there is practically no correlation between darkness of colour and size in *Arcella* shells is an unexpected and rather interesting result, and is apparently contrary to the opinion generally held among zoologists regarding the matter. This result seems to definitely negative any view which would maintain that the *Arcella* shell grows darker with its *cultural* age. Or, to express it in another way, this very low correlation coefficient between colour and size seems to show that we should not expect the $(n+1)$th or $(n+2)$th generation of *Arcella* to be on the average darker in shell colour than the nth. This statement of course assumes what observation clearly warrants, that the shells of the $(n+1)$th and $(n+2)$th generations will be on the average larger than the nth generation shells. It must be repeated, however, that it can only be affirmed that this lack of correlation between colour and size holds for the generations falling within the series of observations here reported. One of the points which impresses one most strongly in following through the history of an *Arcella* culture is that the very minute shells which appear in numbers in the very earliest part of the cultural development are all very light in colour, falling within our colour class A. These extremely small individuals are probably developed from spores and form their shells individually entirely *de novo*. The existence of these small, universally light coloured shells early in the course of the development of the culture would indicate that our result of no correlation between colour and size would not hold for the first few generations formed in the culture.

On the further question as to whether the individual shell changes in colour during the course of its existence, we have at present no evidence to offer.

3. *The rate of change of the average size of shell with the age of the culture.*

As has been stated earlier in the paper the measurements extended over a period of approximately three weeks. Between the time when the first and the last measurements were taken there was evidently an opportunity for the average size of the individuals in the culture to change. The culture was in a flourishing condition during this period, and it seemed a matter of some interest to determine how much of an increase in the average size of the individuals occurred.

The cards on which the measurements were recorded were numbered consecutively in the order of measuring, the card bearing the record of the first individual measured being numbered 1, the next 2, and so on up to the last individual. This consecutive numbering furnishes a method of approximating to the desired determination. The cards recording the measurements of the first 500 individuals were arranged in 10 classes of 50 individuals each. The first class contained the first 50 *Arcellae* measured, the next class the second 50 and so on to the tenth class, which contained the last 50 individuals. Then the cards in each class were arranged into arrays with reference to outer diameter. In this way was formed a double-entry table with position in the order of measurement as the subject character and the outer diameter as the relative character. The mean outer diameter for each of the 10 "age" arrays was then computed. The results are shown in Table IX.

TABLE IX.

Mean Outer Diameters of Arcellae at Successive Stages in the Development of the Culture.

Order of Measurement			Mean Outer Diameter in mikrons
First	50	...	55·48
Second	50	...	56·20
Third	,,	...	55·56
Fourth	,,	...	55·48
Fifth	,,	...	55·44
Sixth	,,	...	56·20
Seventh	,,	...	57·46
Eighth	,,	...	55·20
Ninth	,,	...	54·52
Tenth	,,	...	56·18

It will be seen at once that there is very little definite shift of the mean towards a higher value in the higher "age" classes. The straight line best fitting these means is almost exactly vertical. To get a numerical idea of the amount of the shift, the average of the five means of the first 250 individuals may be compared with the average of the five means of the 250 individuals last measured. The mean diameter for the first 250 individuals measured is 55·632 mikrons; for the last 250 it is 55·912 mikrons. The difference between these values is ·280 mikron, which is evidently a practically insensible difference.

It may be concluded that in a period of time of approximately three weeks the average size of *Arcella* shells in a culture, as estimated by random samples of 50, does not sensibly increase.

IV. SUMMARY.

The present paper comprises the results of a statistical study of variation and correlation in the shell of the common rhizopod, *Arcella vulgaris.*

The points especially investigated were: (i) the variation in the total diameter of the shell, (ii) the variation in the opening of the shell, (iii) the variation in the colour of the shell, (iv) the correlation between the total diameter of the shell and the diameter of the opening, (v) the correlation between the colour and the total diameter of the shell. The frequency distributions and the constants deduced from them, and the correlation tables and coefficients for the various characters are presented.

The results and conclusions may be summarized as follows:

1. Both the diameter of the shell and of the opening show a reasonably high variability. The frequency distributions for both characters are unimodal and markedly skew.

2. The opening in the shell is decidedly more variable in size than is the whole shell, size in both cases being indicated by the respective diameters.

3. The coefficients of variation for the two diameters (opening and total shell) present values in very fair accord with those obtained by Simpson for size characters in *Paramœcium*, showing that these organisms are approximately equally variable in corresponding characters.

4. There is a high degree of positive correlation (coefficient = ·84) between the diameter of the shell and of the opening.

5. Regression and characteristic equations for determining the mean total diameter of the shell associated with a given diameter of the opening and *vice versâ*, are given.

6. There is found to be an extremely small degree of correlation (coefficient = ·01) between the size of the shell and its colour within the size limits of the series of individuals discussed. The correlation is so small as to be practically insensible.

7. There is no sensible change in the average size of the shells as indicated by random samples, within the period of the life of the culture studied. This period covered approximately three weeks in time.

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Explanatory.

THE conclusions drawn by anatomists and anthropologists from examinations of series of crania are at present based upon two types of judgment. The first type of judgment is what we may term the method of appreciation; the anatomist examines the cranial series and by a process of general appreciation classes the series as belonging to a certain race, or throws out certain individuals as belonging to a different race. Sometimes the appreciative inspection will lead the craniologist to classify a series in this way into five or six types or races,—two or three pure types and two or three mixed types. The bases of these appreciative judgments are qualitative, they cannot be criticised by the biometrician, they belong to the *arcana* of the anatomist's training. The biometrician can only place alongside such judgments the results of quantitative investigations, which are deduced by difficult and often very laborious statistical theory. The statistical method in craniology is difficult and laborious because the series dealt with are usually so small that conclusions can only be drawn as the balance of a very fine weighing of probable errors, and accordingly the statistician can frequently only say: "This I think is the balance of probabilities." He is compelled to stand in wonder and admiration before the definite facts which the method of appreciation appears to elicit.

Now qualitative appreciation does not admit of exact statement of the wherefore of the judgment in print; accordingly anatomical craniologists have been in the habit of publishing measurements and dealing with a few statistical arguments based on means or graphical polygons of frequency or even rough tables of correlation. Here the biometrician is at liberty to step in and express a perfectly definite judgment on the statistical processes adopted and the conclusions deduced. In nine cases out of ten he can confidently assert that the anatomical appreciation is only obscured by the statistical arguments used in its favour. The data are either insufficient for any statistical conclusion whatever; or they are unaccompanied by any determination of their probable errors on which alone a judgment could often be based; or the very *principia* of the theory of statistics are clearly unknown to the handlers of the data. It is idle to deny the obvious fact that the numerical arguments used by such authorities as

Virchow, His, Topinard, Broca, Davis, Flower, and a multitude of others, are hopelessly inadequate to maintain the structure reared on them. No valid arguments at all as to race or type can be based on such statistics, and the biometrician can assert this without in the least criticising the appreciative judgment of these great anatomical authorities.

The object of the present series of *Craniological Notes* is to bring home to craniologists the need for the revision of their statistical methods. They will therefore deal principally with recent craniological work, and if they criticise it occasionally with vigour, it will be with the sharpness of the surgeon's knife, which is handled in the real interests of the patient. Our criticism will not be purely negative, but reconstructive.

I. *Professor Aurel von Török's attack on the Arithmetical Mean.*

By Karl Pearson, F.R.S.

A very voluminous craniological memoir has recently been issued by Professor Aurel v. Török assisted by Herr Gabriel v. László. It is entitled:

Ueber das gegenseitige Verhalten der kleinsten und grössten Stirnbreite so wie der kleinsten und grössten Hirnschädelbreite bei Variationen der menschlichen Schädelform. "Zeitschrift für Morphologie und Anthropologie," Band IV. S. 500—88.

This memoir consists of two parts, first a vigorous criticism of the use of the arithmetic mean in craniology, and secondly an investigation of the correlation which exists between certain cranial breadths. The data with which the authors deal are very ample, consisting of 2000 skulls "aus älteren und jüngeren Friedhöfen Ungarns." Presumably the crania were adult and of one sex. This first note deals only with Professor v. Török's attack on the use of the arithmetic mean.

Professor v. Török's attack is of the following character. He says that under the expression "type" can only be understood what is characteristic of the totality of any class of things, something which enables us at once to distinguish them from other things:

> "So kann...die Bestimmung eines Typus in gar nichts anderem bestehen, als dass man unter dem Mehrerlei der Einzelmerkmale dasjenige herausgreift, was in der grossen Ueberzahl anzutreffen ist. Dies ist doch einfach und klar. Nun muss die Frage gestellt werden: kann durch eine arithmetische Mittelzahl das Charakteristische, d. h. das in der grossen Ueberzahl Vorkommende ausgedrückt werden? Dies wäre nur unter der einzigen Bedingung möglich, wenn auf die Werthgrösse der arithmetischen Mittelzahl die überwiegende Mehrheit der Einzelfälle fallen würde. Entweder, oder. Nun soll man eine solche arithmetische Mittelzahl, welche die überwiegende Mehrheit der Einzelfälle in sich vereinigt, doch endlich einmal auch aufzeigen! Es wurde bisher tausend- und tausendmal der Typus mittels der arithmetischen Mittelzahl schon als bestimmt angegeben, aber es soll noch derjenige Anthropologe ausfindig gemacht werden, der sagen könnte, dass er mit seiner arithmetischen Mittelzahl auch wirklich die Ueberzahl der Einzelfälle zum Ausdruck gebracht hat" (p. 511).

Professor v. Török here clearly considers that the type is something which differentiates an *individual* of one population from an *individual* of a second population. As a matter of fact the arithmetic mean may, but does not always, enable us to distinguish one *race* or population from a second *race* or population. Any such population or race is only *fully* defined by a number of statistical constants, means, modes, variabilities, correlations, etc. and will only be distinguishable from a second race provided its constants one or more or all of them differ by quantities sensibly greater than the probable errors of random sampling from those of the second race. A knowledge of the arithmetic means only would never enable one to say of an individual skull that it belonged to one race and *not* to another. A knowledge of the *variabilities and correlations as well* of these races might enable us to state *the degree of probability* that the skull belonged to one race rather than to the other.

If it is necessary for Professor v. Török at this epoch in the history of craniology to define the "type" as that which is the rule, and to demonstrate against the great body of craniologists that the arithmetic mean does not define the type, this science must indeed be in a parlous condition! One can hardly grasp that this can be the state of affairs in any branch of knowledge, or that its workers have never heard of Quetelet, Galton, or even Stieda!

But Professor v. Török perceives that he cannot leave the matter here. He states that the arithmetical mean according to the doctrine of Gauss is that value of the character which most frequently repeats itself in the population, i.e. it coincides with what biometricians are accustomed to term the *mode*. Our authors, merely taking the rough frequencies of their 2000 skulls, show that in three cases out of four the group which corresponds to the arithmetic mean falls short of other groups of frequency in its neighbourhood, and accordingly argue that the arithmetic mean is idle:

> Wie wir sehen, trifft der Fall der häufigsten Wiederholung (Vertretung) der Werthgrösse der arithmetischen Mittelzahl bei den vier Variationsreihen nur ein einziges Mal ein. In der Variationsreihe der grössten Hirnschädelbreite wird sie sogar durch vier andere Einzelwerthe übertroffen. Man frage sich doch angesichts dieser Thatsache, was man Sicheres auf den Beweis einer arithmetischen Mittelzahl behufs unserer Typusforschungen bauen kann? Und wenn man ohne Voreingenommenheit die Sache so wie sie ist beurtheilt, muss man denn nicht zur Einsicht gelangen; dass eben, weil ihre Werthgrösse nie die absolute Mehrheit und auch nicht immer die relative grösste Vertretung aufweist, wir genöthigt sind behufs Aufstellung eines Typus ausser ihr unbedingt noch andere Werthgrössen des betreffenden Maasses in Betracht zu ziehen! (S. 517).

Of course other constants must be dealt with in addition to completely describe a given population. *Only* Professor v. Török's attack fails absolutely, because he has neglected to deal with the variations in his frequency groups due to random sampling, and these as we will show later are such that there is no really *significant* difference between the actual modes,—not his apparent modes (*Scheingipfeln*),—and his means.

Lastly, Professor v. Török says that the position of the arithmetical mean in the whole group of variation must be considered.

> Wie schon der Ausdruck "Mittelzahl" andeutet, erweckt der Begriff einer solcher Zahl in uns die Vorstellung, dass ihre Werthgrösse in der Mitte der betreffenden Variationsreihe (Zahlreihe) steht. Thut sie aber dies? Wenn dies der Fall wäre, so böte sie uns innerhalb der so vielen Veränderlichkeiten unserer kraniometrischen Zahlreihen wenigstens einen fixen Punkt, wo man das Problem einer Typusbestimmung anheben könnte (S. 517).

In other words, to use the biometric term, Professor v. Török questions whether the mean gives the median. By the simple process of taking the characters of his first and last groups of variants adding and halving them, Professor v. Török shows that the arithmetic mean does not give the median! There is not the least attempt to obtain a scientifically accurate median, nor to ascertain whether the difference between that and the mean is a sensible difference or not. The writer has clearly not the most elementary conception of the theory of statistics, nor even a merely physical appreciation of the effects of random sampling, nor again of differences due to different methods of grouping. The whole problem of graduating raw data is a sealed book to him! Yet here are his final conclusions,—sweeping for craniology, revolutionary, indeed, for all science!

> Auch nach den hier vorgetragenen Thatsachen, muss ein jeder selbständig denkender Forscher die Verfehltheit aus der arithmetischen Mittelzahl, einen Typus herausspeculiren zu wollen, doch einsehen ... Es erwächst nun eine unabweisliche Pflicht, und zwar in erster Reihe für die tonangebenden Autoritäten der heutigen Anthropologie: entweder, wenn möglich, ihr bisheriges Verfahren klar und einwurfsfrei zu rechtfertigen; oder, wenn dies ihnen nicht gelingt, mit gutem Beispiel voranzugehen und die Typusfrage von diesem alten Irrthume endlich einmal zu befreien. Ein weiteres Stillschweigen ist gewiss nicht mehr motivirt, die Angelegenheit muss doch einmal geschlichtet werden. Der Kampf um die Wahrheit ist das einzig berechtigte *punctum saliens* für wissenschaftliche Discussionen, neben welchem alle übrigen persönlichen Rücksichten verstummen müssen! (S. 520).

Let us now see exactly in what Professor v. Török's criticism consists. It contains three separate charges against the arithmetic mean:

(i) It does not give the rule, something characteristic of every member of a group termed by craniologists the type.

Obviously and clearly it cannot, and if any craniologist thinks it does, he should begin his studies *ab initio* with a reading of Quetelet's works.

(ii) It is not identical with the *mode*.

(iii) It is not identical with the *median*.

These defects (i), (ii), and (iii) make the mean of no service at all in craniological discussions.

Now let us consider (ii) and (iii) in the light of Professor v. Török's own data treated by an adequate statistical theory. Table I. gives his data for greatest forehead breadth (S. 509) and for greatest skull breadth (S. 509). I selected these two series out of the four given because they looked in Professor v. Török's diagrams the "skewest," and therefore, if there was a sensible distinction between mean, mode and median, I thought it would certainly be evidenced in these.

TABLE I.

Greatest Forehead Breadth				Greatest Skull Breadth			
mm.	Frequency	mm.	Frequency	mm.	Frequency	mm.	Frequency
100	1	*123*	122	*120*	1	*145*	128
101	0	*124*	109	*121*	0	*146*	118
102	0	*125*	80	*122*	0	*147*	115
103	4	*126*	81	*123*	0	*148*	84
104	5	*127*	60	*124*	1	*149*	78
105	5	*128*	45	*125*	0	*150*	52
106	3	*129*	46	*126*	0	*151*	46
107	14	*130*	33	*127*	6	*152*	29
108	13	*131*	26	*128*	1	*153*	30
109	21	*132*	17	*129*	9	*154*	23
110	33	*133*	19	*130*	13	*155*	19
111	50	*134*	4	*131*	20	*156*	8
112	43	*135*	7	*132*	28	*157*	5
113	49	*136*	1	*133*	36	*158*	3
114	89	*137*	2	*134*	36	*159*	5
115	99	*138*	0	*135*	58	*160*	4
116	108	*139*	1	*136*	60	*161*	0
117	124	*140*	0	*137*	88	*162*	1
118	114	*141*	0	*138*	108	*163*	1
119	151	*142*	0	*139*	118	*164*	0
120	148	*143*	1	*140*	119	*165*	0
121	141	*144*	0	*141*	143	*166*	0
122	130	*145*	1	*142*	138	*167*	0
				143	127	*168*	0
				144	140	*169*	1

Now these series are so nearly symmetrical that it seemed sufficient to graduate them with a curve of the type: $y = y_0\left(1+\frac{x}{a}\right)^p e^{-\gamma x}$, where $p=\gamma a$: see *Phil. Trans.* A., Vol. 186, p. 367.

We find :

Greatest Forehead Breadth.	*Greatest Skull Breadth.*
Mean $= 120\cdot0475$ mm.	Mean $= 142\cdot7095$ mm.
S.D. $= 5\cdot8124$ mm.	S.D. $= 5\cdot8374$
$\mu_2 = 33\cdot783,910$	$\mu_2 = 34\cdot075,776$
$\mu_3 = 11\cdot370,311$	$\mu_3 = 17\cdot810,678$
$p = 1192\cdot0125$	$p = 497\cdot9270$
$\gamma = 5\cdot9425$	$\gamma = 3\cdot8264$
$a = 200\cdot5918$	$a = 130\cdot1279$
$y_0 = 137\cdot273$	$y_0 = 136\cdot684$
Mode $= 119\cdot8792$ mm.	Mode $= 142\cdot4482$ mm.

The mode being the origin the equations to the frequency distributions are respectively :

$$y = 137\cdot273\left(1+\frac{x}{200\cdot5918}\right)^{1192\cdot0125} e^{-5\cdot9425x},$$

and :

$$y = 136\cdot684\left(1+\frac{x}{130\cdot1279}\right)^{497\cdot9270} e^{-3\cdot8264x}.$$

The high values of p and a show how very close the distributions are to the Gaussian curve in which mean, mode, and median coincide. Let us consider this matter a little more at length by finding the probable errors of μ_3 and d, the distance between mean and mode. The values of these probable errors are to be found in the paper on the *Probable Errors of Frequency Constants**, and we thus find for :

Greatest Skull Breadth.	*Greatest Forehead Breadth.*
$\mu_3 = 11\cdot3703 \pm 17\cdot3500$	$\mu_3 = 17\cdot8107 \pm 7\cdot5783$
$d = \cdot1683 \pm \cdot1081$	$d = \cdot2163 \pm \cdot1088.$

Thus μ_3 is in one case slightly more than double the probable error and may, perhaps, be significant. In both cases d is less than twice its probable error and is therefore very likely to be insignificant. Actually this is the slender sort of basis upon which Professor von Török would discard the arithmetic mean from craniology ! Looking at the result a little more closely, we see that Professor v. Török having no conception of the probable error of any sub-group of frequency places his mode at the *Scheingipfel* of 121 mm. for the forehead breadth. The actual mode of the graduated observations is at 119·88 mm., only ·17 mm. from the mean at 120·05 mm. ; and the difference between the two is really within the limits of the errors of random sampling. Turning to the maximum skull breadth, the actual mode is at 142·45 mm. and the mean at 142·71 mm., while Professor v. Török points with triumph to *Scheingipfeln* at 141 and 144 as if they subverted the whole theory of the arithmetic mean ! Again we see that the true mode as obtained from a proper system of graduation is quite close to the mean, i.e. within a small fraction of a millimetre, and the difference is quite possibly due to the errors of random sampling.

In order to illustrate these points still further the normal curves for the two series were calculated ; they are :

Greatest Forehead Breadth.	*Greatest Skull Breadth.*
$y = 137\cdot273\, e^{-\frac{1}{2}\left(\frac{x}{5\cdot8124}\right)^2}$	$y = 136\cdot684\, e^{-\frac{1}{2}\left(\frac{x}{5\cdot8374}\right)^2}.$
Origin at mean: 120·0475 mm.	*Origin at mean:* 142·7095 mm.

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* *Biometrika*, Vol. II. p. 273. See also *Phil. Trans.* A., Vol. 191, p. 275.

Table II. gives the observed and calculated values. Applying the method for testing goodness of fit given in *Biometrika*, Vol. I. p. 155, we find in the first case: $\chi^2 = 40{\cdot}429$ and $P = {\cdot}37$, and in the second case, $\chi^2 = 33{\cdot}362$ and $P = {\cdot}69$. Or, supposing the Hungarian data dealt with by Professor v. Török to actually follow the normal or Gaussian curve, in every three samples of 2000 skulls he would on the average have found one fitting theory worse for forehead breadth than his sample actually does; and for skull breadth every two out of three samples of 2000 skulls would on the average give a worse result. In the face of such probabilities as this any sound statistician would not hesitate to say that for skull breadths Professor v. Török's Hungarian data obey the normal law of frequency, exactly as we have proved is the case for many series of other cranial measurements. The accompanying *diagrams* show the observations fitted with the theoretical curves and demonstrate at a glance how idle is any argument based on *Scheingipfeln*.

TABLE II.

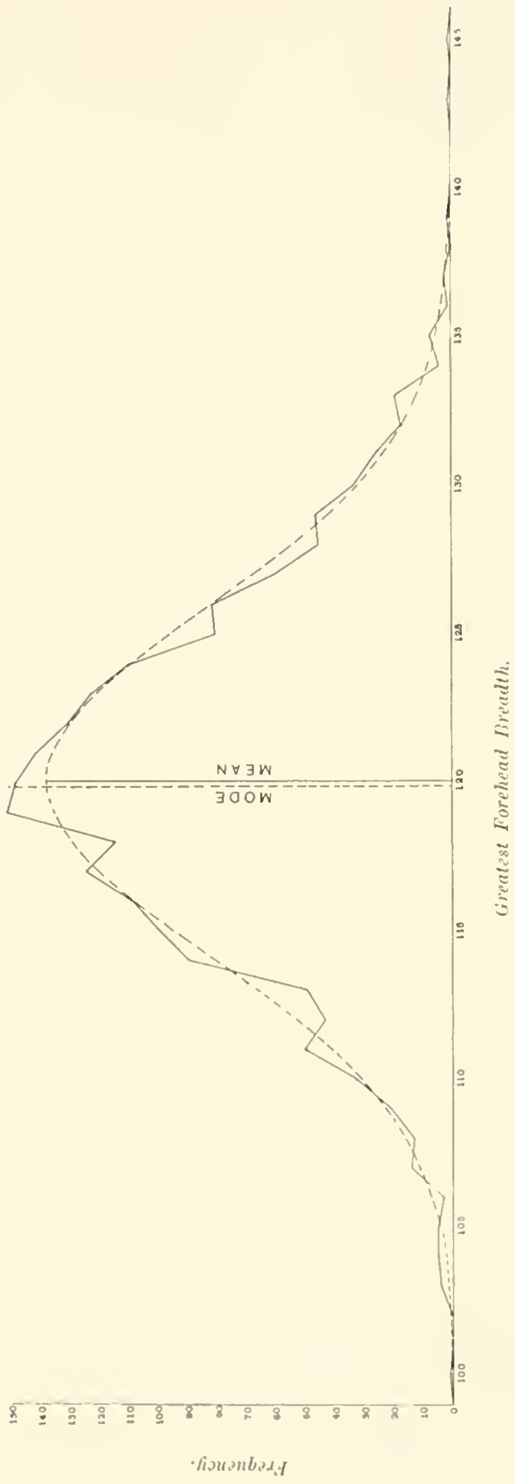
Greatest Forehead Breadth						Greatest Skull Breadth					
mm.	Frequency		mm.	Frequency		mm.	Frequency		mm.	Frequency	
	Observed	Calculated		Observed	Calculated		Observed	Calculated		Observed	Calculated
Below *103*	1	2·5	*125*	80	95·5	Below *127*	2	5·5	*148*	84	90·6
103	4	1·9	*126*	81	81·2	*127*	6	3·7	*149*	78	76·5
104	5	3·1	*127*	60	67·2	*128*	1	5·8	*150*	52	62·7
105	5	4·8	*128*	45	53·9	*129*	9	8·7	*151*	46	49·9
106	3	7·4	*129*	46	42·0	*130*	13	12·9	*152*	29	38·6
107	14	11·1	*130*	33	31·8	*131*	20	18·3	*153*	30	29·0
108	13	16·0	*131*	26	23·3	*132*	28	25·5	*154*	23	21·1
109	21	22·7	*132*	17	16·6	*133*	36	34·3	*155*	19	15·0
110	33	30·9	*133*	19	11·5	*134*	36	45·0	*156*	8	10·3
111	50	40·9	*134*	4	7·8	*135*	58	57·2	*157*	5	6·6
112	43	52·7	*135*	7	5·1	*136*	60	70·6	*158*	3	4·7
113	49	65·8	*136*	1	3·2	*137*	88	84·7	*159*	5	2·8
114	89	79·9	*137*	2	2·0	*138*	108	98·7	*160*	4	1·7
115	99	94·2	*138*	0	1·2	*139*	118	111·6	*161*	0	1·1
116	108	107·7	*139*	1	0·7	*140*	119	122·6	*162*	1	0·6
117	124	119·5				*141*	143	130·8	*163*	1	0·3
118	114	128·9	above *139*	2	0·8	*142*	138	135·5			
119	151	134·9				*143*	127	136·0	above *163*	1	0·4
120	148	137·1				*144*	140	133·5			
121	141	135·2				*145*	128	126·4			
122	130	129·6				*146*	118	116·5			
123	122	120·6				*147*	115	104·3			
124	109	108·8									

Had the mean differed from the mode, modern statistical theory could have deduced the modal value without appealing to *Scheingipfeln*. As it turns out, however, for these skulls' breadths Gaussian theory is perfectly applicable: *mean, mode and median sensibly coincide*, and Professor v. Török's attack on the arithmetical mean fails absolutely, and this because he is attempting, as so many other craniologists do, to advance without any knowledge of statistical theory.

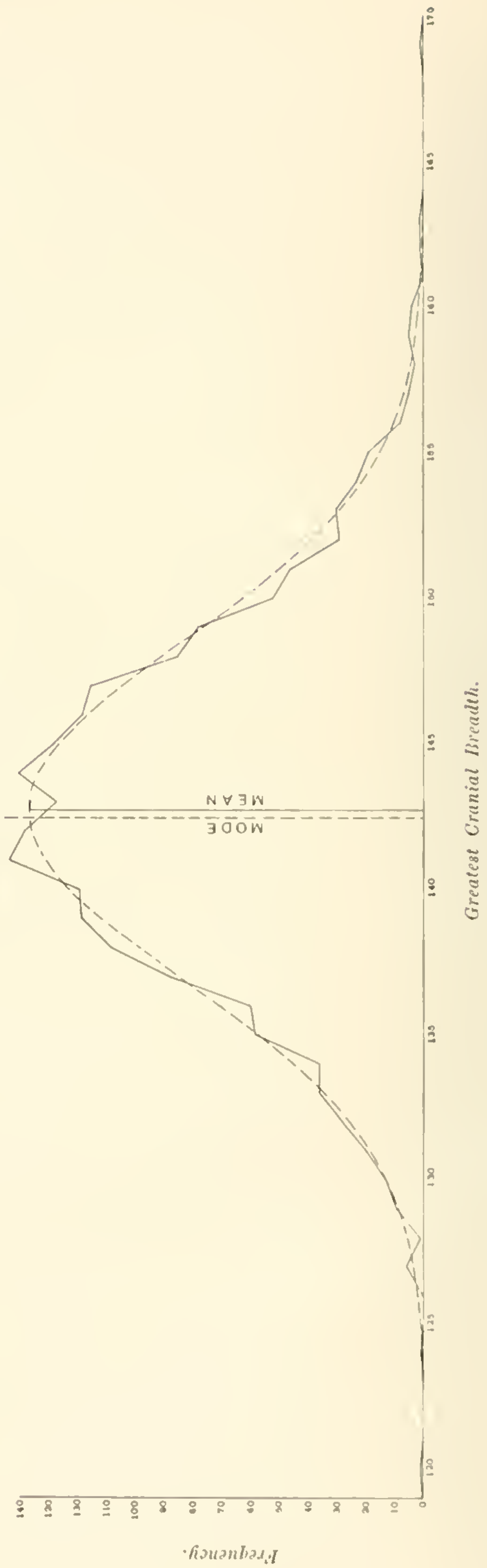
If the median were to coincide with the mean Professor v. Török tells us that there would be a fixed point to start the "Problem einer Typusbestimmung" from. Well, it actually does

2000 ♂ Hungarian Skulls.

I. Greatest Forehead Breadth.



II. Greatest Cranial Breadth.



coincide in Professor v. Török's data within the errors of random samplings. The arithmetic mean can be freed from the charges he brings against it. Can he solve the "Problem einer Typusbestimmung" on these lines? Personally I think not. "Der Kampf um die Wahrheit" in the craniology has for its solely legitimate *punctum saliens* the recognition that craniology is a branch of biometry, and can only be followed profitably when the modern theory of statistics has been properly studied.

II. *Homogeneity and Heterogeneity in Collections of Crania.*

By KARL PEARSON, F.R.S.

In a review[*] by Mr C. S. Myers of a memoir by Miss C. D. Fawcett[†], one of the biometric workers at University College, London, exception is taken to the arguments adduced in favour of the homogeneity of the Naqada prehistoric crania. There are many statements in Mr Myers' review which it would be easy to traverse, but as the writer expresses himself as in sympathy with what he is pleased to term the "new path," a consideration of one of them will suffice to show to what extent he appears to be fit at present to enter upon it.

The statement is as follows:

> The question, moreover, arises, are we entitled to consider either the Naqada or the above English skulls as belonging to people of a single race? The authors think that we are "justified in treating our material as homogeneous and in speaking of a Naqada *race* and not merely of the Naqada crania" (p. 424). "If the [Naqada] material were markedly heterogeneous the variability in length and breadth of skull ought to be large as compared with admittedly homogeneous material" (p. 424). The standard deviations of the male Naqada skulls and of Bavarian, Aino, French, and English male series as regards skull-length are 5·722, 6·088, 5·936, 7·202, 6·146 respectively, and as regards skull-breadth are 4·612, 5·819, 3·897, 6·068, 4·976 respectively. These deviations (and the "coefficients of variability" derived therefrom) are considered by the authors to be small enough to warrant the conclusion that the Naqada crania, the old Bavarian crania of Professor Ranke, and the Whitechapel English crania of Professor Thane constitute each a homogeneous series. They have left neglected the question whether a much larger standard deviation would result, were we to consider a series, say, of forty-nine male skulls of most diverse ethnic types, composed, *e.g.*, of fifteen Australians, seven Guanches, fifteen Eskimos, and twelve Chinese. This is the material which the reviewer set himself to work out, taking the data haphazard from Flower's well-known catalogue of skulls in the Royal College of Surgeons' Museum. The results gave him a standard deviation of 8·389 for the skull-length, and of 7·002 for the skull-breadth. We see, then, how small is the difference of variation between the Naqada skulls of Professor Pearson's series (which are of a "homogeneous character") and a series which is as heterogeneous as it could well be. Are we, then, not justified in considering the Naqada skulls and the others of Professor Pearson's series as if they had sprung from a mixture of races? If not, at least the problem is less simple than the writers appear to think.

The mean of the variabilities of the skull lengths given in the above paragraph is 6·2788 and of the skull breadths is 5·0804. Mr Myers mixing Australians, Guanches, Eskimos and Chinese finds a variability of skull length = 8·389 and of skull breadth = 7·002. He then points to the differences (2·1102 and 1·9216) and triumphantly asks how such small differences can be of any importance! But had Mr Myers had a mathematical training he would know that nothing is "small" absolutely, but only relatively to something else, and had he had a statistical training he would have known that he must compare it with the variability of these variabilities i.e. the standard deviation of the standard deviations of the skull measurements. Now the standard deviation of the above series of skull length variabilities = ·5185 and that of the skull breadth variabilities is ·7996. The first "small" difference is therefore 1·07 times its standard deviation, the second is 2·40 times its standard deviation. In other words whatever sort of group the Naqada, Bavarian, Aino, French, and English male cranial series make, the odds are 42,552 to 1 against such an excess of variability as Mr Myers found for his mixed series of skull lengths belonging to a number of that series, and 121 to 1 against such an excess as he found for the skull breadths occurring in such a series! It is such odds as these the combination of which can hardly fall short of 4,000,000 to 1 and which no sane man in practical conduct could disregard, that amount to "small" differences from the standpoint of the old school of craniologists! But the whole matter deserves further consideration from a wider standpoint. What fairly homogeneous data have we for variability of skull length and breadth? In the first place the list above contains the variability of 860 French male skulls from the Paris catacombs; I have dealt with this series in previous papers and expressed doubts as to its homogeneity, I have published too other cases of variability for much more homogeneous French series, Parisians* and French peasants. The first of these is the larger of the two series, and though both have much less variability than the catacomb skulls I take the Parisians as the better series. We have then the following series, to which I have added three cases of measurements on the living head:

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[*] *Man*, February, 1903, p. 13.

[†] *Biometrika*, Vol. I. pp. 408—467.

Series.	*Length.*	*Breadth.*	*Number.*
Ainos	5·936	3·897	76
Bavarians	6·088	5·849	100
Parisians	5·912	5·214	77
Naqadas	5·722	4·612	139
English†	6·085	4·976	136
Cambridge Undergraduates	6·161	5·055	1000
English Criminals	6·046	5·014	3000
Oraons of Chota Nagpur	5·916	4·397	100

In this series of eight the Naqada crania stand easily first for least variability in length, and third in the order of least variability for breadth.

Now it is impossible to regard these data, which cover, so far as I am aware, practically all the available data for variation in skull or head lengths hitherto worked out, without seeing (i) that variation in head length is practically the same as variation in skull length, and (ii) that both for very different cranial series are extremely nearly six. This corresponds with the coefficient of variation about three, which I have used in this case for many years. Anything that differs much from six—like the Parisian catacomb series—I should *à priori* suspect of heterogeneity. Now if we take the above series we find for the mean variability of head length 5·987 and of head breadth 4·877, and for the standard deviations of these variabilities ·1292 and ·5454 respectively. In this case the deviations of Mr Myers' heterogeneous series are 2·402 and 2·125, or respectively 18·6 and 3·90 times their standard deviations. In other words, the odds against such a highly variable series being of the same character as those we have given above are 6×10^{76} to 1 and 20,832 to 1 respectively! The correlation between length and breadth variations as given by the above data is − ·0197 or *negative*. The combined odds are therefore greater than if the events were independent. In other words the odds against Mr Myers' mixture belonging to the class of cranial series we have grouped together above as homogeneous are more than:

$$12 \times 10^{80} \text{ to } 1\,!$$

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[*] Data provided for me by the courtesy of M. Manouvrier. The other crania were brought to the catacombs from grave-yards which had been in use for several centuries.

[†] Using the results for the glabella maximum length, for which I have to thank Dr W. R. Macdonell, who is at work on these crania. The value cited by Mr Myers is the horizontal length, all that was available for Miss Fawcett's purpose at the time of her memoir.

Mr Myers asks in the face of these odds, the immensity of which is beyond my powers of realisation, "Are we not justified in considering the Naqada skulls and the others of Professor Pearson's series as if they had sprung from a mixture of races?"

There is only one reply to Mr Myers—these or any other series may originally have sprung from a mixture of races, but they possess something which differentiates them *totally* from his mixture of Australians, Guanches, Eskimos, and Chinese, something which the trained biometrician, who realises what is relatively "large" and what is relatively "small," can almost see on inspection. Mr Myers has been, I understand, lecturing on the "new anthropometry." It seems very like the old, and about it we can only cite Mr Myers' own words: "Never again, it is to be hoped, will the old [? new] school be tolerated which collects a few measurements, dissects them, and publishes ill-founded conclusions."

As to the general principles to be deduced from the above figures I think they are these. The heterogeneity of any series the variability of which for skull length exceeds 6·5, or for skull breadth exceeds about the same quantity, should be suspected and the series subjected to close examination. If the variability of the skull length be less than 5·5, or of the skull breadth less than 3·3, then we must suspect that the series is a rather stringently selected sample. This rule will generally enable us to distinguish between heterogeneity due to a mixture of crania from diverse races, and the homogeneity of a single race, which may indeed be the product of a number of generations of cross-breeding such as we may assert of modern English, French or Germans, but hardly with the same certainty of Ainos or Bengal castes.

III. *Preliminary Note on Interracial Characters and their Correlation in Man.*

By S. Jacob, A. Lee, D.Sc., and Karl Pearson, F.R.S.

(1) A distinction has been drawn in this Journal* between *intraracial* and *interracial* correlation, and the present preliminary note is intended to emphasise the importance of this distinction. If we take a race of which we have sufficient data and determine its type† by a number of characteristics, either by forming the means or the modes for the frequency-distributions of these characteristics in the race, we shall find that within the race an individual who diverges from the type of the race for one character will probably do so for a second, and that there is for the total of individuals within the race an interrelationship between these divergences—expressible by their coefficient of correlation. This correlation within the race is an *intraracial* coefficient, it predicts only the probable within the race itself, and must be very cautiously extended from one race to a second without *à priori* justification. The coefficient of correlation thus determined varies as a rule from race to race. Because the correlation between length and height of head of Aino ♂s is ·5, it does not follow that this Aino characteristic may be applied to prehistoric Egyptians or modern Germans. In fact, whereas within these races a long-headed Egyptian or Aino was probably a high-headed individual also, a long-headed German tends to be low-headed. It is accordingly very misleading to predict from observations within one race what are the probable relationships between characters in a second; still less legitimate is it to predict from the coefficient of correlation in one race what would be the probable value of a defective measurement in an individual of a second race‡. Our knowledge at present tends to

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* Vol. I. pp. 460 and 461.

† 'Type' is here defined as what distinguishes one population from a second, and not any member of the first from any member of the second.

‡ An illustration of this may be given from a recent paper by Professor A. v. Török and G. V. László (*Zeitschrift für Morphologie und Anthropologie*, Bd. IV. pp. 500–588). The authors had

show that correlation varies from race to race, and that only for certain special organs (related in a particular way to the manner in which selection has differentiated the two races from a common stock) will the regression coefficients on which prediction depends remain the same for the two races. Only by an extensive tabulation of the differences between regression coefficients shall we be able ultimately to predict the evolution of racial differences, and accordingly there must always be danger in extending intraracial results from one race to a second; we are leaping over the very fence we have ourselves erected when we classified them as *separate* races, for the source of that separation is written from the evolutionary side in the very differences of regression coefficients which we disregard when we predict from one race to a second. But it is not only in predicting from one race to an individual of a second that we need caution. How far may we even assert that what holds within one race holds for the races of the world taken as a whole? A long-headed Aino is probably tall-headed; are the long-headed races of the world tall-headed races? A platyrrhine Naqada was chamaeconchic. Is racial platyrrhiny usually associated with racial chamaeconchy? There are many such problems which can only be answered when a much more ample tabulation of racial types than we have at present has been provided. Still there is an obvious and correct method of approaching and solving such problems; we must correlate the type values of the characters for as many races as possible. Such correlation coefficients have been termed interracial coefficients of correlation, and their discovery must form the basis of an exact theory of race for any species, in particular of a theory of race in man. The present note is only preliminary. Its chief function is by an illustration or two to serve as a caution against the extension of intraracial results to interracial conclusions, or against the application of intraracial results to reasoning on individual organisms belonging to different or possibly quite unknown races. We confine our attention for the present to characters of the human head.

(2) *Illustration I.* **Correlation of Breadths on the Living Head.** As material we took the mean values of 57 castes or tribes from Risley's *Tribes and Castes of Bengal*, selecting those which contained the data for 50 to 100 individuals each. In all instances where Risley's means looked suspicious new averages were struck, and several rather serious errors were in some cases found. Six non-suspicious cases taken at random and tested were found to be sensibly in agreement with Risley's results. The three measurements dealt with were the cephalic breadth, the minimum frontal breadth, and maximum bizygomatic breadth. Let us term these B_C, B_F, and B_Z respectively; let M_C, M_F, M_Z be the corresponding means of the racial means and σ_C, σ_F, σ_Z, the standard deviations of the respective racial means. The correlations will be represented by r_{CF}, r_{CZ}, and r_{FZ} for the interracial relations between cephalic breadth and minimum frontal breadth, between cephalic breadth and maximum bizygomatic breadth, and between minimum frontal breadth and maximum bizygomatic breadth respectively.

apparently splendid material, 2000 Hungarian skulls; they deal with four characters, the maximum and minimum skull breadths, and the maximum and minimum forehead breadths, and their object is to deduce the probable value of one of these breadths from one, two, or three of the others. The direct statistical method was to form six correlation tables, and determine the six coefficients; the multiple regression formulae would then have given the most probable value of any character for given values of the other three. The authors, however, deal at very great length with the arrays and subarrays of the frequency surface for the multiple characters, whereas the correct statistical procedure would have given the type and variability of any subarray at once. Had this been done it would have been possible, for example, to have at once predicted from the least forehead breadth and greatest skull breadth the probable greatest forehead breadth of any Hungarian skull. But beyond this we could not venture to go until we had shown that the regression coefficients calculated for Hungarians closely hold for other races. Professor von Török's application of his Hungarian data to an individual skull of a totally different race, the Neanderthal skull, may give a near result or not, but the application is quite unwarranted by our present knowledge and is full of dangers.

Then we find:

$$M_C = 140\cdot498, \quad M_F = 102\cdot160, \quad M_Z = 132\cdot216,$$
$$\sigma_C = 4\cdot5734, \quad \sigma_F = 3\cdot1034, \quad \sigma_Z = 3\cdot0011,$$
$$r_{FZ} = \cdot5694, \quad r_{ZC} = \cdot6919, \quad r_{CF} = \cdot5415.$$

It will be seen that from the interracial standpoint the cephalic breadth is more closely related to the bizygomatic breadth than either of these to the minimum frontal breadth. Thus a reconstruction for a race of the bizygomatic breadth would be better made from the cephalic than from the frontal breadth. There can be no doubt that we have in Bengal a very great mixture of races, though it may be open to doubt whether it is sufficiently wide to admit of our supposing the above correlation coefficients to be close enough to be true interracial values. In particular a maximum breadth of the living head of 140·5 could hardly correspond to a maximum skull breadth of more than 130 to 132 mm. But such a skull breadth would be extremely small for European races, where, for example, we have: Bavarians 150·5, Whitechapel English 140·0, French 145·2, and Hungarians 142·7. The above values must therefore be looked upon as merely provisional and suggestive.

Now let us compare these values with those within a single tribe. Take first the Oraon tribe of Chota Nagpur. We have:

$$M_C = 139\cdot31, \quad M_F = 101\cdot97, \quad M_Z = 130\cdot46,$$
$$\sigma_C = 4\cdot5093, \quad \sigma_F = 3\cdot2166, \quad \sigma_Z = 3\cdot8273,$$
$$r_{FZ} = \cdot5543, \quad r_{ZC} = \cdot4093, \quad r_{CF} = \cdot5867.$$

Now in the case of skulls the frontal breadth is the character which is usually preserved longer than the bizygomatic or cephalic breadths, we will therefore write down the regression equations for the latter breadths.

Interracial Regression Equations.

(i) $P_Z = 75\cdot961 + \cdot55065\,F.$

(ii) $P_Z = 54\cdot796 + \cdot26421\,F + \cdot35892\,C.$

(iii) $P_C = 58\cdot969 + \cdot79805\,F.$

(iv) $P_C = -7\cdot249 + \cdot31802\,F + \cdot87174\,Z.$

Here P_Z and P_C are the probable bizygomatic and cephalic breadths of a race as deduced from their known mean frontal breadths, or the combination of these and their cephalic or bizygomatic breadths respectively. Applying these equations to predict the mean bizygomatic and frontal breadths of the Oraon tribe we have:

From (i) $P_Z = 132\cdot1$, observed $130\cdot5$,
(ii) $P_Z = 131\cdot7$, ,, $130\cdot5$,
(iii) $P_C = 140\cdot4$, ,, $139\cdot3$,
(iv) $P_C = 138\cdot9$, ,, $139\cdot3$.

We see at once that the mean bizygomatic and cephalic breadths of the tribe are given fairly closely by the interracial regression formulae. But how would these formulae apply to widely different data? Not having measurements on the living head available, let us take cranial measurements, allowing 8 mm. for the flesh on all three breadths. This is probably too much for the frontal breadth and possibly too little for the cephalic breadth, but it will serve our purposes of testing the formulae.

We have the following data for males:

	Naqada	*Bavarians*	*Aino*
M_c	134·87	150·47	141·20,
M_F	91·06	103·70	96·20,
M_z	125·63	135·00	137·30.

Applying our formulae these give:

TABLE I.

Formulae used	NAQADA		BAVARIANS		AINO	
	Calculated	Observed	Calculated	Observed	Calculated	Observed
P_z from (i)	122·5	125·6	129·5	135·0	125·3	137·3
,, (ii)	124·2	125·6	133·2	135·0	127·9	137·3
P_c from (iii)	130·0	134·9	140·1	150·5	131·1	141·2
,, (iv)	132·7	134·9	144·9	150·5	144·5	141·2

Now it is clear that these results fall very wide of the mark—giving with one exception values mostly far too small; or we conclude that: *No satisfactory interracial formulae by which either zygomatic or cephalic breadths could be deduced from frontal breadth can be obtained even from data as varied as that provided by the tribes and castes of Bengal.*

While an interracial formula based on that data gives fairly good results for a special tribe it cannot be applied to such diverse material as the Naqada, Bavarian, and Aino series. In fact our Bengal material has throughout a remarkably small cephalic breadth as compared with the frontal and zygomatic breadths, and we get formulae practically of no service at all for such crania as the German or Aino. We have taken too limited a group of human races. To obtain interracial formulae we must clearly proceed on a much wider basis, and such an investigation we hope to undertake at a later date.

In the next place we turn to an individual race and write down the equations by which the probable cephalic (p_c) and bizygomatic (p_z) breadths might be predicted for an individual head of this race.

For the statistical constants of the Oraon tribe given above we have:

$$p_z = 63{\cdot}21 + {\cdot}65954\,F,$$
$$p_c = 55{\cdot}45 + {\cdot}82245\,F.$$

For a second race we take 52 ♂ Naqada skulls for which the three breadths C, F, and Z are known. We find:

$$M_c = 133{\cdot}68, \qquad M_F = 90{\cdot}52, \qquad M_z = 125{\cdot}39,$$
$$\sigma_c = 4{\cdot}3596, \quad \sigma_F = 4{\cdot}4364, \quad \sigma_z = 5{\cdot}4914,$$
$$r_{Fz} = {\cdot}5695, \quad r_{zc} = {\cdot}0440, \quad r_{cF} = {\cdot}1107.$$

The corresponding formulae for reconstructing the bizygomatic and cephalic breadths from the minimum frontal breadth are:

$$p_z = 61{\cdot}58 + {\cdot}70497\,F,$$
$$p_c = 123{\cdot}84 + {\cdot}10875\,F.$$

The marked feature of the Naqada results is the great drop in the correlations between the cephalic and zygomatic and between the cephalic and the frontal breadths when we pass from the living head results to cranial measurements. Is this something peculiar to the Naqada race or to cranial as distinct from living head measurements? To test this result the cranial constants were worked out for the cases of the English (Whitechapel) skulls* and for the Theban mummies†.

English Crania.

$$M_C = 140{\cdot}70, \quad M_F = 98{\cdot}01, \quad M_Z = 130{\cdot}05,$$

$$\sigma_C = 5{\cdot}2781, \quad \sigma_F = 4{\cdot}0577, \quad \sigma_Z = 5{\cdot}4651,$$

$$r_{FZ} = {\cdot}6238, \quad r_{ZC} = {\cdot}4768, \quad r_{CF} = {\cdot}4075.$$

Theban Mummies.

$$M_C = 136{\cdot}76, \quad M_F = 93{\cdot}82, \quad M_Z = 128{\cdot}32,$$

$$\sigma_C = 4{\cdot}45, \quad \sigma_F = 4{\cdot}31, \quad \sigma_Z = 5{\cdot}22,$$

$$r_{FZ} = {\cdot}342, \quad r_{ZC} = {\cdot}250, \quad r_{CF} = {\cdot}298.$$

Thus we see that while the English crania have high correlations between the breadths, of the same general order as those determined for the living head, yet the Theban mummies give much lower results approaching far nearer to the Naqada low correlations. The fact that the Theban mummies still differ considerably from the closely allied Naqada race is consonant with, indeed confirmatory of, the view expressed by Fawcett and Lee‡ that in the 4000 years which intervened between the two groups, the differentiation that had gone on was especially in the cranial breadths.

In all the intraracial results whether for skull or living head the correlation between zygomatic and cephalic breadths is less than that between zygomatic and frontal breadths—a result sensibly different from that obtained from the interracial Bengal data, where the correlation between zygomatic and cephalic breadths is very sensibly the largest. For the English crania we have the reconstruction formulae:

$$p_Z = 47{\cdot}71 + {\cdot}84016\,F,$$

$$p_C = 88{\cdot}75 + {\cdot}53006\,F.$$

For the Theban crania:

$$p_Z = 89{\cdot}46 + {\cdot}41410\,F,$$

$$p_C = 107{\cdot}90 + {\cdot}30768\,F.$$

Clearly the regression formulae differ immensely as we pass from Naqada to Theban and Theban to English series. Hence also the predictions will vary, and conclusions drawn from one race cannot be extended to a second.

Again it is quite true that we must expect a difference between constants obtained on the living head and on the skull, but we know by experience that certain variations and correlations can be fairly close in the two cases for the same or closely allied races. We see in this case indeed that the correlations and regression coefficients for bizygomatic on minimum frontal breadth are not very widely divergent for Oraon and Naqada, but the correlations between maximum cephalic breadth and the maximum bizygomatic or the minimum frontal breadth are of a totally different order as we pass from the Indian to the Egyptian race. To emphasise this let us take five individual Oraons and five Naqadas§ and predict from their frontal breadths their cephalic and bizygomatic breadths. Using again 8 mm. for difference of living head and skull breadths we have the following Table:

---

* The full measurements on these skulls have now been completed by Dr W. R. Macdonell, and we hope they will shortly be published.

† Data given in the *Anthropologische Sammlungen Deutschlands*, E. Schmidt, "Leipziger Privatsammlung." The constants for these skulls are given provisionally as they have not been revised.

‡ *Biometrika*, Vol. I. pp. 432—3.

§ They were taken absolutely at random, i.e. the 10th, 20th, 30th...... individuals on the list.

TABLE II.

Observed Frontal Breadth	Bizygomatic Breadth			Cephalic Breadth		
	Oraon Formula	Observed	Naqada Formula	Oraon Formula	Observed	Naqada Formula
1st Oraon 103	131	132	137	140	131	142
2nd Oraon 99	128·5	130	134	137	137	142
3rd Oraon 107	134	138	139	143	141	143
4th Oraon 102	130·5	128	136	139	152	142
5th Oraon 103	131	121	137	140	137	142
1st Naqada 90·8	120	123·5	126	129	123·5	134
2nd Naqada 102·0	128	130·0	133·5	138	133·0	135
3rd Naqada 90·5	120	124·0	125	128·5	132·5	134
4th Naqada 92·5	121·5	123·5	127	130	135·0	134
5th Naqada 96·5	124	124·0	130	133	135·5	134

From these results we deduce:

Bizygomatic Breadth:	*Oraon Formula*	*Naqada Formula*
Mean Error of 5 Oraon	3·8	6·8
" " 5 Naqada	2·3	3·3
Cephalic Breadth:		
Mean Error of 5 Oraon	5·2	6·6
" " 5 Naqada	4·4	3·3

It is clear that

(i) The formulae give by no means very close results when we predict the bizygomatic and cephalic breadths of an individual from the formula for his own race, and (ii) may give very divergent results if we predict an individual from the formula for a second race. In other words; *A knowledge of the most probable values of the bizygomatic or cephalic breadths reconstructed from the frontal breadth in the case of one race, is not a knowledge of the most probable values in a second unallied race.*

The coefficients of correlation and the standard deviations of these breadths vary widely from race to race, and accordingly the simple or multiple regression formulae will also vary.

For example, the following values are given for the Neanderthal skull by Professor v. Török*:

$$F = 107 \text{ mm.}, \quad C = 147 \text{ mm.}$$

The cephalic maximum breadth predicted from the minimum frontal breadth would be 144 from the English formula, 143 from the interracial Bengal formula, 142 if predicted from the Oraon tribe, 141 from the Theban formula, and 140 if predicted from the Naqada crania.

---

* *loc. cit.* p. 584.

For the *Pithecanthropus* (Dubois) we have

$$F = 87 \text{ mm.}, \quad C = 133 \text{ mm.},$$

and the reconstructed values of C from F are from the interracial Bengal formula 127, if predicted from the Oraon tribe 126, 135 if predicted from either the English or Theban formula, and if predicted from the Naqada crania 133.

Clearly the accuracy of such predictions varies immensely with the race used in making the prediction. It is not safe to lay very much stress on the prediction of an individual from its own racial formula; it is quite unsafe to judge what will happen in an individual case from the correlation constants of a second and very diverse race. Professor v. Török, without graduating his data—as is actually done in the multiple regression formulae—but by simply arguing on the modes of subarrays of multiple correlation series, has drawn conclusions and comparisons as to skulls of other races*. Until he has shown constancy of regression coefficients for the given breadths for all races of man, any argument from one race to a second is quite invalid. The results already proven and those about to be given show how dangerous is the reasoning by which intraracial results are extended from one race to a second, or by which intraracial and interracial results are interchanged the one with the other.

(3) *Illustration II.* **Correlation of Cephalic and Nasal Indices.** We have dealt with this for two sets of living races in the male. Taking the same series of 57 tribes and castes of Bengal we find:

$$M_{cI} = 75{\cdot}804, \quad M_{NI} = 81{\cdot}282,$$
$$\sigma_{cI} = 2{\cdot}9195, \quad \sigma_{NI} = 6{\cdot}2556,$$
$$r_{cN} = -{\cdot}20160.$$

Taking 51 races from an appendix to Denniker's recent work on the races of man, Dr Alice Lee found †:

$$M_{cI} = 77{\cdot}961, \quad M_{NI} = 75{\cdot}949,$$
$$\sigma_{cI} = 3{\cdot}8231, \quad \sigma_{NI} = 11{\cdot}0081,$$
$$r_{cN} = -{\cdot}32639.$$

We have accordingly the following reconstruction formulae, where i denotes a probable, I an observed index:

Living Head from Bengal Data.

Cephalic Index from Nasal Index: $i_c = 83{\cdot}452 - {\cdot}09409\, I_N$,
Nasal Index from Cephalic Index: $i_N = 114{\cdot}027 - {\cdot}43197\, I_c$.

---

* *loc. cit.* pp. 584–8. Professor v. Török tries to predict from the minimum forehead and maximum cephalic breadths of the Neanderthal skull its maximum forehead breadth, using the ungraduated sub-arrays of the Hungarian correlation data. His process is fallacious (*a*) from the standpoint of the theory of probability, for he ought to have graduated his material by calculating the multiple regression coefficients, and (*b*) from the fact that he extends intraracial conclusions for modern Hungarians to a very different race—that of the Neanderthal man. It would be a most valuable bit of craniological work to deduce the accurate correlations for Hungarian crania from Professor v. Török's material.

† These results give a scientific classification of cephalic index and nasal index. Half the races lie above, half below 78, say. Hence 78 should be the boundary between brachycephaly and dolichocephaly in *living* races. A quarter of the variations are below 75·38 and a quarter above 80·54. Hence indices of 75·5 and 80·5, say, should be taken as the boundaries of hyperdolichocephaly and hyperbrachycephaly. Similarly for the nasal index in *living* races we have the divisions: Below 68·5 hyperleptorrhiny, from 68·5 to 76 leptorrhiny, from 76 to 83·5 platyrrhiny, above 83·5 hyperplatyrrhiny. The Bengal results are for less wide racial ranges and we do not think it desirable to classify the living head indices on their basis. The systematic classification of races will be dealt with at length later when more ample data have been worked out.

Living Head from Denniker's Data.

Cephalic Index from Nasal Index: $i_c = 86\cdot570 - \cdot11335\, I_N$,

Nasal Index from Cephalic Index: $i_N = 149\cdot216 - \cdot93980\, I_c$.

Now these results are extremely interesting. We see that platyrrhine characters are interracially associated with dolichocephally and *vice versâ* in quite a marked degree. We have in each case a quite sensible negative correlation.

The first thing then we are tempted to ask is: whether this is true intraracially? We know already that it is true for the Naqada crania*. The following give the results for two Bengal races for the living head.

Murmi Tribe, Chittagong Hills.	*Oraon Tribe of Chota Nagpur.*
$M_{cI} = 79\cdot914$, $M_{NI} = 74\cdot889$,	$M_{cI} = 75\cdot481$, $M_{NI} = 86\cdot250$,
$\sigma_{cI} = 3\cdot3250$, $\sigma_{NI} = 6\cdot0248$,	$\sigma_{cI} = 3\cdot3632$. $\sigma_{NI} = 7\cdot7830$,
$r_{cN} = -\cdot2284$,	$r_{cN} = -\cdot2392$.

The reconstruction equations from these races are:

Murmi Tribe: $i_c = 89\cdot353 - \cdot12604\, I_N$,

$i_N = 112\cdot983 - \cdot41383\, I_c$,

Oraon Tribe: $i_c = 84\cdot396 - \cdot10336\, I_N$,

$i_N = 128\cdot030 - \cdot55352\, I_c$.

Now these intraracial results are of the same kind as the interracial results, and, considering the smallness of the data dealt with, not widely different in numerical magnitude. We may, we think, conclude accordingly that the association of platyrrhine and dolichocephalic characters is intraracial as well as interracial and the degree of this association has much the same quantitative magnitude in both cases†. It seems legitimate in this instance to draw general conclusions from intraracial to interracial results as far as the living head is concerned.

(4) *Illustration III.* **On the Interracial Relationships of the Orbital to the Cephalic and Nasal Indices of the Cranium in Man (♂).**

Denniker gives for a few races from data of very diverse worth the mean orbital, cephalic and nasal indices. The results obtained by Dr Alice Lee are as follows:

Orbital and Cephalic Index.	*Cephalic and Nasal Index.*	*Nasal and Orbital Index.*
(21 cases)	(16 cases)	(30 cases)
$M_{oI} = 85\cdot786$, $M_{cI} = 75\cdot367$,	$M_{cI} = 74\cdot000$, $M_{NI} = 52\cdot231$,	$M_{NI} = 50\cdot287$, $M_{oI} = 87\cdot023$,
$\sigma_{oI} = 2\cdot9172$, $\sigma_{cI} = 4\cdot0895$,	$\sigma_{cI} = 3\cdot4595$, $\sigma_{NI} = 4\cdot1210$,	$\sigma_{NI} = 4\cdot4205$, $\sigma_{oI} = 9\cdot8969$,
$r_{oc} = \cdot4486 \pm \cdot1176$.	$r_{cN} = -\cdot4460 \pm \cdot1354$.	$r_{No} = -\cdot1088 \pm \cdot1217$.

---

* *Biometrika*, Vol. I. p. 455. The reconstruction formulae in this case are:

$$i_c = 78\cdot054 - \cdot09914\, I_N,$$

$$i_N = 67\cdot206 - \cdot22094\, I_c,$$

equations which naturally differ totally from those for the living head and which are at once seen to be absolutely inapplicable to individual German or Aino crania.

† For example for nasal index $= 100$: Bengal data give cephalic index $= 74$, Denniker's data $= 75$; for nasal index $= 50$: Bengal data give cephalic index $= 79$, and Denniker's data $= 81$. The results are less in agreement if the nasal index be predicted from the cephalic.

The paucity of races dealt with, and the heterogeneity of measurers and of series used, make these results of very small value, and they are only given here tentatively, while larger series of more numerous races are being reduced. But even thus some probable conclusions can be more or less definitely stated, i.e.

(*a*) The general statement that platyrrhine races are dolichocephalic is confirmed by cranial measurements.

Considering the probable error of the correlation of the cranial cephalic and nasal indices, that correlation does not differ very widely from that on the living head obtained from Denniker's data. What do differ widely are the mean and *especially the variability* of the nasal indices for races when measured on the living head and on the skull. It may well be questioned how far this variability is not peculiar to the personal equation of the observer and is therefore not a true anthropological character.

(*b*) There is quite a sensible correlation between orbital and cephalic indices, and it is positive. Or: the brachycephalic races tend to round orbits, and the dolichocephalic races to elongated orbits. This as far as the Naqada crania are concerned has been shown to be an intraracial as well as an interracial association.

(*c*) There is some evidence to show that platyrrhine races have elongated orbits.

The correlation between orbital index and nasal index is negative, but it is not definitely sensible compared with its probable error in the small series at present available. This agrees with the intraracial conclusion drawn from the Naqada crania.

(5) *Illustration IV.* On the Interracial Relationship between the Length and Breadth of the Living Head.

Hitherto although interracial correlations have been shown to differ from intraracial values, we have only dealt with cases in which these correlations were of the same sign, and the reader might be induced to draw the temptingly fascinating conclusion that interracial and intraracial correlations are always qualitatively alike. In order to convince the reader that no conclusion of this kind is allowable we propose to deal in this last illustration with the length and breadth correlations.

Taking our fifty cases from Risley's *Bengal Tribes* we find for maximum length and breadth of living head the interracial results:

$$M_L = 184{\cdot}64, \quad M_B = 140{\cdot}00,$$
$$\sigma_L = 2{\cdot}2368, \quad \sigma_B = 4{\cdot}6005,$$
$$r_{LB} = -{\cdot}2917.$$

Thus we see that a "principle of compensation" applies. A long-headed race is likely to be a narrow-headed race, and a broad-headed race a short-headed race. This conclusion is essentially interracial, and not intraracial. Intraracially we find that the correlation of length and breadth of living head or of skull while varying remarkably is *positive*. Thus it is so, for English (heads and skulls), French, German, Aino, Naqada and other cranial series*.

Or, again take the Oraon tribe of Chota Nagpur, we find:

$$M_L = 184{\cdot}61, \quad M_B = 139{\cdot}31,$$
$$\sigma_L = 5{\cdot}9160, \quad \sigma_B = 4{\cdot}3970,$$
$$r_{LB} = {\cdot}0655.$$

Thus the correlation is small and positive,—far from the very sensible negative interracial value.

---

* See *Biometrika*, Vol. I. p. 457.

Hence we see that the "principle of compensation" which certain anatomists seem to have drawn from the observation of heads in the dissecting room appears to be quite fallacious. The long-headed individual within the race is likely also to be relatively broad, i.e. individuals within the race tend to have generally big or generally small heads. The "principle of compensation" only applies when we compare the mean lengths and breadths of different races. Here the long-headed race is a narrow-headed race and *vice versâ*. This is a curious illustration of how by a false or superficial investigation an erroneous result may be reached,—a result which on an entirely different plane can be justified by a correct theory.

We hope later to publish far more complete intraracial coefficients for the case of man, and lay to some extent the foundation of what we believe to be a scientific theory of race in man. In this preliminary note we are confessedly only touching the fringe of an immense and difficult subject, but we believe that we have indicated the proper method of approaching it.

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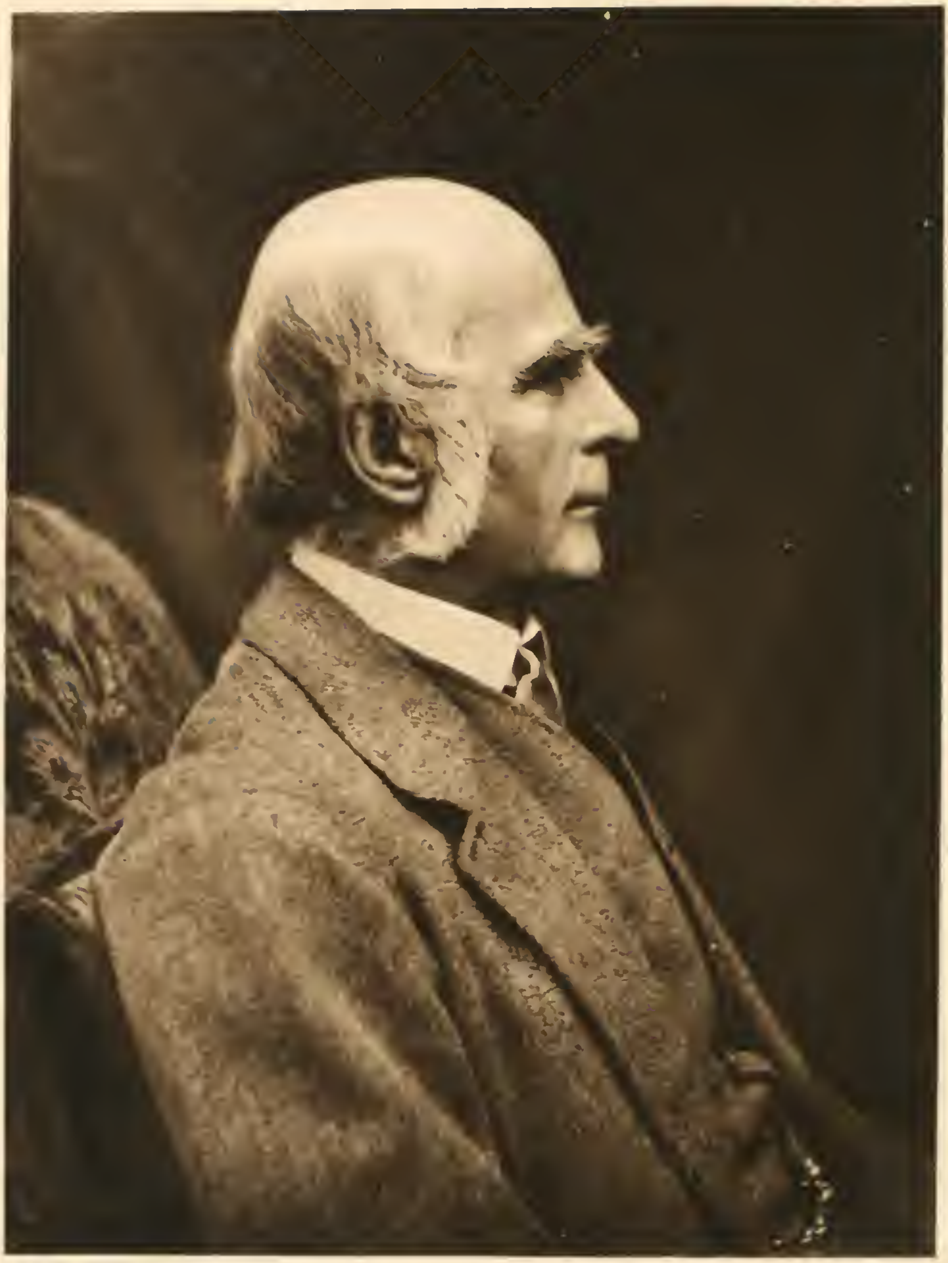

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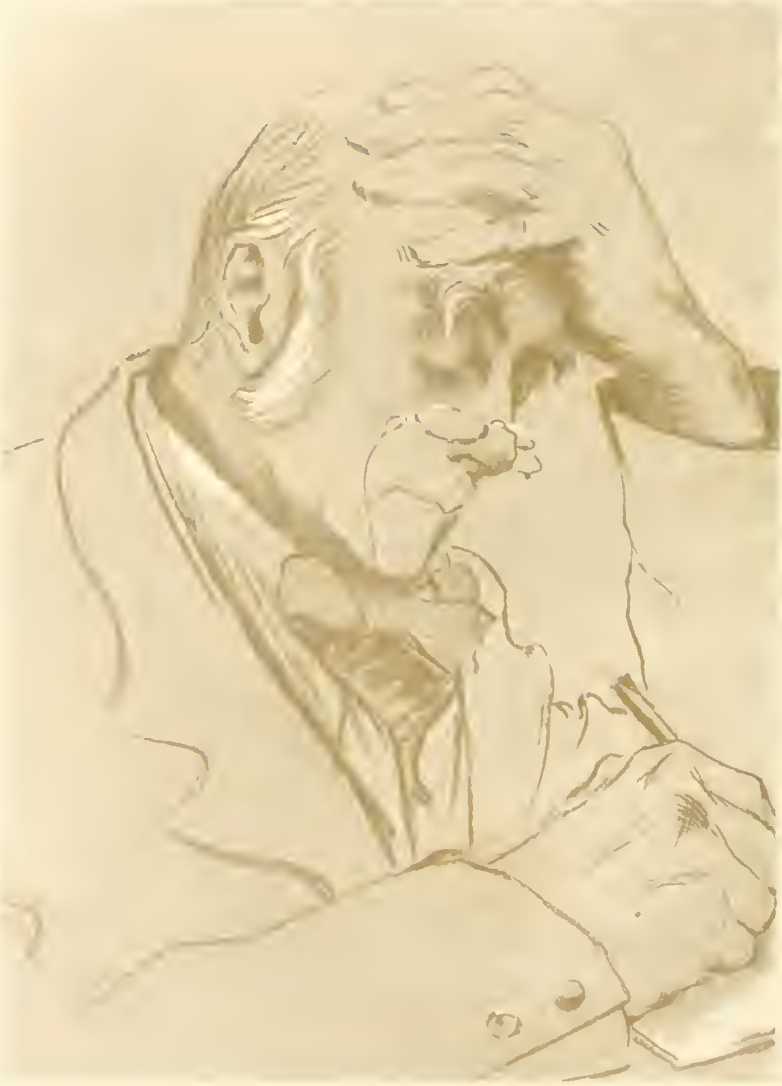

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(i) *Introductory.*

ABOUT eight years ago I determined to supplement the data obtained by Mr Francis Galton for his work *Natural Inheritance* by a rather wider series of measurements on blood relations in man. Mr Galton had most generously placed his original data at my disposal and I had used them as far as stature was concerned in my memoir of 1895[†] and in a joint paper with Dr Lee in 1896[‡]. The eye-colour data of his Family Records were not reduced[§] until after the discovery of a method for dealing with characters not capable of exact quantitative measurement[‖], and it is only recently that the full scheme of relationships back to great-grandparents has been completed[¶]. There were about 200 families in Mr Galton's records and only one measurable character, stature. The conditions

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[*] I must gratefully acknowledge aid in the publication of the elaborate tables which accompany this memoir from a grant made to my department in the University of London by the Worshipful Company of Drapers.

[†] "Regression, Heredity and Panmixia." *Phil. Trans.* Vol. 187, pp. 253—318.

[‡] "On Telegony in Man." *R. S. Proc.* Vol. 60, p. 274 *et. seq.*

[§] "On the Inheritance of Eye Colour in Man." *Phil. Trans.* Vol. 195, pp. 102—121.

[‖] "On the Correlation of Characters not quantitatively Measurable." *Phil. Trans.* Vol. 195, pp. 1—47.

[¶] F. E. Lutz: "Note on the Influence of Change in Sex on the Intensity of Heredity." *Biometrika*, Vol. II. pp. 237—240.

as to age of the measured, or to method of measurement were not, perhaps, as stringent as might now be considered desirable, but Mr Galton's data were amply sufficient to lead him to his great discovery of the general form of the inheritance of blending characters in a stable community. The full significance of this discovery is hardly yet understood, and one constantly notices grave misinterpretations of Mr Galton's theory in the works of non-statistically trained biologists. The constants as determined from Mr Galton's stature data did not seem to me to be final: they were to some extent irregular and were not in full accord with the more uniform eye-colour results. It therefore appeared to me desirable to obtain further data, not only for several physical characters and to compare the results for these characters with those for mental characters, but to deal with both in as wide as possible a system of blood relationships. This was provided for in the following series of observations:

I. *Family Record Series.* About 1893 I drew up in conjunction with my then colleague, W. F. R. Weldon, the directions for family measurement which are described below. The measurements were in great part carried out by college students*, and I largely owe the success of this series to the energy and time devoted to the collection of the data by Dr Alice Lee. In the course of four to five years about 1100 cards were filled in. The tabling of the data on these cards and the calculation of the statistical constants, some 78 tables in all, are due entirely to Dr Lee, and occupied her spare time for nearly two years.

II. *School Record Series.* This series was started some years later and was aided by a grant from the Government Grant Committee. Its object was to record the mental and physical characters in pairs of brothers, of sisters, and of sisters and brothers in schools. About six thousand children were observed and measured, and provided more than 3000 pairs of brethren to illustrate in a great variety of ways the intensity of collateral resemblance in man. This series will only be dealt with incidentally in the first part of this paper, about 150 of the tables have been formed and the correlations deduced from them, but much work remains still to be done on the data for schools.

III. *Cousinship Series.* A third series on the ten kinds of first cousins is now being started with aid from the Government Grant Committee to complete our quantitative conceptions of collateral heredity. But it will be a number of years before the data here desired can be fully collected and still longer before the reductions can be completed. The above series form the material from which it is proposed to obtain quantitative measures of the degree of resemblance between blood relations in man. The present memoir deals primarily with the Family Record Series.

(ii) *Nature of the Family Record Series.*

It seems desirable to give the actual form of the instructions and schedule by aid of which the data were collected.

---

* I must take this opportunity of most heartily thanking the many helpers, who devoted much time and energy to measuring not only single but often 10 or 20 families.

FAMILY MEASUREMENTS.

Professor KARL PEARSON, of University College, London, would esteem it a great favour if any persons in a position to do so, would assist him by making one set (or if possible several sets) of anthropometric measurements on their own family, or on families with whom they are acquainted. The measurements are to be made use of for testing theories of heredity, no names, except that of the recorder, are required, but the Professor trusts to the *bona fides* of each recorder to send only correct results.

Each family should consist of a father, mother, and at least one son or daughter, not necessarily the eldest. The sons or daughters are to be at least 18 years of age, and measurements are to be made on not more than two sons and two daughters of the same family. If more than two sons or two daughters are easily accessible, then not the tallest but the eldest of those accessible should be selected.

To be of real service the whole series ought to contain 1000—2000 families, and therefore the Professor will be only too grateful if anyone will undertake several families for him.

Copies of this paper, together with cards for recording data, may be obtained from

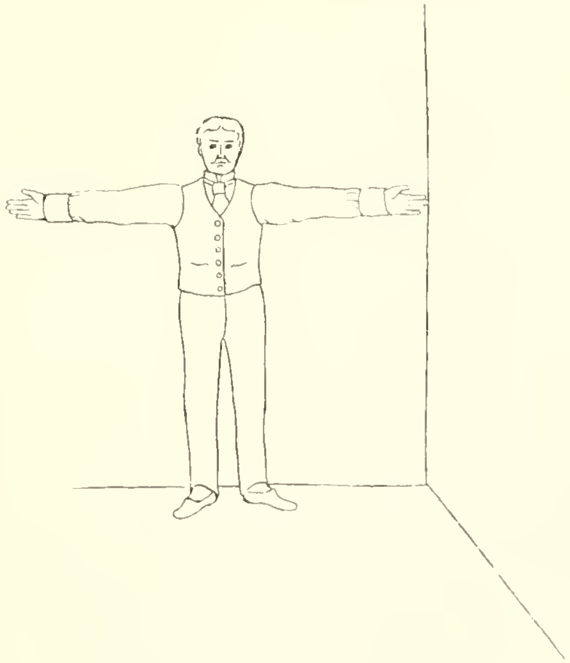
……………………………………………………………………………

or from the above-named Professor.

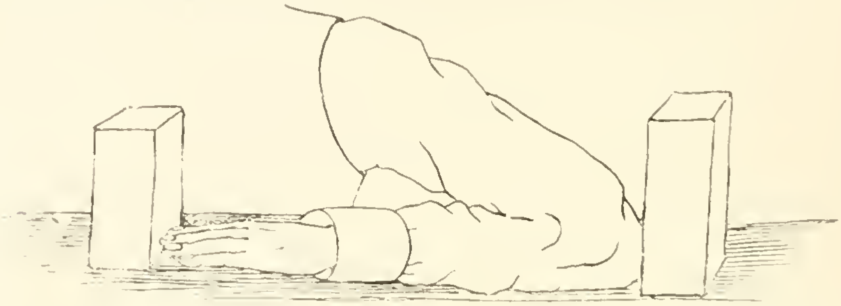
The measurements required in the case of each individual are to be to the nearest quarter of an inch, and to consist of the following:—

(I.) *Height.*—This measurement should be taken, if possible, with the person in stockings, if she or he is in boots it should be noted. The height is most easily measured by pressing a book with its pages in a *vertical plane* on the top of the head while the individual stands against a wall.

(II.) *Span of Arms.*—Greatest possible distance between the tip of one middle finger and the tip of the other middle finger, the individual standing upright against a wall with the feet well apart and the arms outstretched,—if possible with one finger against a doorpost or corner of the room.



III. *The Length of LEFT Forearm.* The arm being bent *as much as possible* is laid upon a table, with the hand flattened and pressed firmly against the table, a box, book, or other hard object is placed on its edge so as touch the bony projection of the elbow, another so as to touch the tip of the middle finger. Care must be taken that the books are both perpendicular to the edge of the table. The distance between the books is measured with a tape.



Or,

The arm being bent *as much as possible* the elbow is pressed against the corner of a room or the doorpost, the hand being flattened and pressed against the wall. The greatest distance from the tip of the middle finger to the corner or doorpost is to be measured.

Sample of filled in Data Card of Family Measurements.

		Height*		Span of Arms		Left Forearm	
One Family only		Feet	Inches	Feet	Inches	Feet	Inches
Father ... ... (Not step-father)		5	9¼	6	1½	1	7¼
Mother ... ... (Not step-mother)		5	0¾	5	2	1	4¼
	Age						
Son ...	26	5	7¼	5	11	1	6½
Son ...	—	—	—	—	—	—	—
Daughter	30	5	4¼	5	5	1	4½
Daughter	24	5	5¼	5	6¼	1	5

Name and Address of Recorder (not to be published in any way, but for convenience of reference).

Miss A. L. Robinson,
Blounts Court Mansions, Kensington, S.W.

....

Both father and mother are absolutely necessary and should not be over 65 years of age.

All the measures are to be recorded to the nearest quarter of an inch. Before measuring read the notice circulated with this card, and kindly return the card as soon as possible to

[Name of individual collector was here inserted]

or to Professor Karl Pearson, University College, London, W.C.

* Put B against numbers if measure is taken in boots. If any person measured has ever broken a leg, arm or collar-bone, put L, A, C against all his or her measurements.

It is not for a moment suggested that the instructions or schedule form are ideal; they are of course open to criticism of a variety of kinds. But they were not settled without considerable thought and a definite reason for each point stated. Thus full growth is not reached at 18 years of age, perhaps not till 25. The growth, however, from 18 to 25 is relatively small, although sensible, and by fixing our limit at 25, we found a very large number of families would be cut off, for both parents would not be surviving, or, if surviving, beyond the age limit fixed for parents. Further, we should have been unable to interest college students effectively in the matter, as the bulk of them fall between 19 and 22. Again, it would have been better to take a lower maximum age for the parents, but in doing so we should again have greatly limited our available material. Better organs might undoubtedly have been selected than stature, span and forearm, e.g. head and finger measurements, but in such cases instruments and greater elaboration are needed, and the difficulty of obtaining upwards of 1000 families, already very great, would have been much intensified. We chose organs easily measured with moderate accuracy and asking for the nearest quarter-inch, only tabulated stature and span to the nearest inch, and forearm to the nearest half-inch. Thus the slight diurnal variations and the errors of measurement of the characters will not sensibly affect the constants calculated from our tables. Only a small percentage were measured in boots; we could not insist that ladies and gentlemen in middle life must remove their boots, or we might have met with a far larger number of refusals to be measured. Still the bulk of the measured did remove boots. After some experimenting on the effect of heels on apparent stature it was found that the subtraction of an inch from the recorded stature fairly represented the average increment due to boots. Hence the small percentage of boot entries was reduced before tabling by one inch.

Of course each family card did not provide us with four children, our maximum number allowed. Thus the number of our parental pairs lies for the different tables between 1000 and 1400, while for the fraternal correlations we have results based on 350 to 1400 pairs, according to the nature of the table. This is due to the fact that it was found far more difficult to get the measurements on *two* adult brothers, than on two sisters*. It was partly this defect in the number of pairs of brothers which led to the wider system of school measurements on brothers. The latter, however, do not modify but only confirm the results obtained from the smaller series in the *Family Records*.

I now propose to deal at length with the results obtained from our material.

(iii) *Theory applied.*

The regression in all cases is essentially linear, i.e. very closely linear within the limits of random sampling. It is impossible to give diagrams of all the 2 × 78 regression lines of the 78 correlation tables, but the following three cases are a fair *random* sample of what actually occurs*.

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* Probably two adult brothers were far more rarely found both living at home, or if at home declined to be submitted to a measurement, which offered no immediate advantage to themselves.

Diagram I. *Stature of Father and Son.*

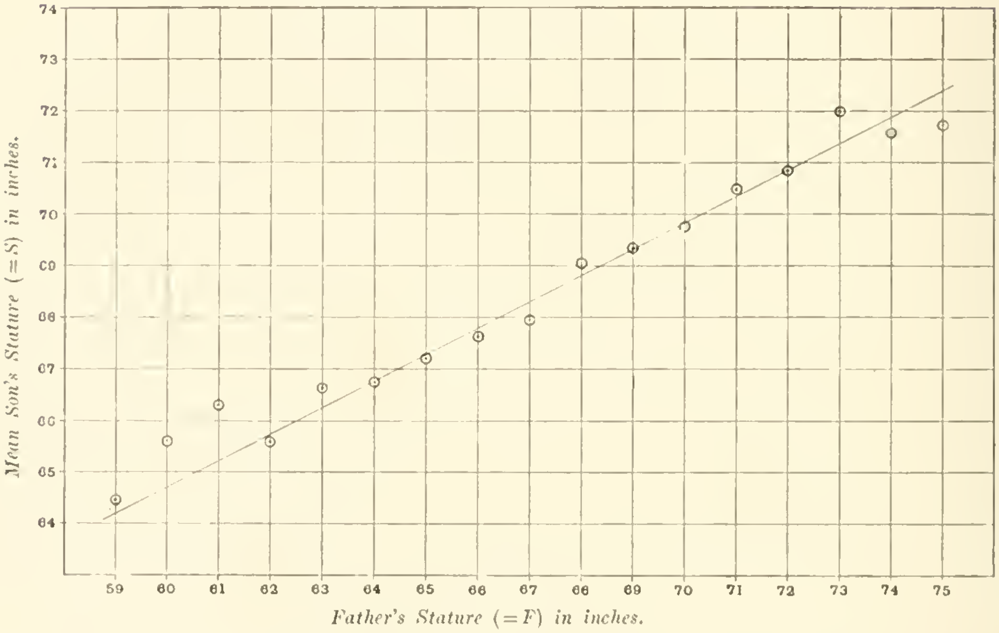
Diagram II. *Span in Mother and Daughter.*

Diagram III. *Brother's Forearm and Sister's Span,*

the latter being an example of a cross-correlation.

Diagram I. *Probable Stature of Son for given Father's Stature.*

Regression Line: $S = 33{\cdot}73 + {\cdot}516\,F$. 1078 Cases.



It will be seen from these cases that, except near the terminals, where the numbers of cases are very few, that the regression is closely linear. We are thus relieved from any difficulties about regression or correlation. We have only to find the ordinary coefficient of correlation r, and the regression coefficient $r\sigma_1/\sigma_2$, and these will suffice to describe the average degree of hereditary resemblance. All this is done without any assumption of the normal curve of frequency. As a matter of fact, however, the normal curve very closely suffices to describe the distribution of many physical characters in a human population. This is illustrated in the accompanying diagrams which are fair samples of stature and span frequencies. In Diagram IV. we have the following data for stature in mothers, plotting frequency observed against theoretical frequency.

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* A further case from the data, that of cubit in Father and Son, was given in *Biometrika*, Vol. II. p. 216.

DIAGRAM II. *Probable Span of Daughter for given Mother's Span.*
Regression Line: $D = 31{\cdot}18 + {\cdot}473\,M$. 1370 Cases.

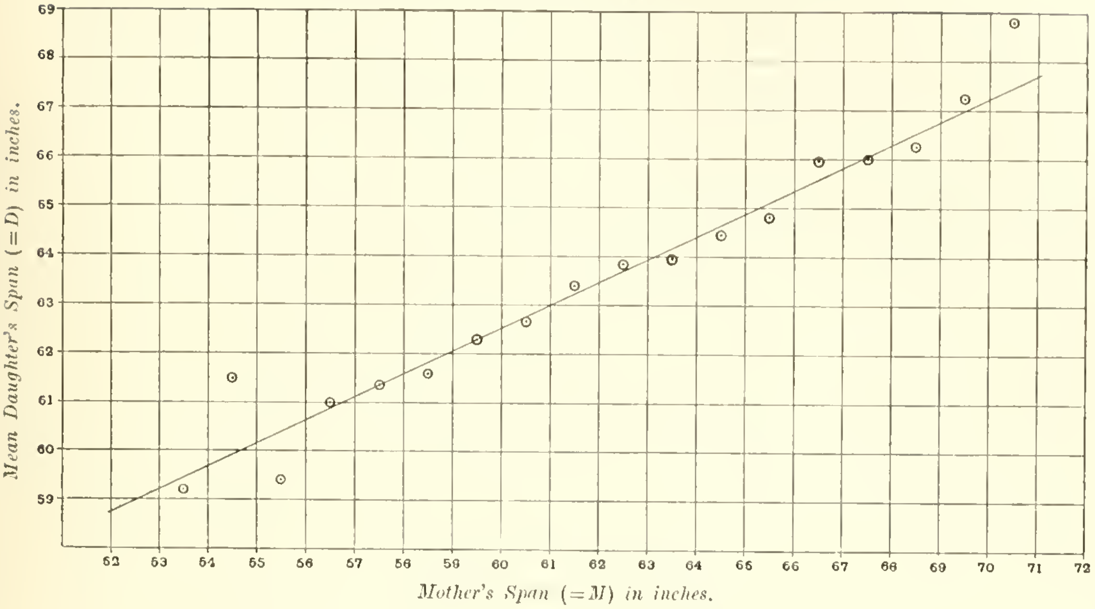


DIAGRAM III. *Probable Span of Sister for given Forearm in Brother.*
Regression Line: $S = 39{\cdot}66 + 1{\cdot}280\,B$. 1399 Cases.

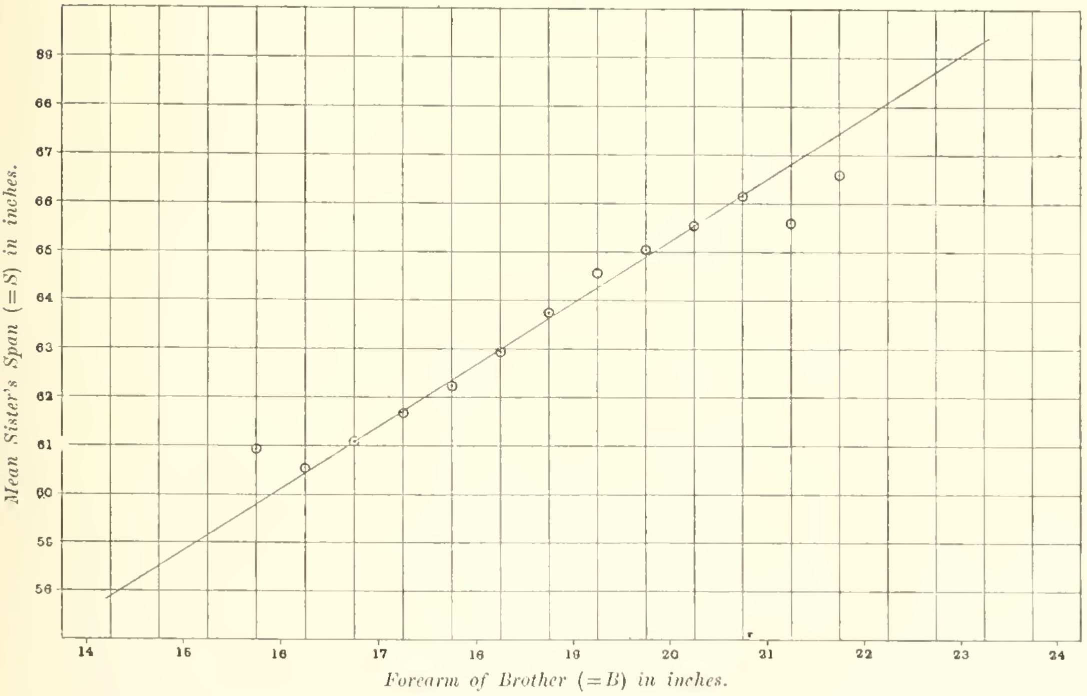
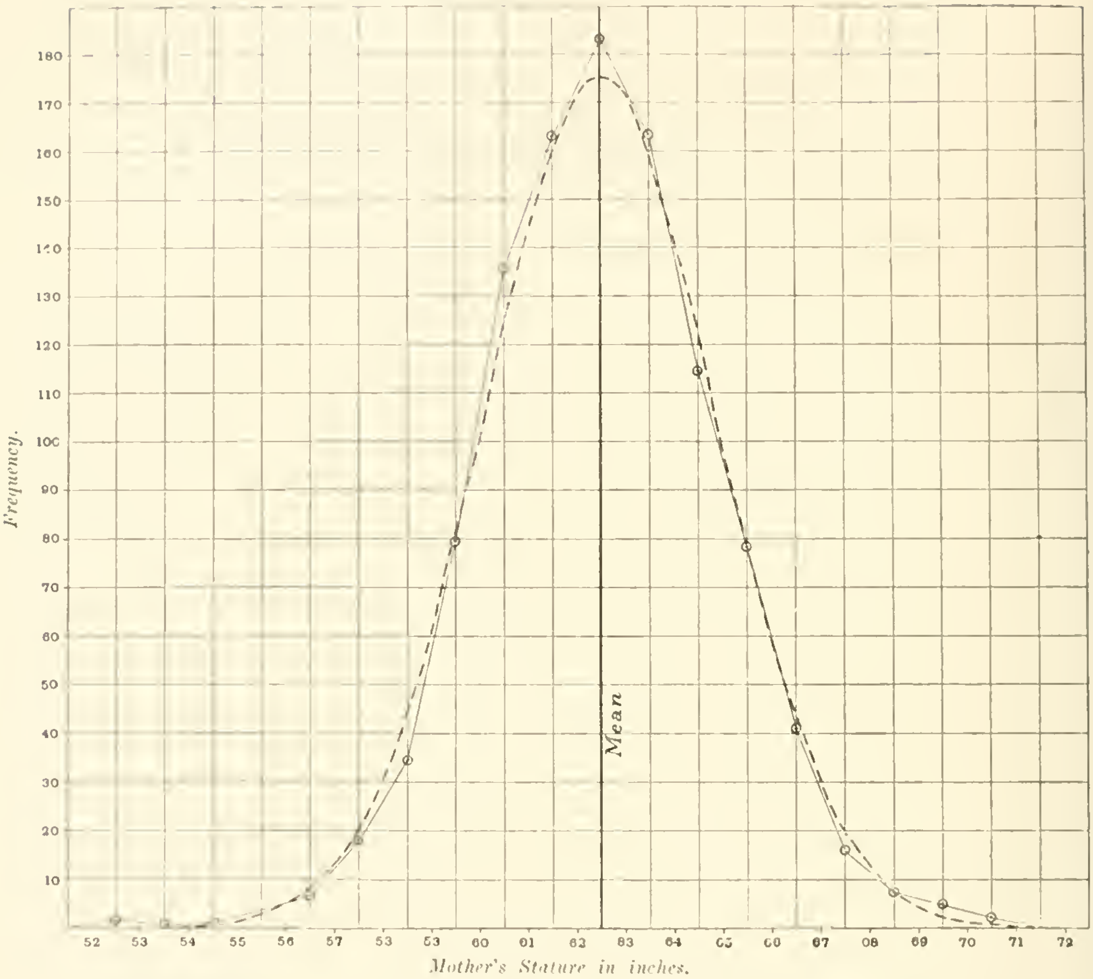


DIAGRAM IV. *Distribution of Stature.*



Stature in Mothers. 1052 *Cases.* *Mean* $= 62''{\cdot}484$, *Standard Deviation* $= 2''{\cdot}3904$.

Stature in inches	52—53	53—54	54—55	55—56	56—57	57—58	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71
Observed Frequency	1·5	·5	1	2	6·5	18	31·5	79·5	135·5	163	183	163	114·5	78·5	41	16	7·5	4·5	2
Normal Frequency	·9			2·6	7·9	20·9	44·5	80·8	124·1	160·3	174·3	159·4	122·8	79·5	43·2	20·1	7·7	2·5	·8

Sheppard's Tables* were used. If we test goodness of fit by my general method†, using Elderton's Tables and notation‡, we find: $\chi^2 = 14{\cdot}47$, and for

---

* *Biometrika*, Vol. II. p. 182 *et seq.* † *Phil. Mag.* Vol. I. pp. 157—175.

‡ *Biometrika*, Vol. I. p. 155 *et seq.*

17 groups this gives $P = \cdot 56$, or, if stature in mothers really obeys a normal law, we should expect worse results by way of fit in 56 out of 100 samples of 1052 mothers. Thus the degree of fit may be considered good. There is some irregularity at the left-hand tail, where I have clubbed three groups together. There is generally an improbable outlier or two in most of these distributions, possibly the result of some slip in measurement, or perhaps special deformity or result of disease not recorded on the cards*.

In Diagram V. we have the following data for span in sons:

Span in Sons. 1156 *Cases.* *Mean* $= 69''\cdot 94$, *Standard Deviation* $= 3''\cdot 0869$.

Span in inches	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	76—77	77—78	78—79	79—80	80—81	81—82
Observed Frequency	·5	·5	·5	3·5	4	13	32	64·5	84·5	105	132·5	153·5	152	125·5	102	76·5	47·5	28	11·5	9	7	1·5	·5	1
Normal Frequency	2·2			3·7	8·3	17·2	31·9	53·4	80·5	109·1	133·2	147·3	146·3	131·1	105·9	76·9	50·6	29·8	15·8	7·6	3·2	1·3	·6	

In this case we have for 21 groups, $\chi^2 = 14\cdot 63$ and $P = \cdot 80$, or in 80 trials out of 100 we should expect the group of 1156 sons to diverge more from the normal distribution than our observed results do. Again we have an excellent fit. It would seem that for stature and span we may fairly use a normal distribution, even as it may be used for cranial and cephalic measurements†. But biometric results are always a field for surprises, partly because of the complexity of causes to be dealt with, partly because we are really only at the beginning of our

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* It is even conceivable that a measurer went out of the way to get a very small man or woman, as in a case where one contributor wrote: "I have got with some difficulty the ——'s, a very tall family."

† Macdonell, *Biometrika*, Vol. I. p. 183 *et seq.* and Fawcett, *Ibid.* Vol. I. p. 443.

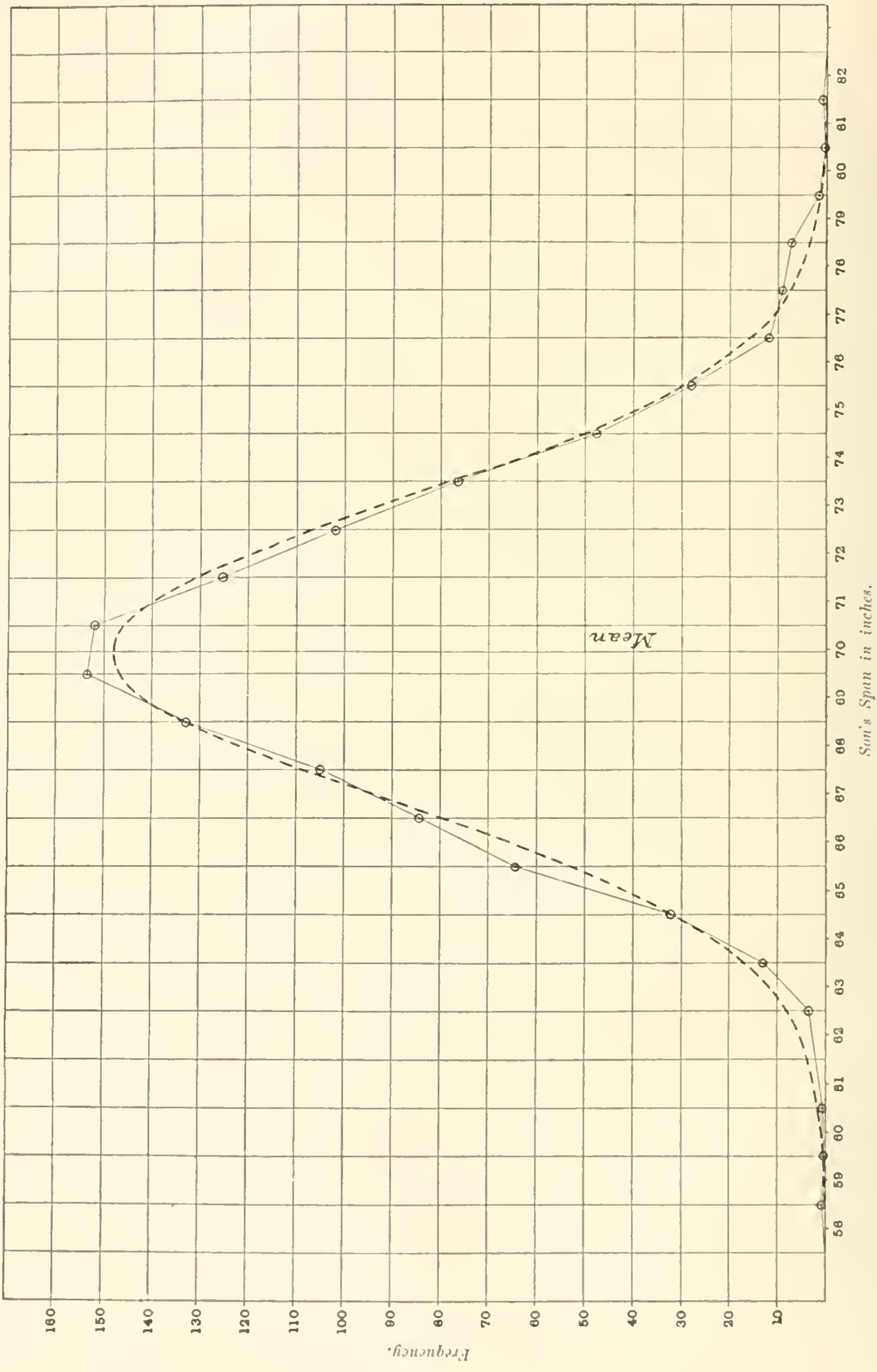

DIAGRAM V. *Distribution of Span.*

collections of data. I may have been unfortunate in my choice of the forearm as more difficult of measurement, or more subject than span to growth influences, but the results for the forearm diverge considerably more from normality than those for stature or span. I give my conclusions for the three cases I have investigated. These are as follows:

Forearm in Fathers: 14 groups, $\chi^2 = 35{\cdot}18$, $P = {\cdot}000$,
„ in Daughters: 14 groups, $\chi^2 = 33{\cdot}51$, $P = {\cdot}003$,
„ in Sons: 15 groups, $\chi^2 = 30{\cdot}76$, $P = {\cdot}007$.

The improbability of the normal distribution is, however, in all these cases chiefly due to a little lump of "outliers" at the "giant" end of the distribution. There are four fathers with excessive forearms, four daughters with the like and four sons also These twelve cases cannot, I feel sure, be in the bulk due to slips of measurement, they may be due to some anomalous growth or to a reversion to an excessive radius. If we remove them we find roughly: $P = {\cdot}45$ for fathers, $= {\cdot}21$ for sons and $= {\cdot}18$ for daughters, i e. we obtain an excellent normal curve fit in the first case, and quite fair ones in the other two. We are therefore forced to the conclusion that forearm in the bulk follows fairly closely a normal distribution, but there appears to exist in man a small abnormal group with excessive forearms, of less than ·5 per cent. The following is the table of observed and theoretical results for forearm in fathers:

Forearm in Fathers. 1050 *Cases.* *Mean* = 18″·31, *Standard Deviation* =″·963.

Forearm in inches	*15—15·5*	*15·5—16*	*16—16·5*	*16·5—17*	*17—17·5*	*17·5—18*	*18—18·5*	*18·5—19*	*19—19·5*	*19·5—20*	*20—20·5*	*20·5—21*	*21—21·5*	*21·5—22*	*22—22·5*
Observed Frequency	1	6·5	17	49	125·5	200	235·5	183·5	127	57·5	31·5	8	3·5	2	2·5
Normal Frequency	1·9	6·7	23·0	59·6	119·0	182·2	214·8	194·3	135·0	72·0	29·5	9·3	2·2	·5	

This is shown in Diagram VI. The mere graphical inspection of such a result as this would hardly lead us to give proper weight to the abnormal group of outliers, which carry P from ·45 to ·000. To some it might seem a good fit, but the trained eye sees at once defects and $P = {\cdot}000$ shows how great they are*.

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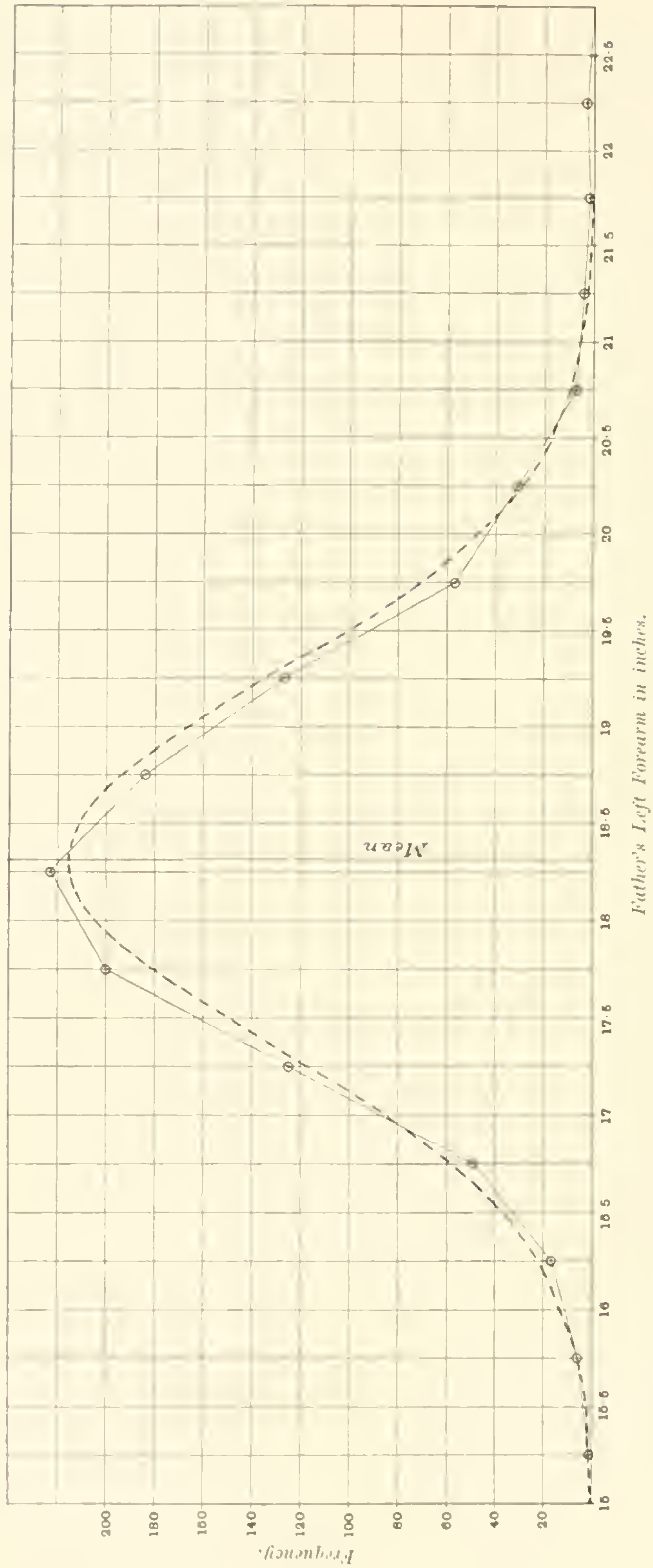
* It is almost in vain that one enters a protest against the mere graphical representation of goodness of fit, now that we have an exact measure of it. As typical cases in which quite recently arguments are based on mere graphical appreciation, I would refer to an article by Thorndyke on "Fertility in Man" (*Popular Science Monthly*, Vol. 63, pp. 64 and 84) wherein the skewness of fertility distributions is denied on graphical appreciation of curves, which are analytically skew by odds of the order of a 1000 to 1! Another transgressor is Johannsen, who in his recent work *Ueber Erblichkeit in Populationen und in reinen Linien*, asserts on mere graphical appreciation that certain data are normal and other non-normal and bases arguments on these assertions, whereas the eye alone cannot possibly judge whether or no his distributions follow the normal law. If biologists use biometric methods, they must be reminded that no vague appreciation will answer biometric problems, they must study sufficient mathematics to apply the necessary tests and criteria on which alone biometric arguments can be safely based.

Diagram VI. *Distribution of Forearm.*



The following is the Table for forearm in sons.

Forearm in Sons. 1156 *Cases.* *Mean* = 18″·52, *Standard Deviation* = ″·983.

Forearm in inches	*14—14·5*	*14·5—15*	*15—15·5*	*15·5—16*	*16—16·5*	*16·5—17*	*17—17·5*	*17·5—18*	*18—18·5*	*18·5—19*	*19—19·5*	*19·5—20*	*20—20·5*	*20·5—21*	*21—21·5*	*21·5—22*	*22—22·5*	*22·5—23*	*23—23·5*
Observed Frequency	1	1	1	5·5	10	39·5	95	177·5	260·5	225	166·5	105	41	14·5	7·5	4·5	—	—	1
Normal Frequency	·2		1·0	4·7	17·1	47·5	102·7	171·7	223·8	225·7	177·3	107·7	51·0	18·7	5·3	1·2	·2		

This Table as well as the previous one suggests that a small but sensible element of skewness in the forearm as well as the outlying group contributes to the divergence from normality.

It will be seen that our present data justifies Mr Galton's original use for stature of the normal curve and the normal surface, i.e.

$$z = \frac{N}{2\pi\sigma_x\sigma_y\sqrt{1-r^2}}\, e^{-\frac{1}{2}\frac{1}{1-r^2}\left\{\frac{x^2}{\sigma_x{}^2} - \frac{2xyr}{\sigma_x\sigma_y} + \frac{y^2}{\sigma_y{}^2}\right\}}$$

(where $z\,\delta x\,\delta y$ is the frequency of a group of relative pairs having characters with deviations from their means lying between x, y and $x+\delta x$, $y+\delta y$; N being the total number of pairs, σ_x, σ_y being the standard deviations, and r the coefficient of correlation of the two characters: see *Phil. Trans.* Vol. 187, A, p. 264 *et seq.*). It also is fully justified for span and even for forearm (if we remember that there exists a small group of "outliers"). The normality of the distribution adds little, however, to our investigation, as long as we can show that the regression is practically linear (see Diagram III.). The practical value of normality arises chiefly when we pass from measurable characters in man to those that are not capable of exact quantitative measurement, for here every exception to normality weakens our general position.

The general linearity of our regression lines enables us in the present case to apply a simple theory, as soon as we have calculated the means, the standard deviations, and the correlations of the various characters.

These will enable us, by using the formulae of simple or multiple correlation, which depend simply on linearity, to predict the probable character in any individual from a knowledge of one or more parents or brethren ("siblings," = brothers or sisters). But without further assumption they do not enable us to test the effect of long-continued selection in establishing stocks; for we have no ancestral correlations, beyond the parental, for the characters dealt with. Ancestral correlations beyond the parental are, however, known for man in eye-colour inheritance (up to great-grandparents), for horses in coat-colour (up to great-great grandparents), and for dogs in coat-colour (up to grandparents). Hence, if the parental correlations for men, horses and dogs are sensibly the same, we shall have small hesitation in assuming that the ancestral correlations for stature, span and forearm in man are closely alike in value to those for his eye-colour and for other characters in horse or dog. We shall thus be able to extend our theory, so as to deduce from our data the rate at which selection, natural or artificial, would establish stocks in man, and further, the limitations there are to the conception of an indefinitely active regression following on the suspension of selection.

It will be found that as far as the actual values are concerned our *Family Records* give values for heredity in man very sensibly larger than Mr Galton's stature data, and much closer to those obtained from his eye-colour data and for coat-colour in horses and dogs.

(iv) *Size and Variability of Characters in the two Generations.*

I will first consider whether there is a sensible change in type between the older and younger generation of our own epoch. The problem is not so easy to answer as it might *à priori* appear to some. We have the following results:

TABLE I. *Alteration in Type.*

MEANS		Stature	Span*	Forearm
1st Generation	Father ...	67″·68 ± ·06	68″·67 ± ·07	18″·31 ± ·02
	Mother ...	62″·48 ± ·05	61″·80 ± ·06	16″·51 ± ·02
2nd Generation	Son	68″·65 ± ·05	69″·91 ± ·06	18″·52 ± ·02
	Daughter...	63″·87 ± ·05	63″·40 ± ·05	16″·75 ± ·02
STANDARD DEVIATIONS		Stature	Span	Forearm
1st Generation	Father ...	2″·70 ± ·04	3″·14 ± ·05	0″·96 ± ·01
	Mother ...	2″·39 ± ·01	2″·81 ± ·04	0″·86 ± ·01
2nd Generation	Son	2″·71 ± ·04	3″·11 ± ·04	0″·98 ± ·01
	Daughter...	2″·61 ± ·03	2″·94 ± ·04	0″·91 ± ·01
COEFFICIENTS OF VARIATION		Stature	Span	Forearm
1st Generation	Father ...	3″·99 ± ·06	4″·64 ± ·07	5″·21 ± ·08
	Mother ...	3″·83 ± ·06	4″·62 ± ·07	5″·21 ± ·08
2nd Generation	Son	3″·95 ± ·06	4″·51 ± ·06	5″·29 ± ·07
	Daughter...	4″·09 ± ·05	4″·71 ± ·06	5″·13 ± ·07
ORGANIC CORRELATIONS		Stature and Span	Span and Forearm	Forearm and Stature
1st Generation	Father ...	·783 ± ·008	·752 ± ·009	·640 ± ·012
	Mother ...	·756 ± ·009	·677 ± ·011	·597 ± ·013
2nd Generation	Son	·802 ± ·007	·758 ± ·008	·686 ± ·011
	Daughter...	·828 ± ·006	·771 ± ·007	·716 ± ·009

---

* We note here a secondary sexual difference, the span on the average is about 1″ greater than the stature in man, and about ·5″ less than the stature in woman.

Now this Table contains a number of most interesting points.

In the first place the probable errors show us that for all three characters in both sexes the younger generation is distinctly larger than the older generation, son than father, daughter than mother. Is this a real progress in type? Taking Mr Powys' diagram for shrinkage in stature*, we should expect our men to reach a maximum at about 28 and our women at 25. Hence, since the average age of our younger generation is not more than 22 years, the younger generation cannot have reached its maximum. On the other hand, our average age of parents must be about 50. Let us suppose them to be 55 even. The difference in age of parents and offspring would thus mark a shrinkage of about ·5″ at a maximum. But the difference between fathers and sons is about an inch for stature and span, and for mothers and daughters about an inch and a half. It seems impossible therefore to attribute the whole change between the two generations to old-age shrinkage. In the next place, can it be due to periodic selection, i.e. only a portion of the younger generation become fathers and mothers? If so, we should expect not only a change in type, but a change in variability between the two generations. Comparing the standard deviations of fathers and sons, we see that fathers and sons are within the limits of random sampling equally variable. On the other hand daughters' standard deviations are in every case sensibly larger than those of their mothers. It would thus seem highly probable that the causes at work in the cases of the two sexes are not entirely the same. Mothers of adult children are a more stringently selected portion of the population than fathers appear to be. Of course some change in type between mothers and adult daughters is undoubtedly due to the fact of child-bearing, independent of any selection in childbed. But it is difficult to see how a physiological effect of this kind could change variability as well as type. I have shown that there is a slight correlation between size and fertility in women†, and this may be partially the source of the observed effect. Whether, however, the result be due to natural or reproductive selection, the change in the variability of the two generations of women seems to me to indicate that there is a selective change going on in the women of the middle classes in this country. The difference in type between fathers and sons,—since there is no change of variability,—might be more likely to be due to improved physical exercise. Of course a portion of the change in the women must also be attributed to this, but the change in variability forbids, I think, its being entirely attributed to this source.

However we judge the matter, whether we consider it due to selection, or to better environment, nourishment, or exercise, there seems no reason to suppose that the population, as far as the middle classes are concerned, is degenerating. In span, stature and forearm the younger generation is sensibly better than its parents.

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* *Biometrika*, Vol. I. p. 47.

† *R. S. Proc.* Vol. 59, p. 303. See also Vol. 66, p. 28 *et seq.*

If we compare the two sexes, we see that except in the matter of stature the married woman is *relatively* as variable as the married man, while in all three characters the young woman is relatively more variable than the young man. The supposed preponderance of male variability is thus again very fully negatived, for large statistics of typical physical characters in mankind*.

Turning to the correlations we see (*a*) that in the older generation the mother is less highly correlated than the father, (*b*) that in the younger generation the son is less highly correlated than the daughter, (*c*) that the younger generation of both sexes is more highly correlated than the older generation. Now the effect of selection is to reduce correlation, hence if selection—a selective death-rate—be a real factor in the case of man and we know it to be so, we should certainly expect the correlations between the ages of youth and of middle life to be reduced. They are thus reduced, but far more markedly so in the case of woman than in that of man. Now as far as our data at present reach we know that the male baby is more variable and more highly correlated than the female†. In youth the woman is more variable and more highly correlated than the man: in adult age after child-bearing she is less highly correlated and perhaps very slightly less variable. It would thus seem that between birth and manhood the male is selected and falls in both variability and correlation below his sister. With womanhood comes her period of selection, sexual selection for wifehood, natural and reproductive selection for motherhood. These act with a little expected intensity and leave mothers of adult families with far less variability and correlation than their husbands have.

Of course these changes in variability and correlation may be partly growth changes, but since on the average the man reaches his maximum size four or five years later than the woman and at least four or five years beyond the average age of our group sons, it is difficult to account for the wide difference in variation and correlation between daughters and mothers as compared with that between sons and fathers by growth changes only.

I am inclined to think Table I. is very illustrative of the nature of selection among mankind, and further that it is also hopeful, not as regards the quantity, of which it takes no account, but as regards the quality of the offspring of a fair sample of the English middle classes.

(v) *Direct Assortative Mating in Man.*

We have seen above that all women, if they indeed become wives, do not become the mothers of adult children, i.e. the mothers of the second generation are not a random sample of their own generation. However it may arise there is certainly a "preferential mating"* taking place. I think we may safely assert that the first factor of sexual selection is active in man. I now turn to the second factor, "assortative mating." If certain women are rejected, at any rate as mothers of adult children, do the remainder mate at random as far as the above three characters are concerned? The answer is most decidedly in the negative, there is a very sensible resemblance in size between husband and wife, which *à priori* I should have said was hardly conceivable. Table II. gives the direct and cross coefficients for assortative mating in man.

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* See *The Chances of Death*, Vol. I. pp. 256—377. A recent criticism by Mr Havelock Ellis of my view that there is no preponderating variability of man over woman seems to need no reply, for the author does not appear to understand what weight is to be given to scientific evidence as compared with vague generalities.

† *R. S. Proc.* Vol. 66, p. 25.

TABLE II.

Assortative Mating. Based on 1000 *to* 1050 *Cases of Husband and Wife.*

	Husband's Character	Wife's Character	Correlation and Probable Error	Symbol
Direct	Stature	Stature	·2804 ± ·0189	r_{12}
	Span	Span	·1989 ± ·0204	r_{31}
	Forearm	Forearm	·1977 ± ·0205	r_{56}
Cross	Stature	Span	·1820 ± ·0201	r_{14}
	Stature	Forearm	·1403 ± ·0204	r_{16}
	Span	Stature	·2023 ± ·0199	r_{32}
	Span	Forearm	·1533 ± ·0203	r_{36}
	Forearm	Stature	·1784 ± ·0201	r_{52}
	Forearm	Span	·1545 ± ·0203	r_{54}

We see at once that between the same physical characters in the husband and wife of adult children there is a correlation of upwards of ·2, a most remarkable degree of resemblance, greater than that of great-grandparents to their great-grandchildren (about ·19†), and probably greater than that of first cousins to each other. We could hardly want stronger evidence of the existence of assortative mating in man, i.e. of the actuality of sexual selection. I had previously found‡ from Mr Galton's *Family Records*, that the correlation in stature between *husband and wife* was ·09 ± ·05, but between *father and mother* of adult offspring was ·18 ± ·02. Considering the comparative smallness of material, the latter result is in very good agreement with the present, but it seems to indicate that a portion of the observed resemblance in the parents of adult offspring is due to reproductive selection, i.e. homogamy being a factor of fertility. If the parents of adult children are on the average more alike than first cousins, then it follows that any evils which may flow from first cousin marriage depend not on likeness of characters, but on sameness of stock§.

That the whole result, further, is not due to a mere general custom of men and women mating with persons not differing widely from them in *stature*, is shown by the sensible correlation there is in eye-colour between husband and wife, i.e. $\cdot 10 \pm \cdot 04$*, which is closely in agreement with the results for stature of husband and wife from the same data.

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* *Phil. Trans.* Vol. 187, p. 253 *et seq.* See especially p. 258.

† *Biometrika*, Vol. II. p. 221.

‡ *R. S. Proc.* Vol. 66, p. 30.

§ I have discussed this point more at length, *R. S. Proc.* Vol. 66, p. 29.

We may, however, estimate how far mating with regard to stature would produce resemblances in span and forearm. Let the subscripts 1, 3, 5 refer to three organs in a male of the population who marries, and 2, 4, 6 to the corresponding organs of a female of the marrying part of the population. Then $r_{13}, r_{35}, r_{51}, r_{24}, r_{46}, r_{62}$ are organic correlations such as we have tabled on p. 370. All correlations such as $r_{12}, r_{14}, r_{16}, r_{32}, r_{34}, r_{36}, r_{52}, r_{54}, r_{56}$ are zero, if we mated pairs at random. Now let them be assortatively mated and let $\rho_{12}, \rho_{34}, \rho_{56}$ represent the degree of resemblance in the sexual selection. Let $\mathbf{r}_{12}, \mathbf{r}_{34}, \mathbf{r}_{56}$ be the apparent correlations of mated pairs; then $\mathbf{r}_{12}$ will not be equal to ρ_{12}, for it is partly due to the degree of assortative mating indicated in ρ_{34} and ρ_{56}, because 3 and 5 are organically correlated with 1, and 4 and 6 with 2; thus the selection of 3's and 5's to associate with 4's and 6's would indirectly influence the relationship of 1 and 2, even if there were no direct associating of 1's and 2's. The relationship of $\mathbf{r}_{12}, \mathbf{r}_{34}, \mathbf{r}_{56}$ to $\rho_{12}, \rho_{34}, \rho_{56}$ may be easily found from my memoir on the influence of selection on variability and correlation†. We have only to put in the formulae of pp. 15—17 the appropriate values for the population described above and we find:

$$\left.\begin{aligned}\mathbf{r}_{12} &= \rho_{12} + \rho_{34} r_{13} r_{24} + \rho_{56} r_{15} r_{26}\\ \mathbf{r}_{34} &= \rho_{12} r_{13} r_{24} + \rho_{34} + \rho_{56} r_{35} r_{46}\\ \mathbf{r}_{56} &= \rho_{12} r_{15} r_{26} + \rho_{34} r_{35} r_{46} + \rho_{56}\end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}.$$

Now suppose that 1 and 2 represent statures, 3 and 4 spans, and 5 and 6 forearms. Then if all assortative mating be due to selection of stature, we might put ρ_{34} and ρ_{56} zero above and we should have:

$$\rho_{12} = \mathbf{r}_{12}, \quad \mathbf{r}_{34} = \mathbf{r}_{12} r_{13} r_{24}, \quad \mathbf{r}_{56} = \mathbf{r}_{12} r_{15} r_{26}.$$

But $r_{13} = \cdot 7829$, $r_{24} = \cdot 7560$, $r_{15} = \cdot 6397$, $r_{26} = \cdot 5968$, and $r_{12} = \cdot 2804$. This leads to

$$\mathbf{r}_{34} = \cdot 1660 \text{ and } \mathbf{r}_{56} = \cdot 1071,$$

as against the observed values:

$$\mathbf{r}_{34} = \cdot 1989 \text{ and } \mathbf{r}_{56} = \cdot 1977.$$

The former values are too small in both cases and, I think, we may safely assert, that the likeness of husband and wife in forearm and span is not *solely* due to a selection of stature.

Another explanation of these high coefficients of assortative mating has been suggested to me, namely that the population of England is built up of a number of local races, and that men and women mate within their locality. Now it appears to me that this argument would be far more valid, if my material was drawn in bulk from local lower middle and artizan classes. But it is very doubtful how far it is true of the middle classes, such as provide the students at the London colleges. The middle classes undoubtedly marry in their own "sets," but these are hardly local sets. Further, a wide series of assortative mating observations have been made on another, wholly different class of characters, in which local race is regarded, and the coefficients come out as high as in the present data. Hence, I think, we are forced to the conclusion that the bulk of the observed resemblance in physical characters between parents is due to a direct, if quite unconscious, selection of like by like, and possibly in a contributory degree to a likeness in parents for the characters under consideration emphasising their fertility.

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* *Phil. Trans.* Vol. 195, A, p. 113. See also pp. 118—150, where it is shown that heterogamy rather than homogamy in eye-colour tends to increased fertility. If this be confirmed, eye-colour differs much in effect from stature.

† *Phil. Trans.* Vol. 200, p. 1 *et seq.*

The amount of "consciousness" in the selection may possibly be measured by the difference between the stature-stature correlation and those for span-span, and forearm-forearm.

(vi) *Cross-Assortative Mating in Man.*

The second part of Table II. gives the cross-coefficients, for example, the correlation between husband's stature and wife's forearm. We might *à priori*, perhaps, anticipate that the correlation between a first organ in the husband and a second in the wife, would be equal to the correlation between the second in the husband and the first in the wife. This is actually the case for span and forearm, and, perhaps, we may consider for stature and span; the results for stature and forearm are less close than we might have anticipated, but the work has been revised without the discovery of any error. Relations such as:

$$\mathbf{r}_{14} = \mathbf{r}_{32}, \quad \mathbf{r}_{16} = \mathbf{r}_{52},$$

do not, however, appear to be theoretically necessary.

The problem now arises: are cross correlations between characters in husband and wife, solely due to selection of direct characters?

I think this may be roughly tested in the following manner. Suppose *only* these organs to be selected and the direct selection coefficients to be ρ_{12}, ρ_{34}, ρ_{56}, as before. They may be found from equations (i)[*] and we have the values:

$$\rho_{12} = \cdot 2374, \quad \rho_{34} = 0053, \quad \rho_{56} = \cdot 1043.$$

Thus there is most immediate selection of stature, a sensible selection of forearm, and practically none of span.

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[*] These give numerically:

$$\begin{aligned} \cdot 2804 &= \rho_{12} + \cdot 5919\rho_{34} + \cdot 3818\rho_{56}, \\ \cdot 1989 &= \cdot 5919\rho_{12} + \rho_{34} + \cdot 5087\rho_{56}, \\ \cdot 1977 &= \cdot 3818\rho_{12} + \cdot 5087\rho_{34} + \rho_{56}. \end{aligned}$$

Now, if there be no *immediate* cross selection of other than these three organs and no immediate direct selection we should expect to find:

$$\left.\begin{aligned}
\mathbf{r}_{32} &= \rho_{12}r_{13} + \rho_{34}r_{24} + \rho_{56}r_{35}r_{26}\\
\mathbf{r}_{14} &= \rho_{12}r_{24} + \rho_{34}r_{13} + \rho_{56}r_{15}r_{46}\\
\mathbf{r}_{52} &= \rho_{12}r_{15} + \rho_{34}r_{35}r_{24} + \rho_{56}r_{26}\\
\mathbf{r}_{16} &= \rho_{12}r_{26} + \rho_{34}r_{31}r_{46} + \rho_{56}r_{15}\\
\mathbf{r}_{36} &= \rho_{12}r_{13}r_{26} + \rho_{34}r_{46} + \rho_{56}r_{35}\\
\mathbf{r}_{54} &= \rho_{12}r_{15}r_{24} + \rho_{34}r_{35} + \rho_{56}r_{46}
\end{aligned}\right\} \quad \dots\dots\dots\dots\dots\dots\dots\dots \text{(ii).}$$

Substituting the ρ's and the organic correlations in (ii) we find:

TABLE III.

Coefficients of Cross Assortative Mating.

Husband's Character	Wife's Character	Observed Value	Calculated Value
Span	Stature	·2023	·2327
Stature	Span	·1820	·2288
Forearm	Stature	·1784	·2171
Stature	Forearm	·1403	·2152
Span	Forearm	·1533	·1929
Forearm	Span	·1545	·1894

We conclude from this Table that: since the calculated values are all larger than the observed, the hypothesis that only direct selection of these three characters takes place is not valid. There must be direct selection of other correlated organs, or in some manner, as yet inexplicable, also an immediate cross assortative mating in man*. Generally, the results given in the present and the previous section for assortative mating, and in section (iv) for preferential mating, indicate that in future a greater degree of attention must be paid to sexual selection. It can hardly be so significant in the case of man, where most people would probably *à priori* suppose it of no account, and yet fail to play an important part in wild life. In particular, experimental enquiry on the relation of homogamy to fertility,—the likeness not being due to in-breeding—would be of very great value. It is clear, that "negative"† natural selection accompanied by the correlation of homogamy and fertility would much aid us in comprehending the origin of species.

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* My own view, for which I have small evidence at present, is that the functions of sex are far more highly correlated with the physical characters in man than is generally supposed, and that the fertility of any given pair is very delicately atuned to the relative proportions of their frames. Hence when we come to correlate the physical characters in the parents of adult children, we find not only high direct but also high cross correlations, which seem inexplicable on any hypothesis of conscious assorting at mating. The only way to test this is to compare the correlations of husbands and wives at marriage with those of parents of adult children. We have already seen that these in certain cases sensibly differ. (See p. 373 and p. 374 footnote.)

† A selection for destruction not survival: see *Phil. Trans.* Vol. 200, A. p. 59.

Although we are unable at present to account for the high coefficients of cross-assortative mating in man, it is possible to give an empirical formula, which will enable us to determine these coefficients in terms of the direct assortative mating coefficients and the organic correlations well within the limits of the probable errors of our results. Clearly the cross-assortative mating coefficients ought to vanish with both direct and organic correlations. Hence, if p, q refer to two organs in the husband and p', q' to the same pair in the wife, we should expect the cross correlation $r_{pq'}$ to be of the form:

$$r_{pq'} = C r_{pp'} r_{p'q'} + C' r_{qq'} r_{pq}.$$

Having satisfied myself that C and C' might be taken as practically equal, I found C as the mean of the last six entries in Table II. There resulted the formulae

$$\left.\begin{aligned} r_{pq'} &= \cdot 5342\ (r_{pp'} r_{p'q'} + r_{qq'} r_{pq}), \\ r_{p'q} &= \cdot 5342\ (r_{pp'} r_{pq} + r_{qq'} r_{p'q'}), \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\text{iii})$$

whence I found the following results.

TABLE III *bis*.

Calculated and Observed Cross Coefficients in Husband and Wife.

Husband's Character	Wife's Character	Observed Value	Calculated Value	Difference
Span	Stature	·202	·198	+ ·004
Stature	Span	·182	·196	− ·014
Forearm	Stature	·178	·159	+ ·019
Stature	Forearm	·140	·157	− ·017
Span	Forearm	·153	·151	+ ·002
Forearm	Span	·155	·151	+ ·004

The differences are well within the probable errors, and the above formulae may I think be safely used, if the cross coefficients are unknown.

(vii) *Direct Parental Inheritance.*

For the resemblance in like organs between offspring and parents we have for our three organs twelve cases. The correlations deduced from Appendix Tables XXII.—XXXIII. are given in Table IV. below.

It is impossible to regard these results without extreme satisfaction, not only as confirmation of the general reliability of the material, but also for the weighty evidence they bring for the nature of inheritance in man. When one remembers the labour of collecting the measurements, the days spent in tabling and reducing it, and the doubts which not unnaturally arose as to its value and the value of the tedious labour spent on it, the sense of satisfaction felt may be considered pardonable. The surprising agreement of the results—well within the probable errors—for each character is to be noted in the first place. Considering that the measurements are made on more than 4000 individuals of different sexes in more than 1000 families, the conviction is complete that these numbers correspond to a reality in nature. From them we may safely draw the following conclusions for the organs examined:

TABLE IV.

Coefficients of Heredity. Parents and Offspring.

Character	Father and		Mother and	
	Son	Daughter	Son	Daughter
Stature	·514 ± ·015	·510 ± ·013	·494 ± ·016	·507 ± ·014
Span	·454 ± ·016	·454 ± ·014	·457 ± ·016	·452 ± ·015
Forearm	·421 ± ·017	·422 ± ·015	·406 ± ·017	·421 ± ·015

(*a*) The son and daughter are equally influenced by their father, and equally influenced by their mother.

While a change of sex does appear to weaken hereditary influence in the eye-colour of man*, it does not appear to have any perceptible influence on the size of the human frame.

(*b*) In their influence on offspring there is no average prepotency of either father or mother, whatever there may be in individual cases.

(*c*) The inheritance of all characters does not appear to be the same.

The inheritance of forearm is for all four cases sensibly less than the inheritance of span, and that of span less than that of stature. We might as a probability put forward the following statement for further investigation.

(*d*) The more complex a character the greater the intensity of hereditary resemblance.

The fact that the correlation falls below ·5 with the simplicity of the character under consideration seems to suggest, however, that the reduction of the intensity cannot be due to an "alternative inheritance" in the case of the simple components of the character†.

For the mean values we have the following results:

Mean parental inheritance, father to son: ·463
" " " " to daughter: ·462
" " " mother to son: ·452
" " " " to daughter: ·460
Mean parental inheritance for both sexes and all characters: ·460.

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* *Biometrika*, Vol. II. pp. 237—240.

† See *R. S. Proc.* Vol. 66, p. 142, and *Natural Inheritance*, p. 139.

I think we may fairly take the intensity of inheritance for measurable characters in man to be ·46, or even for rougher work ·5. It may be as well to put on record here the principal results for heredity in the direct line so far reached. I omit the results obtained in my memoir of 1895*, for I consider my present data to replace that series.

TABLE V.

Parental Inheritance in Different Species.

Species	Character	Mean Value	No. of Pairs used	Source	Remarks
Man	Stature	·506	4886	Present Memoir	
,, ...	Span	·459	4873	ditto	—
,, ...	Forearm	·418	4866	ditto	—
,, ...	Eye Colour	·495	1000	*Phil. Trans.* Vol. 195, p. 106	—
Horse	Coat Colour	·522	4350	*Phil. Trans.* Vol. 195, p. 93	—
Basset Hound ...	Coat Colour	·524	823	*R. S. Proc.* Vol. 66, p. 154	Dams only used
Greyhound ...	Coat Colour	·507	9279	Unpublished data for two characters	Dams and Sires both used
Aphis (*Hyalopterus Trirhodus*) ...	Right Antenna / Frontal Breadth	·439	368	*Biometrika*, Vol. I. p. 129	Ratios only taken to free from growth factor
Daphnia *Magna* ...	Protopodite / Body Length	·466	96	*R. S. Proc.* Vol. 65, p. 154.	

I consider that this table contains the most reliable data we yet have collected and reduced for parental influence on offspring.

The general mean of the whole of these series is ·48 and so far as we have yet gone, we may I think conclude, that:

(*a*) There is no reason for supposing parental heredity to be stronger in one species than a second.

(*b*) Its values lie between ·42 and ·52 and cluster round ·48.

Thus for most practical purposes we may assume parental heredity for all species and all characters to be approximately represented by a correlation of ·5.

In the course of the past 8 years many cases of parental inheritance have been worked out by the biometricians associated with me at University College, some of the most important of these are still unpublished, others have been replaced by far more reliable data; in further cases we know that the material was doubtful, e.g. the cephalic index for *fathers* and children of the North American Indians, or *sire* and offspring in the Basset Hounds. In such cases better material has been sought and our first results modified†. But in the present controversial phase of the theory of heredity, it seems well to hold no material back simply because one knows it to be untrustworthy. I therefore give the frequency distribution for every coefficient of parental correlation I am aware of, neither weighting them with the number of pairs on which they are based, nor remarking on the relative reliability of the data, which covers plants, insects and animals.

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* *Phil. Trans.* Vol. 187, p. 253 *et seq.*

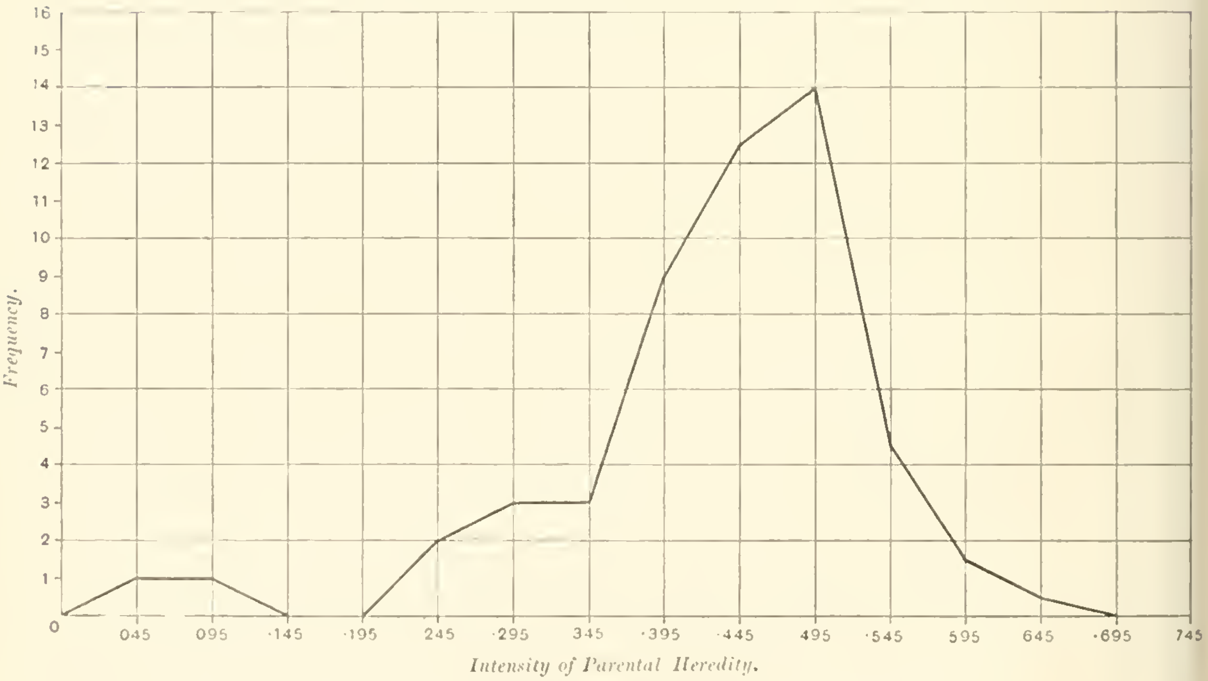
† For example the greyhounds have shown that anomalies of the Basset Hound results were peculiar to the material, the cephalic index is inherited quite normally when we test it on material with reliable parentage, etc. etc.

Frequency Distribution of Correlation Coefficients of Parental Heredity.

Magnitude of Correlation	·02—·07	·07—·12	·12—·17	·17—·22	·22—·27	·27—·32	·32—·37	·37—·42	·42—·47	·47—·52	·52—·57	·57—·62	·62—·67	Total
Frequency	1	1	—		2	3	3	9	12·5	14	4·5	1·5	·5	52

This distribution is represented graphically in the accompanying diagram; the mean and standard deviation of the system are ·430 ± ·010 and ·107 respectively. The fact of the cluster and its quantitative intensity are thus rendered obvious. The four extreme observations on the left are due to the Basset Hound Sires and North American Indian Fathers, both involving doubtful paternity*. If we omit

DIAGRAM VII. *Distribution of Correlation Coefficients in 52 Cases of Parental Heredity.*



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* Both series are also very small, 100 to 400, as compared with the 1000 or more of most of the other series. As to their questionable character see *R. S. Proc.* Vol. 62, p. 414, and Vol. 66, p. 158 and especially footnote p. 159.

these as most certainly questionable the mean result is ·453 ± ·007, the standard deviation being ·071. Thus ·45 may, I think, be justifiably taken in future to represent the approximate value of parental heredity, in cases where no direct observations have been made for the character and species under consideration. I prefer, however, the ·46 to ·5 of the *best* of the above series.

I now pass to the prediction formulae, i.e. the regression lines and planes, from which the probable value of a character in the offspring may be determined when the value of the character in the parentage is known.

If the subscript c denote child and p parent; and m be the mean, C the character; we have for prediction from one parent:

$$C_c = m_c + \frac{r_{cp}\sigma_c}{\sigma_p}(C_p - m_p) \quad \text{.........................(iv)}$$

with a standard deviation for the array of value $\Sigma_c = \sigma_c\sqrt{1 - r^2_{cp}}$.

If we predict from two parents p_1 and p_2, the formula is:

$$C_c = m_c + \frac{r_{cp_1} - r_{cp_2}r_{p_1p_2}}{1 - r^2_{p_1p_2}}\frac{\sigma_c}{\sigma_{p_1}}(C_{p_1} - m_{p_1}) + \frac{r_{cp_2} - r_{cp_1}r_{p_1p_2}}{1 - r^2_{p_1p_2}}\frac{\sigma_c}{\sigma_{p_2}}(C_{p_2} - m_{p_2}) \quad \text{...(v)}$$

with a standard deviation for the array of

$$\Sigma_c = \sigma_c\sqrt{\frac{1 - r^2_{cp_1} - r^2_{cp_2} - r^2_{p_1p_2} + 2r_{cp_1}r_{cp_2}r_{p_1p_2}}{1 - r^2_{p_1p_2}}}.$$

Using these formulae we have the following results[*]:

A. *Stature.*

For Son:

(1) Probable Stature = 33″·73 + ·516 (Father's Stature) ± 1″·56,
(2) Probable Stature = 33″·65 + ·560 (Mother's Stature)[†] ± 1″·59,
(3) Probable Stature = 14″·08 + ·409 (Father's Stature)
+ ·430 (Mother's Stature) ± 1″·42.

For Daughter:

(4) Probable Stature = 30″·50 + ·493 (Father's Stature) ± 1″·51,
(5) Probable Stature = 29″·28 + ·554 (Mother's Stature) ± 1″·52,
(6) Probable Stature = 10″·82 + ·386 (Father's Stature)
+ ·431 (Mother's Stature) ± 1″·33.

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[*] The actual tables of correlation are given in the Appendix and from them it will be seen that all possible pairs were used in each case for determining the correlation. Thus the standard deviations and means vary slightly from table to table, of course well within their probable errors. The formulae here given were, however, obtained by using the means and standard deviations which were adopted for Table I.

[†] If Father and Mother are to contribute indifferently to Son's stature, the parental statures should be in the ratio of about 560 to 516, which is very nearly the ratio of 1·085 to 1, and almost exactly equal to the 1·083 to 1 of ratio of Father's to Mother's average stature.

B. *Span.*

For Son:

(7) Probable Span $= 38''\cdot46 + \cdot458$ (Father's Span) $\pm 1''\cdot89$,

(8) Probable Span $= 38''\cdot38 + \cdot511$ (Mother's Span) $\pm 1''\cdot88$,

(9) Probable Span $= 18''\cdot04 + \cdot375$ (Father's Span) $+ \cdot423$ (Mother's Span) $\pm 1''\cdot70$.

For Daughter:

(10) Probable Span $= 34''\cdot20 + \cdot425$ (Father's Span) $\pm 1''\cdot77$,

(11) Probable Span $= 34''\cdot18 + \cdot473$ (Mother's Span) $\pm 1''\cdot77$,

(12) Probable Span $= 14''\cdot70 + \cdot355$ (Father's Span) $+ \cdot394$ (Mother's Span) $\pm 1''\cdot61$.

C. *Forearm.*

For Son:

(13) Probable Forearm $= 10''\cdot65 + \cdot430$ (Father's Forearm) $\pm ''\cdot60$,

(14) Probable Forearm $= 10''\cdot88 + \cdot463$ (Mother's Forearm) $\pm ''\cdot60$,

(15) Probable Forearm $= 5''\cdot58 + \cdot362$ (Father's Forearm) $+ \cdot383$ (Mother's Forearm) $\pm ''\cdot56$.

For Daughter:

(16) Probable Forearm $= 9''\cdot43 + \cdot400$ (Father's Forearm) $\pm ''\cdot56$,

(17) Probable Forearm $= 9''\cdot40 + \cdot445$ (Mother's Forearm) $\pm ''\cdot56$,

(18) Probable Forearm $= 4''\cdot50 + \cdot334$ (Father's Forearm) $+ \cdot371$ (Mother's Forearm) $\pm ''\cdot51$.

On the right is given in each case the probable error of the prediction*. We see from these formulae that with the selection of one parent only, the offspring rise to within 40 to 50 per cent. of the selected value; with the selection of *both* parents to within 70 to 80 per cent. of it. The diminution of the variability of the array due to two selected parents, is however only slightly less than that due to the selection of one parent only.

If we selected for two generations we should have offspring the same very nearly as the selected ancestry†. With our values for parental correlation, it is obvious that two or three generations of selection will suffice to bring the average of the offspring sensibly up to the selected ancestry, and the regression after this, if the stock mates with its like, will be very small or zero*. We cannot unfortunately on the present material determine absolutely its value; for, in the first place we have no correlations with grandparents or higher ascendants, and in the second place the assortative mating coefficients are so large, that we cannot afford to neglect them to a first approximation, as we have done for the case of eye-colour in man and coat-colour in horses†.

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* In using these formulae for prediction, those not fully conversant with statistical theory, must bear in mind that they give only the mean or most probable results of a whole array of offspring due to *all* parents of definite characters. The validity of the formulae cannot be tested on merely individual cases. This warning is necessary because I have so often had *individual* cases in man or dogs cited as upsetting the whole of the ancestral law!

† "The Law of Ancestral Heredity," *Biometrika*, Vol. II. pp. 221—6.

What, however, the present investigation impresses upon one is this: Parental correlation being from ·45 to ·5 in value is much higher than we could anticipate from Mr Galton's *Natural Inheritance* data. Hence selection is far more rapid in its effects than we supposed a few years ago, two to four generations suffice to effect what we originally considered would need 6 to 8. Further, the regression after such selection may well be zero. I have not worked out yet the multiple regression formulae allowing for assortative mating; they present considerable difficulty owing to the complexity introduced by the correlations between relations-in-law due to such mating. But neglecting for a moment the effect of assortative mating, the series of ancestral correlations,

·46 ·32 ·23 ·16

proceeding by a factor ·7 would give a zero-regression and not differ widely from the ancestral correlations we know for eye-colour in man‡. I lay no stress on these particular numbers, but I wish to emphasise the point that a few generations of selection in the case of man suffice to establish a breed, and that regression for this breed may well be insensible.

(viii) *Cross Parental Inheritance.*

I have defined *cross* heredity to be the correlation of two different organs in two blood relations§. We are now for the first time in a position to estimate its magnitude.

We see at once that these coefficients of cross heredity are for some cases almost as large as the coefficients of direct heredity, and on the whole sensibly larger than the values which but a few years ago were supposed to be those of the

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[*] The physical aspect of this is perfectly easy to understand. When we select one parent the offspring advance 35 to 40 p.c. on the general population, when we select two parents 70 to 80 p.c. If we select two parents and four grandparents, there is another percentage increase which brings us up into the 90 per cents., and if we select for three generations we have nearly the 100 per cent. of the required character. Now suppose the selected stock to inbreed or otherwise mate with its likes for this special character. Why will the regression now that selection ceases be zero or insensible? For this simple reason, that while we cease to select within the stock, yet each new generation has an additional selected generation of ancestry behind it, and the influence of this ancestry balances the regressional tendency. This is the simple verbal explanation of the cessation of regression with selection. The algebraical expression of its possibility was first given in my memoir of 1898: see *R. S. Proc.* Vol. 62, p. 401.

[†] *Ibid.* p. 224. See also *R. S. Proc.* Vol. 62, p. 388.

[‡] *Biometrika*, Vol. II. p. 222. ϵ and ρ of p. 224 would be ·56 and ·44 respectively.

[§] *Phil. Trans.* Vol. 187, A, p. 259. See also *R. S. Proc.* Vol. 62, p. 410.

direct coefficients. We have thus, if it were needed, still further evidence that the original estimates of the strength of heredity were far too low.

TABLE VI.

Cross Parental Heredity Coefficients.

Parent	Parent's Character	Offspring	Offspring's Character	Correlation and Probable Error
Father ...	Stature	Son	Span	·418 ± ·017
,, ...	Span	,,	Stature	·399 ± ·017
,, ...	Stature	,,	Forearm	·370 ± ·018
,, ...	Forearm	,,	Stature	·355 ± ·018
,, ...	Span	,,	Forearm	·399 ± ·017
,, ...	Forearm	,,	Span	·400 ± ·017
Father ...	Stature	Daughter	Span	·423 ± ·015
,, ...	Span	,,	Stature	·407 ± ·015
,, ...	Stature	,,	Forearm	·341 ± ·016
,, ...	Forearm	,,	Stature	·383 ± ·016
,, ...	Span	,,	Forearm	·382 ± ·016
,, ...	Forearm	,,	Span	·396 ± ·015
Mother ...	Stature	Son	Span	·424 ± ·017
,, ...	Span	,,	Stature	·390 ± ·017
,, ...	Stature	,,	Forearm	·356 ± ·018
,, ...	Forearm	,,	Stature	·344 ± ·018
,, ...	Span	,,	Forearm	·345 ± ·018
,, ...	Forearm	,,	Span	·365 ± ·018
Mother ...	Stature	Daughter	Span	·431 ± ·015
,, ...	Span	,,	Stature	·385 ± ·016
,, ...	Stature	,,	Forearm	·387 ± ·015
,, ...	Forearm	,,	Stature	·318 ± ·016
,, ...	Span	,,	Forearm	·370 ± ·016
,, ...	Forearm	,,	Span	·362 ± ·016

One of the most difficult points to be sure about is the theoretical relationship which is to be expected between the intensities of direct and cross inheritance. If (i) all organs and characters were inherited at the same rate, and (ii) the organic correlations in younger and older generations were the same, and (iii) the variabilities of these generations, as measured by their coefficients of variability, were the same, then it follows that the mean of two corresponding coefficients of cross heredity is the product of the coefficient of direct heredity into the organic correlation*. But none of the three conditions stated above is accurately fulfilled, as we have seen, in the present material. Notably we find sensible divergence from the first. We may possibly attempt to allow for the first disturbing factor in the following manner: the cross-correlations should vanish (*a*) when the direct heredity is zero, and again (*b*) when the organic correlations are zero. Hence we might, if 1, 2 represent organs in one of a pair, and 1′, 2′ the same organs in the other of a pair of relatives, expect to find:

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* *R. S. Proc.* Vol. 62, p. 411.

$$r_{12'} = c\ r_{11'} r_{1'2'} + c'\ r_{22'} r_{12},$$
$$r_{1'2} = c'' r_{22'} r_{1'2'} + c''' r_{11'} r_{12},$$

where c, c', c'' and c''' are at present indeterminate.

Hence:

$$\tfrac{1}{2}(r_{12'} + r'_{1'2}) = r_{1'2'}\left(\frac{c r_{11'} + c'' r_{22'}}{2}\right) + r_{12}\left(\frac{c' r_{22'} + c'' r_{11'}}{2}\right).$$

Now if heredity were constant for all characters, we should have $r_{11'} = r_{22'}$, and we should reach the above proposition by putting $c = c' = c'' = c''' = \cdot 5$. Thus we should expect the c's to be equal to ·5 plus functions of $r_{11'}$, $r_{22'}$, r_{12} and $r_{1'2'}$, which vanish when $r_{11'} = r_{22'}$ and $r_{12} = r_{1'2'}$. What those functions may be it would probably be hard to determine. I therefore propose to write simply

$$\left.\begin{aligned} r_{12'} &= C(r_{11'} r_{1'2'} + r_{22'} r_{12}) \\ r_{1'2} &= C(r_{22'} r_{1'2'} + r_{11'} r_{12}) \end{aligned}\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(vi)}$$

and determine the values of C. These are given in the Table VII. below. We see at once that C is always greater than ·5, its mean value is ·5683. If we adopt this value we should have the following *empirical* formula to determine a cross heredity coefficient:

$$r_{12'} = \cdot 5683\,(r_{11'} r_{1'2'} + r_{22'} r_{12}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(vii)}.$$

But since the numerical factor is greater than ·5, and $r_{11'}$ and $r_{22'}$ as a rule somewhat less, we ought to get *rough* values of the cross coefficients from

$$r_{12'} = \tfrac{1}{4}(r_{1'2'} + r_{12}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ \ldots\ldots\ \text{(viii)}.$$

The values calculated from these empirical formulae are given in Table VIII. below with the differences.

The probable errors of these coefficients of cross correlations are given in Table VI. Formula (vii) gives 13 values above and 11 below the corresponding probable error. Formula (viii) gives 11 above it and 13 below it. The mean deviation of (vii) is ·019 and (viii) is also ·019. Thus the formulae are practically equally good so far. But (vii) gives 10 above and 14 below, while (viii) gives only 3 above and 21 below the observed values. Thus as an empirical formula (vii) is somewhat better than (viii), which is really based on the equality of all inheritance coefficients and their approximation to a value of ·5, assumptions only roughly true.

Practically either (vii) or (viii) would suffice for most purposes, and the manner in which they smooth the observed results, especially in making what we might *à priori* expect, near equality* between the pairs of corresponding cross correlations is itself an argument in their favour. Hence I should say that when the

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* See *R. S. Proc.* Vol. 62, p. 411.

TABLE VII.

Values of C from (vi).

Father	Son	*C*	Father	Daughter	*C*	Mother	Son	*C*	Mother	Daughter	*C*
Stature	Span	·5445	Stature	Span	·5439	Stature	Span	·5711	Stature	Span	·5660
Span	Stature	·5205	Span	Stature	·5250	Span	Stature	·5270	Span	Stature	·5081
Stature	Forearm	·5948	Stature	Forearm	·5425	Stature	Forearm	·6125	Stature	Forearm	·6299
Forearm	Stature	·5727	Forearm	Stature	·6029	Forearm	Stature	·5999	Forearm	Stature	·5280
Span	Forearm	·6039	Span	Forearm	·5729	Span	Forearm	·5553	Span	Forearm	·5840
Forearm	Span	·6057	Forearm	Span	·5934	Forearm	Span	·5914	Forearm	Span	·5741
Means	... ...	·5737	—		·5634	—		·5762	—		·5650

TABLE VIII.

Calculated and Observed Values of Cross Correlations.

Organs		Father and Son				Father and Daughter				Mother and Son				Mother and Daughter			
		From (vii)		From (viii)		From (vii)		From (viii)		From (vii)		From (viii)		From (vii)		From (viii)	
		Value	Δ	Value	Δ	Value	Δ	Value	Δ	Value	Δ	Value	Δ	Value	Δ	Value	Δ
Stature	Span	·436	– ·018	·396	·022	·442	– ·019	·403	·020	·422	·002	·390	·034	·433	– ·002	·396	·035
Span	Stature	·436	– ·037	·396	·003	·441	– ·031	·403	·004	·421	– ·031	·390	·000	·431	– ·046	·396	– ·011
Stature	Forearm	·354	·016	·332	·038	·357	– ·011	·339	·002	·330	·026	·321	·035	·349	·038	·328	·059
Forearm	Stature	·352	·003	·332	·023	·361	·022	339	·041	·326	·018	·321	·023	·343	– ·024	·328	– ·010
Span	Forearm	·375	·024	·378	·021	·378	·001	·381	·001	·353	– ·008	·359	– ·014	·360	·010	·362	·008
Forearm	Span	·375	·025	·378	·022	·379	·017	·381	·015	·351	·014	·359	·006	·358	·004	·362	·000

Δ = Observed – Calculated Value.

organic correlations and direct heredity coefficients are known the cross heredity coefficients may be found very closely from formula (vii). If the organic correlations are known, but no heredity coefficients at all, then the direct heredity coefficients may approximately be taken as equal to ·5 and the cross heredity coefficients approximately found from the organic correlations by formula (viii).

(ix) *Direct Fraternal Resemblance.*

I now turn to the observed degree of resemblance between brothers and sisters for the three characters we have measured in our Family Records. We have the following results:

TABLE IX.

Correlation Coefficients for Direct Fraternal Heredity.

Character	Brother and Brother	Sister and Sister	Brother and Sister	Mean
Stature	·511 ± ·028	·537 ± ·022	·553 ± ·013	·534
Span	·549 ± ·026	·555 ± ·021	·525 ± ·013	·543
Forearm	·491 ± ·029	·507 ± ·023	·440 ± ·015	·479
Mean	·517	·533	·506	·519
Eye Colour*	·517 ± ·020	·446 ± ·023	·462 ± ·022	·475
Total mean	·517	·511	·495	·508

Now there are certain differences in the entries in this Table, thus resemblance in siblings seems greater for stature and span, than for forearm or eye-colour, and again the resemblance of brother and sister seems on the average slightly smaller than the resemblance between siblings of the same sex. There are also certain irregularities, which I have no means of accounting for, and which seem larger than can be explained by random sampling†. Still there cannot be the least doubt from the above table that ·5 measures very closely the average degree of hereditary resemblance in human siblings, and that the correlation clusters closely round this value. As in the case of parental heredity we see a most marked increase in the intensity of hereditary resemblance when we compare our results with those obtained for stature some years ago‡. If we compare our Family Records with the School Records, of which I only cite at present the results for brothers, we find good agreement in the ·5 value. In the Table below each series involves 1000 to 2000 cases.

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* Francis Galton's eye-colour record reduced for my paper on Heredity in Man in *Phil. Trans.* Vol. 195, A, p. 106.

† The material, as the reader will see by consulting the actual tables, is far less.

‡ *Phil. Trans.* Vol. 187, A, p. 281. It was the difficulties associated with the data used in 1895 (see especially pp. 283—5 of above memoir) that led to the collection of heredity data which has been in progress since 1894.

TABLE X.

Fraternal Resemblance in Boys at School.

Character	Correlation	Character	Correlation
General Health... ...	·520	Cephalic Index	·486
Eye Colour	·539	Head Length	·501
Hair Colour	·621	Head Breadth	·593
Straightness of Hair ...	·498	Auricular Height	·554

Mean of Eight Characters ·539.

This compares well with ·517 of the above Family Records for Brothers. At the same time several of the above results are under revision, namely the absolute measurements of the head. In this case every boy was reduced to a standard age 12 by adding to or subtracting from his age the *average* growth found to take place between his age at measurement and the age of 12. A more subtle method of determining the allowance to be made for growth has been recently given by me in a paper published this year, only it involves a very great amount of numerical work, i.e. five instead of two correlation tables, and thus we have only been able so far to modify the correlation in the matter of head length*. I consider it better, however, to place above the correlations as found by one uniform method until we are in position to publish all the results revised. I anticipate that both head breadth and auricular height will ultimately be found to be really nearer to ·5 than appears above. Meanwhile it seems quite safe to sum up our results for fraternal correlation in man as follows:

(i) The degree of resemblance of brethren is closely the same for all characters.

(ii) The two sexes appear to be equally influenced by heredity.

(iii) The intensity of fraternal correlation in man is close to ·5, possibly slightly greater. But for practical purposes we may conveniently work with $\frac{1}{2}$ as a round number.

I now turn to what personally I consider one of the most obscure points in the quantitative determination of inheritance, namely: the manner in which fraternal resemblance varies from species to species, while paternal inheritance remains fairly constant. If we look at Table V. we see that within moderate limits parental influence approximates to the same value for very different species and very diverse characters. This cannot be asserted with the same accuracy of fraternal correlation. I have found values of it ranging from ·4 to ·7 for large and apparently very trustworthy data for different species. I attribute this, although I have not been able at present to verify it, to *prepotency**. In dealing with prepotency I think it important to distinguish *ab initio* between three kinds: *sex-prepotency, unit prepotency,* and *intermittent prepotency.* By *sex-prepotency* I understand that the offspring of one or other sex or of both sexes are more like the male or the female parent as the case may be. Its existence is demonstrated by showing that the correlation for one parent with all the offspring or with one class of offspring is greater than for the other parent. An examination of Table IV. seems to prove that in man for stature, span and forearm there exists no sex-prepotency. On the other hand in eye-colour in man, there does appear to be a *differential* sex-prepotency, fathers are prepotent over mothers for eye-colour in sons, and mothers are prepotent over fathers for the same character in daughters*. If the paternal record were trustworthy in the case of Basset Hounds—which I am very doubtful about—then there would be a large sex-prepotency for all offspring of the dam over sire in coat-colour†. From this sex-prepotency must be distinguished an individual prepotency which I term *unit prepotency,* and which is independent of sex. In unit prepotency one or other unit in a mating is prepotent owing to the possession of some physical character, other than a sexual character. This physical character may or may not be that in which the prepotency shows itself in the offspring. Thus it is conceivable that a dark-eyed parent of either sex might have a unit prepotency over a light-eyed parent, not necessarily in eye-colour or in eye-colour only, but possibly in hair-colour, or stature or mental characters. The unit prepotency may, however, in no way depend upon a simple observable character like this, but on a subtle combination of physical factors producing individual prepotency in one unit of the pair. To demonstrate the latter form of unit prepotency will always be a difficult problem; it could possibly be attacked by considering the reduction of variability in the array of offspring of supposed unit prepotent matings below the average variability of arrays in which such prepotency is supposed not to exist. This method would hardly be possible in the case of man where the number of offspring is too small to get the variability of an array free from a very large probable error. It might be effective in the case of snails, moths, many insects and plants with numerous offspring. When unit prepotency is supposed to be associated with the possession of a definite physical character, it is perfectly possible to attack the problem by the method of association, i.e., investigating the association between the presence (or absence) of this character in a parent and the ratio to total offspring of offspring in the array who do (or do not) possess this character, or some other character of the parent in question. If unit prepotency were absolute we should have the case of "dominance" as originally propounded by Mendel.

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* *R. S. Proc.* Vol. 71, pp. 290—4.

While we suppose unit prepotency,—the tendency of one individual out of a pair to be prepotent,—to be *chronic*, there is another form of prepotency which we may describe as *intermittent*. One or other parent may at a particular mating, or may in certain individual offspring of one and the same mating, be prepotent. On another occasion, or in other offspring of one and the same mating, it may not be prepotent or even the other parent may be prepotent. Such prepotency might exhibit itself in "alternative" or "exclusive" inheritance*, and is distinct from any unit prepotency or absolute or partial dominance. It does not depend on the possession by one mate of certain characters, but on the condition of the parents and other circumstances peculiar to a special mating.

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* *Phil. Trans.* Vol. 195, A, p. 106. See also F. Lutz, *Biometrika*, Vol. II. p. 234.

† *R. S. Proc.* Vol. 66, p. 157.

Now the fundamental point to be borne in mind is this, that apart from sex-prepotency, neither unit prepotency nor intermittent prepotency need in any way influence the parental correlations. The average resemblances of offspring to either parent will not be affected if in some matings the mother, in other the father is prepotent. Nor again will it be affected, if occasionally the two parents are intermittently prepotent. But such types of prepotency will largely influence the degree of resemblance between brethren. If, either invariably or intermittently, one parent is prepotent, the offspring of all matings of these parents or the offspring of one litter will be more alike, than the offspring of another species in which such prepotency does not exist. When therefore we find parental correlation the same for a number of species and fraternal correlation different, I am strongly of opinion that this will be found to be due to differing amounts of unit prepotency or of intermittent prepotency or of both combined in diverse species. I have already insisted on this effect of prepotency in disturbing fraternal correlation†, but it seemed necessary again to refer to it as the probable explanation of the great differences observable in the fraternal correlations given below in Table XI.

TABLE XI. *Fraternal Correlation in Different Species.*

Species	Characters	Brother and Brother	Sister and Sister	Sister and Brother	All Siblings
Man	Family Records. Mean of three characters ...	·517	·533	·506	·519
"	Eye Colour	·517	·446	·462	·475
"	School Records. Mean of sixteen characters ...	·520	·519	·518	·519
Basset Hound ...	Coat Colour for same litter	—	—	—	·508
Greyhound ‡ ...	Amount of Red in Coat, same litter	·683	·710	·707	·700
" ...	Amount of Black in Coat, same litter	·642	·680	·659	·660
Thoroughbred Horse	Coat Colour	·623	·693	·583	·633
Daphnia (*Magna*) ...	Ratio of Protopodite to Body Length	—	—	—	·693
Aphis (*Hyalopterus Trirhodus*) ...	Ratio of Right Antenna to Frontal Breadth ...	—	—	—	·589

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* *R. S. Proc.* Vol. 66, p. 141, etc. and *Phil. Trans.* Vol. 195, A, p. 89 *et seq.*

† *R. S. Proc.* Vol. 66, p. 152, and *Phil. Trans.* Vol. 195, A, p. 101.

‡ Unpublished results, tabled by Miss A. Barrington from Mr Howard Collins' data, reduced by Dr A. Lee.

I have not placed in this table the results for stature as found from Mr Galton's Family Data, nor those for Cephalic Index for North American Indians, because I consider that the results for both these characters are replaced by the larger series we have now at our disposal, and which are included under "man" in the above list. Otherwise it embraces nearly all the data we have as yet at our disposal. Now it is clear that the value for man is about ·5 and agrees well with the value found for Basset Hounds, and indeed with that for the Shirley Poppy, assuming complete cross fertilisation*. On the other hand the horses and greyhounds, *while agreeing well with man for the parental correlations* (see Table V. p. 23), show a much increased fraternal correlation of the same order as that between the parthenogenetic offspring of Daphnia and Aphis†. Now how far is this due to such factors as unit prepotency or intermittent prepotency? All we can do at present is to suspend our judgment on this point. In the case of dogs, intermittent prepotency might manifest itself by the offspring of the same parents for the same litter being more alike than for different litters. Now will this account for the high values of the greyhound results? Unfortunately our records contain only greyhounds of the *same* litter, all members being recorded, while the volumes of the greyhound stud-book contain only a *selection* of all dogs born, colour undoubtedly being a selected character. Further it is very difficult from those volumes to extract a sufficient number of brethren of full blood from different litters. Still we hope to be able to throw some light on the problem of at least intermittent prepotency in the case of greyhounds. It is remarkable that the fraternal correlation in the Basset Hounds, while according closely with that in man, is the same sensibly in intensity for siblings from the same and from different litters. The case of the thoroughbred horses is somewhat different, but here we propose to draw up separate tables for twin foals and foals from the same parents in different years, and thus if possible differentiate intermittent prepotency, if it really exists. The high values, however, found for half-siblings in the case of the thoroughbreds seem to indicate that we must look rather to unit prepotency than intermittent prepotency for the source of the high value of fraternal as compared with parental correlation in the case of the horse.

What is quite clear is that we badly want the *measurement* of further characters for siblings in both mammals and insects. The present results show that while the value ·5 has overwhelming evidence for it in the case of both measurable and unmeasurable characters in man, we are yet without like data for the measurable characters in horse, dog or any other mammal. Should these ultimately be found to agree with the results given above for the quantitatively unmeasurable characters, I personally hold at present, that the solution for equal parental and unequal fraternal correlations in these different species should first be sought in a fuller study of unit and intermittent prepotency.

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* *Biometrika*, Vol. II. p. 81.

† In the case of these insects differential environment may, of course, have emphasised the resemblance.

(x) *Cross Fraternal Resemblance.*

I turn to the relationships between different organs in pairs of siblings. These are tabulated below.

TABLE XII.

Correlation Coefficients for Cross Fraternal Heredity.

1st Sibling	Character	2nd Sibling	Character	Correlation and Probable Error
Brother	Stature	Brother	Span	·441 ± ·021
,,	Stature	,,	Forearm	·368 ± ·023
,,	Span	,,	Forearm	·451 ± ·021
Sister	Stature	Sister	Span	·471 ± ·017
,,	Stature	,,	Forearm	·438 ± ·018
,,	Span	,,	Forearm	·453 ± ·017
Brother	Stature	Sister	Span	·478 ± ·014
,,	Span	,,	Stature	·456 ± ·014
,,	Stature	,,	Forearm	·399 ± ·015
,,	Forearm	,,	Stature	·412 ± ·015
,,	Span	,,	Forearm	·419 ± ·015
,,	Forearm	,,	Span	·423 ± ·015

The same general remark must again be made here, i.e these cross-correlations are remarkably high,—as high as a few years ago we anticipated that the direct fraternal correlations would be.

The series being rather short—three to four hundred brothers*—the results are more irregular than we might have hoped for. In particular the cross-correlation between brother's stature and brother's forearm is distinctly less than we might have expected. A result of slightly over ·4 would clearly be more consonant with the other results, but I have not been able to discover any slip in the arithmetic. In the brother-sister correlations we find that within the limits of the probable errors of random sampling the cross-correlation coefficients are pair and pair equal, e.g. the relation of brother's stature to sister's forearm is sensibly that of brother's forearm to sister's stature.

To obtain an empirical formula, I assumed that as in (vi) p. 385 we should have

$$r_{12'} = C\,(r_{11'}\,r_{1'2'} + r_{22'}\,r_{12}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ix)}.$$

I determined C from the twelve series and found for its mean value ·5585. This is within two per cent. of the value of C found for the cross-parental relationships, and I think the agreement is as close as we could hope for†.

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* See Appendix of correlation tables.

† To obtain the desirable end of using one formula instead of two. I am not convinced that equality is *à priori* to be expected.

Using the formula

$$r_{12'} = \cdot 5585\,(r_{11'}r_{1'2'} + r_{22'}r_{12}) \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\;(\text{x}),$$

I find the following results given in Table XIII. The agreement of the observed and calculated results is not as close as in the previous case of cross-parental heredity, but the series from which the observed values are determined are not half as large. Further, the calculated values depend on the coefficients of direct collateral inheritance, and in working out these we have always correlated elder with younger brother. On the other hand it did not seem worth while in calculating the cross-coefficients to separate our rather small amount of material up into two groups and distinguish between the relationship of, say, stature of elder brother to span of younger brother, and again, stature of younger brother to span of elder brother. This difference of treatment is no doubt a source of some of the observed irregularity, but the bulk of it is due to the smallness of our group of brothers.

The mean error of the results from (vii) is ·019 and from (x) is ·020, but (vii) has errors of ·043, ·043 and ·039 larger than the maximum ·034 reached by (x). The first formula gives seven values greater and five less, the second formula gives six greater and six less than the corresponding probable errors in Table XII. Thus on the whole Formula (x) is slightly the better, but the advantage is so small that for practical convenience (vii) might be well used for both. I do not see why the numerical factors in (x) and (vii) should necessarily be equal or nearly equal; still less is there any reason why the factors in these blood relationship formulae should be nearly equal to the value of the factor in (iii), the empirical formula for assortative mating. But it is worth noting that for most practical purposes a common formula with a mean numerical factor of ·555 will give results quite within the limits of the probable errors of our material.

It thus appears that my original proposition as to cross-heredity, based on the assumptions of equality of all inheritance-coefficients and of the corresponding organic correlations in the pair of relatives, is not correct; the factor of ·5 in the original proposition has in the case of man to be replaced by a value lying between ·5 and ·6, the mean value being ·555. We have not at present material enough to test how far this number has any validity beyond cross-heredity in man*. The cases I have data for, however, do show an excess over ·5 of the same order as we find in the case of man, and I hope shortly to publish further results for cross-heredity, closely bearing on this point.

(xi) *General Conclusions.*

If readers of the present paper feel that on certain points it is inconclusive, I think this must be largely attributed to the inherent difficulties of the subject. The further we advance, the more complex the problem becomes, and the wider

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* A short series in *Aphis* has been dealt with by Dr Warren: see *Biometrika*, Vol. I. p. 142. The value of the factor there given is ·5 for one character and ·68 for the second, giving a mean *parental* factor of ·59 for *Aphis* as against ·56 for man.

TABLE XIII.

Observed and Calculated Values of Cross Fraternal Coefficients.

Pair of Characters	Brother and Brother				Sister and Sister				First Brother and Second Sister			
	From (vii)		From (x)		From (vii)		From (x)		From (vii)		From (x)	
	Calculated	Δ	Calculated	Δ	Calculated	Δ	Calculated	Δ	Calculated	Δ	Calculated	Δ
Stature and Span ...	·483	– ·039	·475	– ·031	·511	– ·043	·505	– ·034	·499	– ·021	·491	– ·013
Span and Stature ...	—	—	—	—	—	—	—	—	·499	– ·043	·490	– ·034
Stature and Forearm	·391	– ·023	·384	– ·016	·425	+ ·013	·417	+ ·021	·397	+ ·002	·390	+ ·007
Forearm and Stature	—	—	—	—	—	—	—	—	·395	+ ·017	·388	+ ·024
Span and Forearm ...	·448	+ ·003	·410	+ ·011	·432	+ ·021	·425	+ ·028	·420	– ·001	·412	+ ·007
Forearm and Span...	—	—	—	—	—	—	—	—	·419	+ ·004	·412	+ ·011

Δ = Observed – Calculated Value.

the range of new problems which suggest themselves for solution. Yet I think each large mass of material statistically reduced places a further stratum of firm ground beneath us. In particular, this first paper on inheritance of the physical characters in man, has, I hold, enabled us to reach some very definite results. Indeed, I believe them sufficient repayment for the years spent by my helpers in measuring upwards of a thousand families and tabling and reducing the data*. Of the special results obtained I would refer in particular to the following.

(i) We have very definite evidence that the normal curve suffices to describe within the limits of random sampling the distribution of the chief physical characters in man.

This confirms the conclusions of Galton, Macdonell, Fawcett and other workers in anthropometry, and is of special value when we come to extend our results to the inheritance of characters not quantitatively measurable.

(ii) The regression curve between pairs of blood relations, whether we deal with direct or cross-heredity, is within the limits of random sampling *linear*. This had been already suggested by Galton on the basis of the theory of normal distribution, and confirmed by his researches on stature. I think we may safely assume in future that the dimensions of the human body give linear regression-lines†.

(iii) There is an apparent change in type going on in man, especially evidenced in the female, but also sensible in the male. The young adult differs in magnitude, variability and correlation from the old adult and the difference appears to be significantly beyond growth changes.

We cannot at present determine whether this change is:

(*a*) Environmental, due to change in physical training and food between the young and old generations.

(*b*) Due to natural selection, the young adults being reduced nearer to the old adult type by deaths of a selective character in the intervening 20 to 30 years. If the change of type is due to a selective death rate, it may be either periodic, occurring in each generation, or secular, i.e., a progressive change.

(*c*) Due to reproductive selection, out of young adults a certain class have a differential fertility and become in bulk the parents of adult offspring.

But although we are not in a position to effectually discriminate at present between the amount of change due to (*a*), (*b*) and (*c*), our results immensely emphasise the view that even in apparently unessential characters mankind is, even at the present day, not in a stable condition, but that a change of type is very probably taking place owing to natural or reproductive selection or environmental influence, and the change is of a magnitude, which would accumulate, if it be secular, within a comparatively few centuries into most significant differences.

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* When it is remembered that the whole work of measurement was done by volunteer aid, and without assistance from any public fund, I think other workers may take heart, who imagine that problems in heredity are necessarily confined to extensive breeding experiments of an expensive nature.

† This is really a very important point. In a forthcoming memoir on skew correlation, I deal with non-linear regression and show how fairly frequent it is and how complex it renders the treatment of correlation.

(iv) There is a quite unexpected amount of sexual selection even of the physical characters in man. There is probably preferential mating, there is most certainly a large amount of assortative mating, and this not only in the same but in cross characters. The *modus operandi* of this assortative mating is not clear; it can hardly in any great part be due to conscious selection; it may be the result of reproductive selection, i.e. a subtle combination of physical characters in male and female being most likely to give a pair with a number of adult children. Be the source what it may, the existence of this assortative mating most substantially modifies the form of biparental inheritance, and its existence can hardly in future be neglected when we are considering the problems of heredity.

(v) The coefficient of parental heredity varies to some extent from character to character in man, having a mean value of about ·46. This value is, however, in quite close agreement with the results obtained for other species, and we may roughly say that parental heredity in the species hitherto dealt with is close to ·5.

(vi) Fraternal correlation for the physical characters of man dealt with in this memoir is also close to ·5. This is in good agreement with the result obtained for eight physical and eight mental characters compared in pairs of brothers in schools. While, however, parental correlations are in good accord for different species, fraternal correlations in such species have a much wider range. This curious result is being further considered, but the tentative suggestion is made that it is due either to different degrees of unit or of intermittent prepotency in the members of these species.

(vii) For the first time in this paper statistics of an extensive kind are given for answering the problems of cross heredity, and an empirical formula is given for determining cross heredity from direct heredity and organic correlations. Some years ago, I proposed for cross heredity a formula which amounted in the notation of the present memoir to

$$r_{12'} = \tfrac{1}{2}\,(r_{11'}\,r_{1'2'} + r_{22'}\,r_{12})$$

where 1, 2 are the organs in the first, 1′, 2′ the like organs in the second relative. This formula was based on a theory involving the constancy of the heredity coefficient for all characters (cf. (v) above). It is shown in the present paper that on the average $\frac{1}{2}$ must be replaced by ·57 for parental and by ·56 for fraternal cross correlations. With these numbers we obtain from the direct and organic correlations values of the cross heredity coefficients well within the errors of random sampling. Thus at any rate in the case of man, we are in a position to determine cross-heredity for the physical characters without direct investigation. A further research on cross-heredity will, I hope, shortly be published.

(viii) The values of the parental correlations determined for man, show that two or three generations of selection would suffice to raise the mean of the offspring to the selected standard. Further with quite reasonable values of the grandparental correlations no regression would take place, and the stock breed true.

The result is of extreme importance, for two reasons:

(*a*) It illustrates the absurdity of the prevalent biological conceptions of regression as a constant factor, only restrained by the action of persistent selection.

(*b*) It emphasises the all-important law that with judicious mating human stock is capable of rapid progress. A few generations suffice to modify a race of men, and the nations which breed freely only from their poorer stocks will not be dominant factors in civilisation by the end of the century.

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TABLE I.

Father's Stature and Span (Organic).

Father's Stature.

Father's Span.	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	Totals
59—60	·25	·25	·5	—	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	2
60—61	·25	1	1·75	1	·5	·75	·25	·5	·5	—	—	—	—	—	—	—	—	—	6·5
61—62	—	·75	·5	2	2·5	2	1·25	1·25	·25	—	—	—	—	—	—	—	—	—	10·5
62—63	—	—	·25	2	5·25	3·5	1	3·75	·75	—	—	—	—	—	—	—	—	—	16·5
63—64	—	·25	·25	1	5	7	7	4	2·75	·25	·5	—	—	—	—	—	—	—	28
64—65	—	·25	·75	1·25	9·75	12·5	13·25	16	5·75	2	1·5	—	·5	—	—	—	—	—	63·5
65—66	—	—	·5	·5	4	5·75	15·5	20	12·75	6·25	2	2·5	1·25	—	—	—	—	—	71
66—67	—	—	—	·5	3	4	15·75	24·75	35·75	16·75	10·25	1·75	1·5	—	—	—	—	—	114
67—68	—	—	—	·25	1·75	2·75	10·25	20·25	31	27·5	21·25	10·5	2·75	1·25	—	—	—	—	129·5
68—69	—	—	—	—	·5	2·5	5·75	18·25	27	29	31·75	14·25	7	1·5	—	·5	·5	—	138·5
69—70	—	—	—	—	·25	1·25	·75	7	12·25	25	38·25	23	15·75	3	—	—	—	—	126·5
70—71	—	—	—	—	—	·5	·25	4	7·5	21·25	24·5	24	15·25	8·5	3·25	—	—	—	109
71—72	—	—	—	—	—	—	·25	1·5	4·25	4·25	13·75	16·25	19·5	13·5	6·25	·5	—	—	80
72—73	—	—	—	—	—	—	·25	·5	1	2	6·75	15·75	19·5	12·75	6·75	1·75	—	—	67
73—74	—	—	—	—	—	—	—	·25	5	1·25	4	7	8	6·75	4·25	3·5	2	·5	38
74—75	—	—	—	—	—	—	—	—	—	—	—	3·5	7	3·75	5·5	2·25	—	·5	22·5
75—76	—	—	—	—	—	—	—	—	—	—	—	—	2·5	4	4·5	3	1	1·5	16·5
76—77	—	—	—	—	—	—	—	—	—	·5	—	—	1·5	·5	1	·75	1·25	·5	6
77—78	—	—	—	—	—	—	—	—	—	·5	—	—	·5	—	—	·25	·25	—	1·5
78—79	—	—	—	—	—	—	—	—	—		—	—	—	1	1	—	—	—	2
79—80	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
80—81	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	—	1
Totals	·5	2·5	4·5	8·5	32·5	42·5	71·5	122·5	142·5	136·5	154·5	118·5	102·5	56·5	33	13	5	3	1050

TABLE II.

Mother's Stature and Span (Organic).

Mother's Stature.

Mother's Span.	52—53	53—54	54—55	55—56	56—57	57—58	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	Totals
52—53	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
53—54	—	—	—		—	·25	·75	·5	—	—	—	—	—	—	—	—	—	—	—	1·5
54—55	—	—	—	·25	·25	·75	·75	—	·5	—	—	—	—	—	—	—	—	—	—	2·5
55—56	·5	·5	—	·75	1·5	2·5	2·25	1·25	1·25	1	—	—	—	—	—	—	—	—	—	11·5
56—57	—	—	·5	·5	1·75	5	4·75	8·25	2·75	1·5	1·25	·25	—	—	—	—	—	—	—	26·5
57—58	—	—	·5	·5	1·5	6·25	6·5	8·75	9	4	2·5	·5	—	—	—	—	—	—	—	40
58—59	—	—	—	—	·5	1·25	6·25	18·25	20·25	13·25	7·75	2·25	·25	1	—	—	—	—	—	71
59—60	—	—	—	—	1	1·75	9	15·75	35·25	33·5	15·25	9·25	1·25	·25	·25	1	—	—	—	123·5
60—61	—	—	—	—	—	·25	2·5	16	35	35·75	29·5	12·5	2·75	1	1·75	1	—	—	—	138
61—62	—	—	—	—	—	—	·75	5·25	19·5	34·75	42	28·25	10	5·75	2·75	—	—	—	—	149
62—63	—	—	—	—	—	—	·75	3·25	6	22	42·5	40	23·75	6·75	3	·5	—	—	—	148·5
63—64	—	—	—	—	—	—	·25	1·25	2·75	10·75	24·25	34·5	30	14·25	3	·5	—	—	—	121·5
64—65	—	—	—	—	—	—	—	·5	1·5	3·5	11·25	23·25	24·25	19	6·5	1·75	—	—	—	91·5
65—66	—	—	—	—	—	—	—	·5	·75	1	5·25	8·75	10·5	15·5	14·5	2·5	·25	—	—	59·5
66—67	—	—	—	—	—	—	—	—	·75	·75	·5	1·5	7	6·75	5	3	1·25	—	—	26·5
67—68	—	—	—	—	—	—	—	—	·25	·25	—	1	2·5	4	3·5	2·5	2	—	—	16
68—69	—	—	—	—	—	—	—	—	—	·5	·5	1	1	1·5	·75	1·75	2	2·5	—	11·5
69—70	—	—	—	—	—	—	—	—	—	·5	·5	—	·75	2·25	—	1·25	1·75	1	·5	8·5
70—71	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	—	·25	·25	1	1·5	3·5
71—72	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	—	—	—	—	—	·5
Totals	1·5	·5	1	2	6·5	18	34·5	79·5	135·5	163	183	163	114·5	78·5	41	16	7·5	4·5	2	1052

TABLE III.

Brother's Stature and Span (Organic).

Brother's Stature.

Brother's Span.	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	76—77	77—78	78—79	79—80	Totals
58—59	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
59—60	·25	1·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1·5
60—61	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
61—62	·25	·75	·5	1·25	·25	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	3·5
62—63	—	—	—	1·25	·25	1	—	—	1·25	·25	—	·25	·25	—	—	—	—	—	—	—	—	4·5
63—64	—	—	·5	1·5	5·25	2·5	2·25	1	—	—	—	·25	·25	—	—	—	—	—	—	—	—	13·5
64—65	—	—	·5	—	4·5	8·25	4·75	6	6	2	—	—	—	—	—	—	—	—	—	—	—	32
65—66	—	—	—	1	7·5	13·75	13·75	15·25	7·75	3·5	2	·5	·5	—	—	—	—	—	—	—	—	65·5
66—67	—	—	—	1	6·5	12	21·75	19	17·25	4·5	1·5	·5	·5	—	—	—	—	—	—	—	—	84·5
67—68	—	—	—	1·75	5	11·5	17·5	32·25	23	12·75	1·75	1	—	—	—	—	—	—	—	—	—	106·5
68—69	—	—	—	·75	1·75	6	9	21·5	43·5	31	13·5	3	2	—	—	—	—	—	—	—	—	132
69—70	—	—	—	·5	—	1	6·75	21	39	44	29	12·75	1·5	·5	—	—	—	—	—	—	—	156
70—71	—	—	—	·5	—	—	2·25	7·25	23·75	37·25	33	32·75	11·25	1·5	·5	·5	·5	—	—	—	—	151
71—72	—	—	—	—	—	—	—	2·5	9	30·75	19·5	33·25	22	5·25	3·25	1	·5	—	—	—	—	127
72—73	—	—	—	—	—	—	·25	·5	4·75	13	17	29·75	18·25	13	4·25	1·5	·25	—	—	—	—	102·5
73—74	—	—	—	—	—	—	·25	·75	3	6·5	8	17·75	17·5	11·5	7·5	3·5	·25	—	—	—	—	76·5
74—75	—	—	—	—	—	—	—	—	—	1·75	8·25	3·5	11	12·25	7·75	3·5	·5	—	—	—	—	48·5
75—76	—	—	—	—	—	—	—	—	—	1·5	1·25	1	5·25	5	9	2	1	·5	—	—	—	26·5
76—77	—	—	—	—	—	—	—	—	—	—	·25	·75	2	3	2	2·5	·5	1·5	—	—	—	12·5
77—78	—	—	—	—	—	—	—	—	—	—	1	—	·75	·5	4·25	1·5	·5	·5	—	—	—	9
78—79	—	—	—	—	—	—	—	—	—	—	1	—	—	—	·5	·75	2·25	1	·5	·5	—	6·5
79—80	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	—	·25	·75	—	1·5
80—81	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·75	—	1
81—82	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	1	2·5	1·5	9·5	31	56	78·5	127	178·5	189	137	137	93	52·5	39	17	6·5	3·5	1	2	1	1164

TABLE IV.

Sister's Stature and Span (Organic).

Sister's Stature.

Sister's Span.

	52—53	53—54	54—55	55—56	56—57	57—58	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	Totals
53—54	—	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
54—55	—	—	—	—	1	1	·25	·25	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	3
55—56	—	—	—	—	—	1	·5	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	2
56—57	—	—	—	1	1·25	2·5	1·25	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	6·5
57—58	—	—	—	—	·5	2·75	3	3·5	6·75	2·5	·5	—	—	—	—	—	—	—	—	—	—	—	19·5
58—59	—	—	—	—	1·5	4·25	5·25	17·5	14·75	7·25	2	·75	·25	—	—	—	—	—	—	—	—	—	53·5
59—60	—	—	—	—	·75	2·25	3·25	15·25	27·75	18·75	13·75	7·25	3·5	—	—	—	—	—	—	—	—	—	92·5
60—61	—	—	—	—	—	1·25	·5	6·75	25·75	39·75	38	14	6·75	1·25	—	·5	·5	—	—	—	—	—	135
61—62	·5	·5	—	—	—	—	1·25	6·25	14·25	33·5	53	34	14	6	1·25	—	—	—	—	—	—	—	164·5
62—63	—	—	—	—	—	—	—	1·75	8·5	26·75	44·75	59	29·75	12·25	6·5	1·25	—	—	—	—	—	—	190·5
63—64	—	—	—	—	—	—	—	—	1	16·25	30·5	58·25	52·25	22·25	11·25	3·25	·5	—	—	—	—	—	195·5
64—65	—	—	—	—	—	—	—	—	·75	3·5	9·5	28·25	54·75	45·75	25·25	8·25	·5	—	·25	·25	—	—	177
65—66	—	—	—	—	—	—	—	—	·5	1·5	4·75	15·75	34	51·5	26·75	11·25	3·25	·5	·5	·25	—	—	150·5
66—67	—	—	—	—	—	—	—	—	—	·25	1·25	3·75	14·75	17·75	31·5	15·75	8·75	2	·25	—	—	—	96
67—68	—	—	—	—	—	—	—	—	—	—	1	1·5	4	6·25	27·25	19·75	11	1·75	·5	—	—	—	73
68—69	—	—	—	—	—	—	—	—	—	—	—	·5	1	3·5	13·25	11·25	10	6·5	3	·5	—	—	49·5
69—70	—	—	—	—	—	—	—	—	—	—	—	—	—	2	6·75	6·75	4·25	5·75	1·5	·5	—	—	27·5
70—71	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1·75	2·5	1·75	2	1·75	·25	—	—	11
71—72	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	·75	2	2·25	—	—	6
72—73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	—	—	—	·5
73—74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
74—75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	·5	1
75—76	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	·5	1
Totals	·5	·5	—	1	5	15	15·5	52	101	150	199	223	215	169·5	151·5	81·5	40·5	19·5	10	5	—	1	1456

TABLE V.

Father's Forearm and Span (Organic).

Father's Forearm.

Father's Span.	*15—15·5*	*15·5—16*	*16—16·5*	*16·5—17*	*17—17·5*	*17·5—18*	*18—18·5*	*18·5—19*	*19—19·5*	*19·5—20*	*20—20·5*	*20·5—21*	*21—21·5*	*21·5—22*	*22—22·5*	Totals
59—60	—	·5	·25	·75	·5	—	—	—	—	—	—	—	—	—	—	2
60—61	—	1·25	2	1·25	2	—	—	—	—	—	—	—	—	—	—	6·5
61—62	—	1	1·25	3·5	4·25			·25	·25	—	—	—	—	—	—	10·5
62—63	—	1	3	4·75	1	1·25	·5	·25	·5	·25	—	—	—	—	—	15·5
63—64	·25	·75	2·5	6·75	10·75	5·25	1	·25	·25	·25	—	—	—	—	—	28
64—65	·25	·5	3	15·25	23	11·5	4·75	1·75	—	—	—	—	—	—	—	63
65—66	—	—	·25	5·75	23·75	28·5	11	3	1·25	—	—	—	—	—	—	73·5
66—67	·5	·75	·75	6	32·25	37·5	28	7	1·25	—	—	—	—	—	—	114
67—68	—	·5	2·75	3·25	15	15·25	42	14·5	4·5	1	·75	—	—	—	—	129·5
68—69	—	·25	·25	·75	5·75	32·75	51·75	26·25	11	2·5	·75	1	—	·5	·5	137
69—70	—	—	·5	1	3·75	20	43·75	33·25	19·25	4·5	·5	·5	—	—	—	127
70—71	—	—	·5	—	·5	11·75	33·5	37·75	19·5	3·5	4·5	·25	·25	—		109
71—72	—	—	—	—	—	2	11·5	32·25	20·5	7	2·25	1	·5	·25	·75	78
72—73	—	—	—	—	—	·75	3·75	18·5	21	12·5	5·5	·75	1	·5	·75	68
73—74	—	—	—	—	—	·5	1	6	14·75	9	5·75	·25	·5	·25	—	38
74—75	—	—	—	—	—	—	—	1·5	6·25	7·5	5·5	1	·25	·25	·25	22·5
75—76	—	—	—	—	—	—	—	—	2·25	5	6·5	2·25	·5	·25	·25	17
76—77	—	—	—	—	—	—	—	·5	1·5	3·25	1	·25		—	—	6·5
77—78	—	—	—	—	—	—	—	·5	—	·25	·5	·25	—	—	—	1·5
78—79	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	2
79—80	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
80—81	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	—	1
Totals	1	6·5	17	49	125·5	200	235·5	183·5	127	57·5	31·5	8	3·5	2	2·5	1050

TABLE VI.

Mother's Forearm and Span (Organic).

Mother's Forearm.

Mother's Span.	*13—13·5*	*13·5—14*	*14—14·5*	*14·5—15*	*15—15·5*	*15·5—16*	*16—16·5*	*16·5—17*	*17—17·5*	*17·5—18*	*18—18·5*	*18·5—19*	*19—19·5*	*19·5—20*	Totals
52—53	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	1
53—54	—	·25	·25	1	—	—	—	—	—	—	—	—	—	—	1·5
54—55	—	·25	·25	1·25	1	·25	—	—	—	—	—	—	—	—	3
55—56		—	—	1·25	5·25	5	·5	—	—	—	—	—	—	—	12
56—57	—	—	1·25	6·25	9·75	4·25	1·75	2·5	·75	—	—	—	—	—	26·5
57—58		—	1·5	4	10·75	12	6·75	1	1·5	—	—	—	—	—	40·5
58—59	·25	·25	1·25	5·5	12·5	24	15·75	7	2·5	·75	·25	—	—	—	70
59—60	·25	·25	1·25	4	14	42·25	10·75	12·5	3·5	1·5	1	·5	·25	—	122
60—61	—	—	·25	2	7	44	51	26·25	5·25	1·25	1·25	·5	·25	—	139
61—62		—	·5	1·75	3·75	20·5	55	43·75	17·5	4·5	1·25	—	—	—	148·5
62—63	—	—	—	1·25	2·25	10·25	16	52·25	27·25	6·75	2	—	—	—	148
63—64	—	—	—	·5	1·25	5·5	26·25	41·25	35·5	7·25	3	—	—	—	120·5
64—65	—	—	—	·25	·5	3·25	6·5	30	31·75	14·75	3·5	1·25	1·75	—	93·5
65—66	—	—	—	—	—	2	2	10·75	21·75	15·5	5·75	1·5	·25	—	59·5
66—67	—	—	—	—	—	·25	·25	2·75	7·25	8·25	7	·5	·25	—	26·5
67—68	—	—	—	—	—	—	—	1	4·5	5·25	3·25	2·25	·25	—	16·5
68—69	—	—	—	—	—	—	1	1·25	1·25	3·25	3	1	·25	—	11
69—70	—	—		·25	·25	—	—	·25	·5	1·25	2·75	1·75	1	—	8
70—71	—	—	—	·25	·25	·25	·25	—	·25	·25	1	·25	·75	·5	4
71—72	—	—		—	—	·25	·25	—	—	—	—	—	—	—	·5
Totals	·5	1·5	7	29·5	68·5	174	254	235·5	161	70·5	35	9·5	5	·5	1052

TABLE VII.

Son's Span and Forearm (Organic).

Son's Span.

Son's Forearm.	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	76—77	77—78	78—79	79—80	80—81	81—82	Totals
14—14·5	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
14·5—15	—	—	·25	·25	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
15—15·5	—	—	·25	·25	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
15·5—16	—	—	—	—	—	·25	1·5	1·75	—	·75	·75	—	·25	·25	—	—	—	—	—	—	—	—	—	—	5·5
16—16·5	—	—	—	·5	1	1·75	·75	2·5	·25	1	·75	—	·5	·5	·25	·25	—	—	—	—	—	—	—	—	10
16·5—17	·25	·25	—	1·5	1·5	6·25	5	10·25	6·25	3·75	1·25	1	·5	·5	·75	·5	—	—	—	—	—	—	—	—	39·5
17—17·5	·25	·25	—	·25	·75	3·25	11	19·5	27·25	14·25	9·75	4·25	2	1·5	·5	·25	—	—	—	—	—	—	—	—	95
17·5—18	—	—	—	·5	·5	1	7·5	19·75	27·75	39	39	27·25	8·75	3·75	1·75	1	—	—	—	—	—	—	—	—	177·5
18—18·5	—	—	—	·25	·25	·5	3·75	6·75	14·5	35	52	64	42·75	25·25	9·25	5·25	·5	·25	·25	—	—	—	—	—	260·5
18·5—19	—	—	—	—	—	—	·5	1·75	5	8	20·75	39·5	58·5	37·5	31	14·25	5·5	1·5	·5	·25	·5	—	—	—	225
19—19·5	—	—	—	—	—	—	—	1·25	2·25	3	6·25	10·5	30	34·75	33·75	23	15	3·25	1·75	·75	1	—	—	—	166·5
19·5—20	—	—	—	—	—	—	—	·75	1	·25	1	4	8	16·5	17·5	22·5	17·75	11·75	2	1·5	·5	—	—	—	105
20—20·5	—	—	—	—	—	—	—	·25	·25	—	·75	1·75	·75	3·5	6	8·25	5·75	6·25	4·5	2·25	·5	·25	—	—	41
20·5—21	—	—	—	—	—	—	—	—	—	—	·25	·75	—	·25	1	1	2	3·25	1·5	2	1·75	·5	·25	—	14·5
21—21·5	—	—	—	—	—	—	—	—	—	—	—	·5	—	·25	·25	·25	1	·75	·5	1·75	1·5	·5	·25	—	7·5
21·5—22	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	1·25	·25	—	1	4·5
22—22·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
22·5—23	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
23—23·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	—	—	—	1
Totals	·5	·5	·5	3·5	4	13	32	64·5	84·5	105	132·5	153·5	152	125·5	102	76·5	47·5	28	11·5	9	7	1·5	·5	1	1156

TABLE VIII.

Sister's Span and Forearm (Organic).

Sister's Span.

Sister's Forearm.	*53—54*	*54—55*	*55—56*	*56—57*	*57—58*	*58—59*	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	*74—75*	*75—76*	Totals
13·5—14	·25	·25	—	—	—	·25	·25	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1·5
14—14·5	·25	1·25	—	—	—	·75	1·25	·25	·75	·5	—	—	1	—	—	—	—	—	—	—	—	—	—	6
14·5—15	—	1·25	1·5	2·75	2·75	3·5	2·5	2	·5	1·25	—	·5	·5	—	—	—	—	—	—	—	—	—	—	19
15—15·5	—	·25	·5	2·25	8·25	19·25	17·75	12·25	5·5	3	—	—	·75	·25	—	—	—	—	—	—	—	—	—	70
15·5—16	—	—	—	1·25	3	21·75	35	43·75	42·5	22·75	8·5	1·5	2·5	·75	·25	·5	—	—	—	—	—	—	—	184
16—16·5	—	—	—	1	4·25	6·25	24·75	55	63·25	75·75	54	19·5	11·5	1·75	2	1	—	—	—	—	—	—	—	320
16·5—17	—	—	—	·25	1	1	6	19	39·5	57	76·75	67	33·75	15	2	·25	—	—	—	—	—	—	—	318·5
17—17·5	—	—	—	—	·25	·75	2·5	4·75	11·75	24·75	42·5	62·75	52·5	36·5	15	4·75	2·5	·25	—	—	—	—	—	261·5
17·5—18	—	—	—	—	—	—	·5	·5	·25	3·75	10	16·5	33·75	29·75	34·5	21·25	7·5	1·25	—	—	—	—	—	159·5
18—18·5	—	—	—	—	—	—	—	—	—	1·5	3·5	8·75	11·25	10·5	14·75	13·75	8·25	3·75	2·5	·5	—	—	—	79
18·5—19	—	—	—	—	—	—	—	—	—	·75	1	·25	·25	1·25	4·75	5·75	6·5	4	1·5	—	—	—	—	26
19—19·5	—	—	—	—	—	—	—	—	—	·5	·25	·5	1,25	·5	·5	1·5	2·75	·75	2	—	—	·75	·75	12
19·5—20	—	—	—	—	—	—	—	—	—	·5	—	·25	1·25	·5	·25	·25	—	—	—	—	—	·25	·25	3·5
20—20·5	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	·25	·25	—	·5	—	—	—	—	—	1·5
20·5—21	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	—	·5	—	—	—	—	—	1
Totals	·5	3	2	7·5	19·5	53·5	90·5	137·5	164·5	192	196·5	177·5	150·5	97	74·5	49·5	27·5	11	6	·5	—	1	1	1463

TABLE IX.

Father's Stature and Forearm (Organic).

Father's Stature.

Father's Forearm.	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	Totals
15—15·5	—	—	—	—	—	—	—	—	·75	·25	—	—	—	—	—	—	—	—	1
15·5—16	·5	1·5	—	·5	1·25	·75			1	·75	·25		—		—	—		—	6·5
16—16·5	—	1	·5	2·5	3	2·25	1	1·5	1·5	3	·5	·75	·5	—	—	—	—	—	18
16·5—17	—	·25	1	1·5	8	9·5	8	10·75	5	2·5	1·25	1	·25	—	—	—		—	49
17—17·5	—	·25	1	2·75	8	13	22·5	28·25	26·75	10	8·75	3·5	·75	—	—	—	—	—	125·5
17·5—18	—	·25	·75	·75	6·5	9	22·75	35·5	43·5	33·25	27·25	10	7·25	2·75	·5	—	—		200
18—18·5	—	·25	·25	·25	3·25	6	11·75	27·75	37·5	40·25	48·75	32·75	16·25	8	2·5	—	—	—	235·5
18·5—19	—	—	—	·25	1·75	1·25	6	13·25	19·25	29	33·25	34	26·75	13·5	5	·25	—		183·5
19—19·5	—	—	—	—	·75	·75	·5	5	8·75	10·5	23	22	23	19	9	3·75	1		127
19·5—20		—	—	—	—	—	—	1	1·5	2·75	7·5	8·5	14·75	8·25	7·25	2·5	2·25	1·75	58
20—20·5	—		—	—	—	—		·5	·5	·75	1·5	5·5	8	2·75	6·25	3·5	1·5	1·25	32
20·5—21	—	—	—	—	—	—	—	—	—	1	·5	·5	2	·5	1	2·25	·25	—	8
21—21·5	—	—	—	—	—	—	—	—	—	—	·5	·5	·5	·75	·5	·75	—	—	3·5
21·5—22	—	—	—	—	—	—	—	—	—	—	—	·25	·25	1	·5	—	—	—	2
22—22·5	—	—	—	—	—	—	—	—	—	—	—	·25	1·25	·5	·5	—	—	—	2·5
Total	·5	3·5	3·5	8·5	32·5	42·5	72·5	123·5	146	134	153	119·5	101·5	57	33	13	5	3	1052

TABLE X.

Mother's Stature and Forearm (Organic).

Mother's Stature.

Mother's Forearm.	52—53	53—54	54—55	55—56	56—57	57—58	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	Totals
13—13·5	—	—	—	—	—	—	—	—	—	—	—	·5	—		—	—	—		—	·5
13·5—14	·5	—	—	—	—	·25	·25	—	—	—	—	·5	—	—	—	—	—	—	—	1·5
14—14·5	·5	—	—	—	·25	1	1·75	·5	·5	1	·75	·25	—	·5	—	—	—	—	—	7
14·5—15	—	—	·25	·25	1·25	2·25	4·25	7·75	6	1·75	1·5	2·25	1	·5	—	·25	·25	—	—	29·5
15—15·5	—	—	·5	1·25	1·75	5·25	8·5	13·75	16·5	11	3·5	4	1·75	·25	—	·25	·25	—	—	68·5
15·5—16	·5	·5	·25	1	2·75	5·75	9·75	22	43·25	38·75	26·75	17·5	4·25	1	—	—	—	—	—	174
16—16·5	—	—	—	—	·5	2·5	5·75	20	37·5	58·5	62·75	40·5	17·25	5	2·25	·75	·25	—	—	253·5
16·5—17	—	—	—	—	—	1	2·5	12	17·5	33·5	54	46·5	37·75	17·75	9·75	2·5	·25	—	—	235
17—17·5	—	—	—	—	—	·5	1	2·75	8·5	14·25	26·5	31	32	26·75	13·25	2·5	1·5		—	160·5
17·5—18	—	—	—	—	—	—	·75	1·25	1·5	2·75	5·25	14·75	10·5	16·25	9	5	2	1·5	—	70·5
18—18·5	—	—	—	—	—	—	—	—	·75	1	·5	4·25	7·5	7·25	6·25	3·5	1·5	1·5	1	35
18·5—19	—	—	—	—	—	—	—	·25	·75	·25	·25	·75	2·25	1·75	1	1·25	1	—	·5	10
19—19·5	—	—	—	—	—	—	—	·25	·25	·25	·25	·25	·25	1·5	—	5	·5	·5	·5	5
19·5—20	—	—	—	—	—	—	—	—	—		—	—	—	—	—	—	—	·5	—	·5
Totals	1·5	·5	1	2·5	6·5	18·5	34·5	80·5	133	163	182	163	114·5	78·5	41·5	16·5	7·5	4	2	1051

TABLE XI.

Brother's Stature and Forearm (Organic)

Stature.

Forearm.	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	76—77	77—78	78—79	79—80	Totals
14·5—15	·25	·25	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	1
15—15·5	·25	·25	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	1
15·5—16	—	—	—	—	—	·75	2	·5	·75	1	·5	—	—	—	—	—	—	—	—	—	—	5·5
16—16·5	—	·25	·25	1	2	·75	2	1	1·75	1	1	—	—	—	—	—	—	—	—	—	—	11
16·5—17	·25	·5	·75	2·5	5·75	8	6·25	7·5	4	2·5	·75	·5	·25	—	—	—	—	—	—	—	—	39·5
17—17·5	·25	·25	·5	1	11	22	21·5	16·25	14·25	5·75	1	1·25	1	—	—	—	—	—	—	—	—	96
17·5—18	—	—	—	2·5	6·25	15·75	24·5	39·75	37	28·5	11·75	7·25	2·5	·25	—	—	—	—	—	—	—	176
18—18·5	—	—	—	1·5	6·25	5·5	17·25	37·75	59·75	60	32·75	25·75	10	1·5	1·5	·25	·25	—	—	—	—	260
18·5—19	—	—	—	—	·25	2·25	3·25	15·75	36·75	48·75	38·25	11	20·25	9·25	3·5	2	·75	—	—	—	—	225
19—19·5	—	—	—	1	—	·5	·75	5·25	15·25	25·75	29·25	38	28	12·25	6·5	4	·5	—	—	—	—	167
19·5—20	—	—	—	—	—	—	—	1·5	3·5	10·75	16	16	19·75	17·75	12·25	4·5	1·25	·75	·5	—	—	101·5
20—20·5	—	—	—	—	—	—	—	·25	1·5	3·5	3·75	2·25	7·75	7·75	7·75	1·75	1·5	·75	—	—	—	41·5
20·5—21	—	—	—	—	—	—	—	—	—	·75	·75	1·25	·25	2·75	1·5	1	2	1·25	—	·5	—	15
21—21·5	—	—	—	—	—	—	—	—	—	·25	·25	·75	·25	·5	2	1	·25	·75	·5	1	—	7·5
21·5—22	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	·5	·5	·5	3·5
22—22·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
22·5—23	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
23—23·5	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
Totals	1	1·5	1·5	9·5	31·5	55·5	77·5	126	175	189·5	136	138	91	52	38	17·5	6·5	3·5	1·5	2	·5	1155

TABLE XII.

Daughter's Stature and Forearm (Organic).

Daughter's Stature.

Daughter's Forearm	*52—53*	*53—54*	*54—55*	*55—56*	*56—57*	*57—58*	*58—59*	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	Totals
13·5—14	—	—	—	—	—	—	·25	·25	·25	·25	—	·25	·25	—	—	—	—	—	—	—	—	—	1·5
14—14·5	—	—	—	—	·5	·5	·25	1·25	·75	·5	·75	·25	·25	1	—	—	—	—	—	—	—	—	6
14·5—15	—	—	—	1	2	3	2·25	1·75	2·5	2·75	1	2·25	—	—	—	·25	·25	—	—	—	—	—	19
15—15·5	·25	·25	—	—	1·5	5·25	5	13·25	18·25	12·5	8·25	4·25	1·25	·5	·25	·5	·25	—	—	—	—	—	71·5
15·5—16	·25	·25	—	—	1	3·75	5·25	17·75	32·75	44·5	39·25	20	12·75	4·25	2	·25	·75	·25	—	—	—	—	185
16—16·5	—	—	—	—	—	1·75	1·75	13·25	32	54·5	78·75	68	44·75	16·5	7·75	1·5	·75	·25	—	—	—	—	321·5
16·5—17	—	—	—	—	—	·5	·75	4·5	10·5	24·5	47·5	72·5	70·5	47	28·5	10·25	1	—	—	—	—	—	318
17—17·5	—	—	—	—	—	·25	·5	1·5	5	8	22	41	55·75	56·25	42	17	10	1·25	—	—	—	—	260·5
17·5—18	—	—	—	—	—	—	—	—	·5	1·5	4	8·25	19·75	30·75	50	28·5	12·25	4·5	1·5	—	—	—	161·5
18—18·5	—	—	—	—	—	—	—	—	—	·75	1·5	3·5	8·25	11·5	19·75	15	8	6·5	3·75	·5	—	—	79
18·5—19	—	—	—	—	—	—	—	—	—	·25	·5	·25	—	2	4·75	5	3·75	6	2·75	1·25	—	—	26·5
19—19·5	—	—	—	—	—	—	—	—	—	—	—	·25	·75	1·25	1	1	1·75	1·5	2	2	—	·5	12
19·5—20	—	—	—	—	—	—	—	—	—	—	—	·25	·75	—	—	·25	·5	·5	·5	·25	—	·5	3·5
20—20·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	1	·25	—	—	—	—	1·5
20·5—21	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·75	—	—	—	—	—	1
Totals	·5	·5	—	1	5	15	16	53·5	102·5	150	203·5	221	215	171	156	80	41	21	10·5	4	—	1	1468

TABLE XIII.

Father's Stature and Mother's Stature.

Father's Stature.

Mother's Stature.	*58—59*	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	*74—75*	*75—76*	Totals
52—53	—	—	—	—	—	—	—	1	·5	—	—	—	—	—	—	—	—	—	1·5
53—54	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	·5
54—55	—	—	—	—	—	—	—	·25	·25	—	·5	—	—	—	—	—	—	—	1
55—56	—	—	—	·5	1	—	—	·25	·25	—	·5	—	—	—	—	—	—	—	2·5
56—57	—	—	—	—	·75	1·25	—	1	1·75	1·75	—	—	—	—	—	—	—	—	6·5
57—58	—	—	—	·25	1	1·25	1·5	4	3·25	2·5	3	1·25	·5	—	—	—	—	—	18·5
58—59	—	·25	·75	1·25	1·25	2·75	4	7	5·75	4·5	3·75	1·25	2	—	—	—	—	—	34·5
59—60	—	1·25	1·25	1	4	4·5	7·75	10	15	16·75	9	5·5	3·25	1·25	1	·5	—	—	82
60—61	·25	·25	·5	2	4·25	4·5	18	16	24	14·75	23·25	12·75	7·25	5·75	4·25	·75	—	—	138·5
61—62	·25	·25	—	—	8	8·25	15	17·25	25	20·75	24	14·25	14·25	10	4	·75	·5	—	162·5
62—63	—	·5	·5	1·25	4·75	7·75	10	26	21·25	28	28	23	14·25	10·75	4·5	2	1	·5	184
63—64	—	—	·25	2	3·5	4·5	9	21	15·75	20·75	19·5	24	22·5	10·75	4	2·25	2·25	·5	162·5
64—65	—	—	1·25	·75	2	6	6·5	9·75	16	18·25	23	16·75	13·75	6·75	4·75	2·25	·25	1·5	129·5
65—66	—	—	—	·25	1·5	1·5	3·25	5·5	9·75	7	15·5	12·75	10·5	6·25	4·25	1·75	·25	—	80
66—67	—	—	—	·25	1	·75	·5	3·5	5	3	7·25	7·75	7	3·5	2·75	1·5	·25	—	44
67—68	—	—	—	—	—	—	—	1	2·5	1·5	2·75	3·25	2·75	1·5	1	·5	·25	—	17
68—69	—	—	—	—	—	—	—	—	—	1	2·5	1·25	1·25	·5	1	·25	·25	—	8
69—70	—	—	—	—	—	—	—	—	—	—	—	·25	2·25	—	2	—	—	—	4·5
70—71	—	—	—	—	—	—	—	—	—	—	—	—	1	—	·5	—	—	—	1·5
Totals	·5	2·5	4·5	9·5	33	43	75·5	123·5	146·5	140·5	162·5	124	102·5	57	34	12·5	5	2·5	1079

TABLE XIV.

Father's Span and Mother's Span.

Mother's Span.

Mother's Span.	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	76—77	77—78	78—79	79—80	80—81	Totals
52—53	—	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—		—	—	—	—	1
53—54	—	—		—	—	—	·5	·5	—	—	·25	·25		—	—	—	—	—	—	—	—	—	1·5
54—55	—	—	—	—	—	·5	·5	—	—	—	·25	·25	1·5	—	—	—	—	—	—	—		—	3
55—56	—	—	—	—	2·25	1·25	1	1·25	2·75	—	1·5	·5	·5	—	—	—	—	—	—		—	—	11
56—57	—	—	·5	2·25	1·5	2·75	1·5	2·25	2·75	4·5	4	1·25	1·75	—	·5	·5	·5	—	—	—		—	26·5
57—58	—	—	·5	·25	·75	3	9	4·25	5	8·25	3	·5	2·25	1·25	·5	—	·5		—	—	—		39
58—59	—	—	1·25	·75	2·5	3	4·5	10·5	7·75	10·25	7·25	4	5·5	2·75	1	·5	—	—	—	—	—	—	61·5
59—60	·5	1·5	·5	1·75	6·5	8	7	16·75	14·75	14·75	15·25	11·75	9	4·25	1·75	1	·5	—	—	—		—	115·5
60—61	·75	1·75	·25	1·25	2·75	7·75	9	22·5	19·75	13	14	11·5	12·5	8·5	4·5	3·25	1	—	—	1		—	135
61—62	·75	1·75	3	2·5	2·25	12	6·5	14·25	15	22·75	18	13·25	8·25	9·5	7	2·75	2·5	1	—	—	—	—	143
62—63	—	—	1	1	1·5	9·5	10·75	10·75	19·25	17·75	20	17	8	11·25	5·25	4·25	2·75		—	—		—	140
63—64	—	—	1·25	2·25	2·25	5	8·5	12·25	9	14·5	14·5	12·5	14·5	8·5	5·5	3·75	1·5	2	·75	—	—	·5	119
64—65	—	1	1·5	2·25	1·25	4·25	1·5	7·5	12·5	11·75	9·25	14	5	3·75	2·5	3·5	1·75	1·25	·5	—		·5	88·5
65—66	—	·5	·25	·25	3	1	2·25	4·75	5·5	11	7·25	5·75	2·75	3·75	2·5	1	1·5	1·25	·25	—	—	—	54·5
66—67	—		—	—	·5	·5	2	·75	2·75	3	4	4·75	1·75	3	1·5	·5	1	·5		·5	—	—	27
67—68	—	—	·25	·75	·5	·5	2	·25	2	1·25	1·5	1·25	·25	2·5	·5	·25	·25	·5	—	·5	—	—	15
68—69	—	—	·25	·75	—	—	—	—	1·25	·25	2	1·5	·5	1	1·5	·75	·75	—	—	—		—	10·5
69—70	—	—	—	—	—	—	·5	1	1·5	—	1·5	—	·5	1·25	·25	·5	1	—	—	—	—	—	8
70—71	—	—	—	—	—	—	—	—	—	—	·25	·25	1·5	·25	·25	—	1·5		—	—	—	—	4
71—72	—	—	—	—	—	—	—	·5	—	—	·25	·25		—	—	—	—	—	—	—	—	—	1
72—73	—		—	—	—	—	—	·5		—	—	—	—	—	—	—	—	—	—	—	—	—	·5
Totals	2	6·5	10·5	16	28	59·5	70	110·5	121·5	133	124	100·5	76	61·5	35	22·5	17	6·5	1·5	2	—	1	1005

TABLE XV.

Father's Forearm and Mother's Forearm.

Father's Forearm.

Mother's Forearm.	*15—15·5*	*15·5—16*	*16—16·5*	*16·5—17*	*17—17·5*	*17·5—18*	*18—18·5*	*18·5—19*	*19—19·5*	*19·5—20*	*20—20·5*	*20·5—21*	*21—21·5*	*21·5—22*	*22—22·5*	Totals
13—13·5	–	—	—	—	—	—	–	—	—	—	·25	·25	—	—	—	·5
13·5—14	—	—	—	—	—	—	·25	·25		·25	·5	·25	—	—	—	1·5
14—14·5	·5	·5	·5	1	·25	·75	·75	1·25	—	·5	·5	·25	·25	—	—	7
14·5—15	·5	·75	1	1·25	5	8	3·75	3·75	1·75	·5	·25	·25	·25	·25	·25	27·5
15—15·5	—	·25	1	5·25	12·5	16	13·25	8	3·5	·75	1	·25	·25	·25	·25	62·5
15·5—16	—	1·5	5·5	9·5	22·5	35·5	34·25	30·75	14·75	5·25	3·75	·75	·5	—	—	164·5
16—16·5	—	1·25	5·25	11·5	32·75	43·5	58·5	41·25	29·75	11·75	4·75	1·25	1	—	—	242·5
16·5—17	-	1·25	2·25	11·5	23·75	40·75	51	43·5	29·25	13·5	4·75	·75	·25	·25	·25	223
17—17·5	--	·25	1·25	4·25	10·75	24	41·5	27·25	22	8·75	5·75	3	·5	·5	·25	150
17·5—18	—	·25	·75	1·5	7	12	11·5	11·5	11·25	6·75	3·75	1	·5	·5	·25	68·5
18—18·5	—	--	-	·5	3·25	4·5	6·75	5·75	6	4·75	3·5	—	—	·25	·25	35·5
18·5—19		—	-	·5	·5	1	1·25	1·5	2·25	2	2			—	—	11
19—19·5	—	—	—	·25	·25	—	·75	1·25	1·25	1	·75	—			—	5·5
19·5—20	—	—	—	—	—	—	—	--	·25	·25	-				—	·5
Totals	1	6	17·5	47	118·5	186	223·5	176	122	56	31·5	8	3·5	2	1·5	1000

TABLE XVI.

Father's Stature and Mother's Span.

Father's Stature.

Mother's Span.	*58—59*	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	*74—75*	*75—76*	Totals
52—53	—	—	—	—	—			1		—	—	—	—	—	—	—	—	—	1
53—54	--	—	—	—	—	—	1	-	—	·5	-	—	—	—	—	—	—	—	1·5
54—55	—	—	—	—	·5	—	—	—	·5	·5	1	—	·5	—	—	—	—	—	3
55—56	—	—	—	·5	1·25	·5	2·75	1·5	2	·75	1·75	·5	·5	—	—	—	—	—	12
56—57	--	—	—	1	1·75	2·5	2·25	4·25	4·75	3	4	·5	1	—	—	1	—	·5	26·5
57—58	—	—	—	·5	1	1·75	3·5	8·75	5·5	7·25	6·5	3	·25	·75	1	·25	—	·5	40·5
58—59	—	·25	·25	2·25	2·5	2·5	6·5	4·75	11	13·5	9·5	6·25	2·5	3·5	2	·5	·75	—	68·5
59—60		·25	1·25	·25	3·25	8·25	8·75	18·25	20·75	14·5	18·25	12·5	8	4·75	2·5	·5	1	—	123
60—61	·25	·25	1·5	1	4·75	4·75	11·75	18·75	26·75	17	13·5	13·5	11·25	7·5	7·25	1·5	·25	—	141·5
61—62	·25	1·25	·5	·5	2·75	5·75	10·25	19·75	20·25	18·25	23·75	14·5	10·5	11·75	4·25	2·25	·5	·5	147·5
62—63	-		-	1·25	6·25	4	6·5	15·5	17·25	23·25	25	19·75	13·75	9·75	2·25	1·5	1	1	148
63—64	—	—	—	·25	3·5	5·5	7	9·75	12·5	10·5	24·75	19·5	14·5	6	5	2	·25	—	121
64—65	—	·5	1·25	·75	2·5	2·75	5·5	11·75	10·5	11·25	9·25	12·25	14	4·5	4·25	1·5	·5	—	93
65—66	—	—	·25	·25	2	2·25	1·25	5	7	9	7·25	8·25	9·5	3·75	1·5	1·5	·5	·25	59·5
66—67	-	—		—		·5	1·75	1·25	3	3·5	4·5	4	4·25	2·5	·75		·25	·25	26·5
67—68	—	—	—	—	·5	1	1	1	1·75	2·75	1·5	2	3·75	·5	·75	—	—	—	16·5
68—69	—	—	—	-	—	·5	·75	·25	-	·5	1·5	1·5	3	1	2·5		—	—	11·5
69—70	—	—	—	—	—	—	1·5	1·5	1·5	—	2	·5	2	·5			—	—	9·5
70—71	—	—	-	—	—	—	-	--	·25	·25	·5	—	1·75	·75	·5		—	-	4
71—72	—	—	—	—	—	—	—	--	·25	·25		—		—	—	—	—	—	·5
Totals	·5	2·5	5	8·5	32·5	42·5	72	123	145·5	136·5	154·5	118·5	101	57·5	34·5	12·5	5	3	1055

TABLE XVII.

Father's Stature and Mother's Forearm.

Father's Stature.

Mother's Forearm.	*58—59*	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	*74—75*	*75—76*	Totals
13—13·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	—	·5
13·5—14	—	—	—	—	—	—	—	·5	—	·5	—	—	—	—	—	·25	·25	—	1·5
14—14·5	—	—	—	·5	·25	·75	·5	·5	1·5	1·5	—	—	·25	·25	·5	·5	—	—	7
14·5—15	—	—	—	·5	1·25	1·75	1	3·5	6	4	4·75	1·5	2	1·25	1	·5	—	—	29
15—15·5	—	—	—	1·75	3	2·75	6·25	11·5	14·75	8·25	7·25	2·5	3·75	3·25	2	—	—	·5	67·5
15·5—16	—	·5	·5	1·75	7	9·5	16·5	23·5	25·25	24·25	22·5	19	10·5	9·25	3·75	1·25	·5	·5	176
16—16·5	—	·5	·5	2	7	10·75	22·25	30	32·75	35·75	35·25	33·5	16·5	13·25	5·5	1·5	1·5	1·5	250
16·5—17	·5	1·25	1·75	2	6·5	8·5	10·75	28	31·25	32·75	37·75	25	26·25	11·75	5·75	4·25	1·5	—	235·5
17—17·5	—	·25	·25	1·25	4·25	4·5	7	12·25	20	20·5	28·75	20·25	24·25	8·5	5	3·75	·25	·5	161·5
17·5—18	—	—	—	·25	1·75	1·25	4·25	7·75	9·25	6·75	8	9	10	6	3·5	1	·75	—	69·5
18—18·5	—	—	—	—	·5	1·5	2·25	3·25	4·75	2·5	5·5	5·25	2·75	2·75	3·25	·75	—	—	35
18·5—19	—	—	·25	·25	·5	·75	·25	·25	·5	·25	2·75	1·5	1·75	·25	·75	—	—	—	10
19—19·5	—	—	·25	·25	·5	—	—	—	—	—	1·5	·75	2·25	—	—	—	—	—	5·5
19·5—20	—	—	—	—	—	—	—	—	—	—	—	·25	1·25	—	—	—	—	—	1·5
Totals	·5	2·5	3·5	10·5	32·5	42	71	121	146	137	154	118·5	101·5	56·5	31	14	5	3	1050

TABLE XVIII.

Father's Span and Mother's Stature.

Father's Span.

Mother's Stature.	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	76—77	77—78	78—79	79—80	80—81	Totals
52—53	—	—	—	—	·5	·5	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	1·5
53—54	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	·5
54—55	—	—	—	—	—	—	—	—	·75	·25	—	—	—	—	—	—	—	—	—	—	—	—	1
55—56	—	—	—	—	—	·75	·75	—	·75	·25	—	—	—	—	—	—	—	—	—	—	—	—	2·5
56—57	—	—	—	—	·25	1·5	·5	1·5	1·75	1	—	—	—	—	—	—	—	—	—	—	—	—	6·5
57—58	—	—	—	—	1·25	3·25	1·5	2·25	1·25	2	·75	2	3·25	—	—	—	—	—	—	—	—	—	17·5
58—59	—	·5	—	1·25	3·25	1·75	5·5	5·25	3·5	4	3·75	3·5	2	·25	—	—	—	—	—	—	—	—	34·5
59—60	·25	1·5	2·25	1·75	2·75	6·25	8·25	10	10·75	12·25	11	4·75	4·25	2	·75	1·5	·25	—	—	—	—	—	80·5
60—61	·5	·75	2·25	1	3·5	11·5	9·75	13·5	21·25	22·25	11·75	11	9·5	7·5	3·5	1·75	·75	·5	—	1	—	—	133·5
61—62	·25	1·25	·5	2·25	5	11·25	9	23·75	18·75	21·75	20·25	16·75	12·25	9·25	5·75	3·25	1·25	·5	—	—	—	—	163
62—63	—	·5	2·5	4	3·25	8·5	12	24·5	25	16	25·5	18	12·5	11·25	11·25	2·5	3·25	1·75	·75	—	—	—	183
63—64	·25	1·25	1	1	3·25	10·25	9·25	11·5	19·5	21	21·75	21	11·5	16	6	5	1·5	·5	·5	1	—	—	163
64—65	·75	·75	1	2·25	2	5·75	6·5	8·5	12·75	17·5	13·75	15·75	10·75	9	2·5	1·25	3·25	·25	·25	—	—	—	114·5
65—66	—	—	·75	1·75	2	1·25	4·25	7·25	6·75	7·5	8·75	11	11·25	4·75	2·75	4·75	2	1·25	—	—	—	1	79
66—67	—	—	·25	·5	·25	—	3·75	6	3·75	7·75	4·25	3·75	·75	3·25	1·75	2	2·75	1·75	—	—	—	—	42·5
67—68	—	—	—	·75	·75	·5	1·5	1	2·5	2	4	·25	·25	1	1·25	·25	—	—	—	—	—	—	16
68—69	—	—	—	—	—	—	1	—	·5	·5	1·5	·25	·25	1·25	2	·25	—	—	—	—	—	—	7·5
69—70	—	—	—	—	—	—	—	—	—	—	—	1	1·5	1·5	·5	—	—	—	—	—	—	—	4·5
70—71	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	—	—	2
Totals	2	6·5	10·5	16·5	28	63	73·5	115	129·5	137	127	109	80	67	38	22·5	17	6·5	1·5	2	—	1	1053

TABLE XIX.

Father's Span and Mother's Forearm.

Father's Span.

Mother's Forearm.	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	76—77	77—78	78—79	79—80	80—81	Totals
13—13·5	—	—	—	—	—		—	—	—	·5	—	—		—	—		—	—	—	—	—	—	·5
13·5—14	—	—	—	—	·25	·25	—	—	—	·5	·25	·25	—	—	—	—		—	—	—	—	—	1·5
14—14·5	—	—	—	·5	·25	·75	1	1·5	—	·5	·25	·25	·5	·5	·25	·75	—	—		—	—	—	7
14·5—15	·25	·25	—	·5	·75	1·5	3	5·5	5·75	3·25	1·5	1·75	2·75	1	1	·75	—	—	—		—	—	29·5
15—15·5	·25	1·25	1·75	1	2·25	5·25	7	10	10	7·5	6·25	4·5	6	2	1·5	·25	·75	—	—	—		—	67·5
15·5—16	—	1·25	1·75	2·75	6·5	14·5	13	22·75	21·75	24·5	17·5	16	14·75	8	4·5	2·5	1	·25	·25	·5	—	—	174
16—16·5	—	1·25	1·75	3·5	6·75	19·5	17·75	27·5	34·25	34	35	24	15·75	13	9·25	6·25	2·5	·25	·25	·5	—	·5	253·5
16·5—17	1·5	3	3	3·5	4·25	12	14·5	19·5	25·75	30·5	30·25	32	17	16·5	8·25	5·25	5·25	2·5	—	—	—	·5	235
17—17·5	—	—	·5	1·25	3·75	6	9	18	17·25	24·25	20	18·25	13·75	13·5	5·75	2·75	3·5	2·25	·75	·5	—	—	161
17·5—18	—	—	·25	1·25	1·75	2·5	5·75	5·5	8·75	7·5	10·25	7·75	5·25	4·75	4	1·5	1·25	1·25	·25	·5	—	—	70
18—18·5	—	—	·25	1	1·5	·75	2	2·75	4·75	3·75	2·75	3·75	2·25	5·5	2·75	1·5	1·25			—	—	—	36·5
18·5—19	—	—	·75	·75	—	—		·5	1·25	1·25	1·75	·25	—	2·25	1·25	·75	·75	—	—	—	—	—	11·5
19—19·5	—	—	·5	·5	—	—	—	—	—	—	1·25	·25	1·5	—	—	·25	·75	—	—	—	—	—	5
19·5—20	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	·5
Totals	2	7	10·5	16·5	28	63	73	113·5	129·5	138	127	109	80	67	38·5	22·5	17	6·5	1·5	2	—	1	1053

TABLE XX.

Father's Forearm and Mother's Stature.

Father's Forearm.

Mother's Stature.	*15—15·5*	*15·5—16*	*16—16·5*	*16·5—17*	*17—17·5*	*17·5—18*	*18—18·5*	*18·5—19*	*19—19·5*	*19·5—20*	*20—20·5*	*20·5—21*	*21—21·5*	*21·5—22*	*22—22·5*	Totals
52—53	—	—	—	—	—	·5	·5	·5	—	—	—	—	—	—	—	1·5
53—54	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	·5
54—55	—	—	—	—	·25	·25	·25	·25	—	—	—	—	—	—	—	1
55—56	—	—	—	·5	·75	·75	·25	·25	—	—	—	—	—	—	—	2·5
56—57	—	—	—	1·25	·5	2·5	2	·25	—	—	—	—	—	—	—	6·5
57—58	—	—	—	·25	4	5	3·75	2·5	1·5	·25	·25	—	—	—	—	17·5
58—59	·5	1·25	2	1·5	3·5	9	8·75	5·25	2·75	·25	·25	·25	·25	—	—	35·5
59—60	—	1·25	3·75	7·5	11·5	17·5	15·75	11·5	6·75	·75	1·5	·75	·5	—	—	79
60—61	—	·5	1·75	5·25	18·75	29·25	30·75	23·25	11·75	6·75	2·75	1	·75	—	—	133
61—62	—	1·25	2	7·75	25·75	26·75	31·75	34	19·5	7·5	2·75	—	—	—	—	162
62—63	—	1	3	9·25	21·75	34·5	40·75	31·25	22	12	4·75	1·25	1	—	—	182·5
63—64	·25	·75	1·75	4·25	15·75	26·75	39	29	26·5	9·25	6·5	1·25	·5	1·25	1·25	164
64—65	·25	·5	2·5	6·5	7·75	18·75	28·25	23·25	13·5	6·75	4·5	1·5	—	·25	·75	115
65—66	—	·25	1·5	1·5	8·25	13·5	16	13·5	9·5	8·75	3·75	1	·5	·5	·5	79
66—67	—	·25	·25	·75	3·75	7·75	9	7·25	7·25	2·25	2	1	—	—	—	41·5
67—68		—	—	1	2	4	3·5	1·25	2·5	1·25	·5		—	—	—	16
68—69	—	—	—	1·25	·25	·75	1	1	2	·75	·5	—	—	—	—	7·5
69—70	—	—	—	—			·75	·75	1·5	1	·5	—	—	—	—	4·5
70—71	—	—	—	—	—	—	—	—	—	·5	1·5	—	—	—	—	2
Totals	1	7	18·5	48·5	124·5	198	235	185	127·5	58	32	8	3·5	2	2·5	1051

TABLE XXI.

Father's Forearm and Mother's Span.

Father's Forearm.

Mother's Span.	*15—15·5*	*15·5—16*	*16—16·5*	*16·5—17*	*17—17·5*	*17·5—18*	*18—18·5*	*18·5—19*	*19—19·5*	*19·5—20*	*20—20·5*	*20·5—21*	*21—21·5*	*21·5—22*	*22—22·5*	Totals
52—53	—	—	—	—	—		·5	·5		—	—	—	—	—	—	1
53—54	—	—	—	—	·5	·5		—	—	·25	·25	—		—	—	1·5
54—55	—	—	—	—	·25	·75	—	·25	1·25	·25	·25	—	—	—	—	3
55—56	—	—	—	·75	3·25	4·75	2	·5	·75	—	—	—	—	—	—	12
56—57	—	—	1·25	1·25	5·75	7·25	5·25	2·5	·75	—	·5	·5	·5	—	—	25·5
57—58	—	—	·75	3	4·75	9·5	10	7·5	2·5	1	1	—	—	—	—	40
58—59	·25	·25	2	4·75	7·25	13·75	17·75	13	5·5	1·75	1	·25	—	—	—	67·5
59—60	·25	·75	2·5	6·5	15·5	27·5	28	23·75	11·75	4·25	1·75	·5	—	—	—	123
60—61		1	3·75	6·75	21·5	27·5	25	25	16·5	7·25	5·5	·75	·5	—	—	141
61—62	·25	2·25	3·75	7·75	17	23	33·25	26·25	19·25	9	3·25	1·25	·75	·5	·5	148
62—63	·25	1·25	1·5	3·5	17·5	23·75	34·75	28	23·25	9·25	2	·75	·75	·25	1·25	148
63—64	—	·5	1·5	6·25	10	20·25	27·75	23·25	19·25	6·5	2·75	·75	·75	1	·5	121
64—65	—	1	1·5	2·5	11·5	16·5	21·5	16·25	11·75	5·5	3·5	1·25	·25	·25	·25	93·5
65—66	—	—	—	2·5	5·25	13·5	15·25	8·25	6·5	3·5	3·25	1·5		—	—	59·5
66—67	—	—	—	·5	·5	4·5	9	5·5	2	3	1·5	—	—	—	—	26·5
67—68	—	—	—	1·75	1·25	3·25	2·25	1·5	2	2·25	2	·25		—	—	16·5
68—69	—	—	—	·75	·75	·75	1·25	1	2·75	2	1·5	·25	—	—	—	11
69—70	—	—	—	—	1·5	1·75	1	1·25	·75	1	·75	—	—	—	—	8
70—71	—	—	—	—	—	·5	·75	·25	·5	·75	1·25	—	—	—	—	4
71—72	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	·5
Totals	1	7	18·5	48·5	124	199·5	235·5	184·5	127	57·5	32	8	3·5	2	2·5	1051

TABLE XXII.

Father's Stature and Son's Stature.

Father's Stature.

Son's Stature.	58·5—59·5	59·5—60·5	60·5—61·5	61·5—62·5	62·5—63·5	63·5—64·5	64·5—65·5	65·5—66·5	66·5—67·5	67·5—68·5	68·5—69·5	69·5—70·5	70·5—71·5	71·5—72·5	72·5—73·5	73·5—74·5	74·5—75·5	Totals
59·5—60·5	—	—	—	—	·5	·5	1	—	—	—	—	—	—	—	—	—	—	2
60·5—61·5	—	—	—	—	·5		—	—	1	—	—	—	—	—	—	—	—	1·5
61·5—62·5	—	·25	·25	—	·5	1	·25	·25	·5	·5	—	—	—	—	—	—	—	3·5
62·5—63·5	—	·25	·25	2·25	2·25	2	4	5	2·75	1·25	—	·25	.25	—	—	—	—	20·5
63·5—64·5	1	—	1·5	3·75	3	4·25	8	9·25	3	1·25	1·5	·75	1·25	—	—	—	—	38·5
64·5—65·5	2	·1	·5	2	3·25	9·5	13·5	10·75	7·5	5·5	3·5	2·5	—	—	—	—	—	61·5
65·5—66·5	—	·5	1	2·25	5·25	9·5	10	16·75	17·5	16	5·25	2	2·5	1	—	—	—	89·5
66·5—67·5	—	1·5	2	4.75	3·5	13·75	19·75	26·5	25·75	19·5	12·5	13·75	3·25	·5	1	—	—	148·0
67·5—68·5	—	—	1·5	2	7·5	10	10·25	24·25	31·5	23·5	29·5	13·25	8·5	9·5	2·25	—	—	173·5
68·5—69·5	—	—	1	—	5·25	5	12·75	18·25	16	24	29	21·5	10	3·5	2·25	—	1	149·5
69·5—70·5	—	—	—	—	1	2·5	5·75	18·75	11·75	19·5	22·5	19·5	14·5	6·25	3·5	1·5	1	128·0
70·5—71·5	—	—	—	—	—	3·25	5	8·75	10·75	19	14·75	20·75	10·75	8	5	1	1	108·0
71·5—72·5	—	—	—	—	—	·25	3	1.25	7	7·75	10·75	11·25	10	8·5	2·75	·5	—	63·0
72·5—73·5	—	—	—	—	—	—	·75	·75	2·5	7·5	6·5	6	7·5	6·25	3·25	·5	·5	42·0
73·5—74·5	—	—	—	—	1	—	1·5	1·5	—	5·25	2·25	2·5	6·5	3·25	3·25	—	2	29·0
74·5—75·5	—	—	—	—	—	—	—	—	—	1	2	—	2·5	·75	1·75	·5	—	8·5
75·5—76·5	—	—	—	—	—	—	—	—	—	1·25	·25	—	·5	1	1	—	—	4·0
76·5—77·5	—	—	—	—	—	—	—	—	—	1·25	·25	1	—	—	1·5	—	—	4·0
77·5—78·5	—	—	—	—	—	—	—	—	—	—	1	1	—	·25	·75	—	—	3·0
78·5—79·5	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	—	—	·5
Totals	3	3·5	8	17	33·5	61·5	95·5	142	137·5	154	141·5	116	78	49	28·5	4	5·5	1078

TABLE XXIII.

Father's Span and Son's Span.

Father's Span.

Son's Span.	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	76—77	77—78	78—79	79—80	80—81	Totals
58—59	—	—	—	·25	·25	—	—		—	—		—	—	—	—	—	—	—	—	—	—	—	·5
59—60	—	—	—	·25	·25	—	—	—	—	—	—	—			—	—	—	—	—	—	—	—	·5
60—61	·25	·25	—	—	—	—	—	1	—	—	—	—	—	—	·5		—	—	—	—	—	—	2
61—62	·25	·25	—	—	—	—	1	—	—	·5	·5	—	—	—	·5	—	—	—	—	—	—	—	3
62—63	—	—	—	—	·25	1·25	·5	·5	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3·5
63—64	—	—	—	1	1·75	2·25	2·5	1·5	·75	1·5	·25	·5	·5		—	—	—	—	—	—	—	—	12·5
64—65	—	1	·25	·25	2	3·25	3·5	3·25	6·5	3·75	·75	1	·75	·75	—	—	—	—	—	—	—	—	27
65—66	·25	·75	3·25	3·25	3·25	9·75	5	9·5	7·25	5·5	5·5	3	1·75	2	—	—	—	—	—	—	—	—	60
66—67	·25	1·25	2·5	4·5	3·75	11	10·5	13·25	7·25	9·75	4·75	5·75	1·25	1·5	·25	—	·5	—	—	—	—	—	78
67—68	—	1·25	2	·75	3·5	11·25	9·5	14·75	16·25	13·5	9·5	5·25	5·75	4·5	1·5	·5	·75	—	—	—	—	—	100·5
68—69	—	1·25	1·75	·75	2·25	10·75	6·75	15	17·75	21·5	15·25	12	9·25	4·25	2·75	1·5	·75	—	—	—	—	—	123·5
69—70	·5	1	·75	1·5	2·25	6·25	11·75	20·5	18·75	19·75	19·5	16	8	5	1·5	1	·5	1	—	1	—	—	136·5
70—71	—	—	1	1	1·75	7·5	10·25	13	17·25	20·5	22	19·25	12	6·5	6·5	—	1·5	—	—	·5	—	—	140·5
71—72	—	—	·25	·25	·25	3·75	6·5	8	13	13·5	17	15·25	14·25	9	5	2	1·5	1	—	·5	—	—	111
72—73	—	—	·25	1·75	1	2·5	3·75	6·75	8·5	12·25	10·5	11	12	8	6·75	5·5	1	·75	·25	—	—	—	92·5
73—74	—	—	—	·75	1·25	·5	3·5	3	6·5	9·75	8·5	8·25	9·75	8·5	7·75	3·5	1·5	1·75	·25	—	—	—	75
74—75	—	—	—	2·25	·25	·5	·5	1	4·25	3·25	5·75	4·5	3	4·25	7·25	4·75	2·5	·5	—	—	—	—	44·5
75—76	—	—	—	—	—	1·5	—	·5	1	·5	4·5	3	2·5	4	2·5	1·5	3	1·5	—	·5	—	—	26·5
76—77	—	—	—	—	—	—	—	·5	—	1·25	1	1·75	1·75	1	·5	·75	1	—	—	·5	—	—	10
77—78	—	—	—	—	—	—	—	—	—	1·25	·25	—	2·5	1·5	·5	1·5	—	—	—	—	—	—	7·5
78—79	—	—	—	—	—	—	—	—	1	—	·5	·5	1·5	1·25	·25	—	1	—	—	—	—	—	6
79—80	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	·25	·25	—	—	—	·5	2
80—81	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	·25	·25	—	—	—	·5	2
Totals	1·5	7	12	18·5	21	72	75·5	112	127	138	128	107	86·5	62	44	22·5	16	7	0·5	3	—	1	1065

TABLE XXIV.

Father's Forearm and Son's Forearm.

Father's Forearm.

Son's Forearm.	*15—15·5*	*15·5—16*	*16—16·5*	*16·5—17*	*17—17·5*	*17·5—18*	*18—18·5*	*18·5—19*	*19—19·5*	*19·5—20*	*20—20·5*	*20·5—21*	*21—21·5*	*21·5—22*	*22—22·5*	Totals
14—14·5	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
14·5—15	·5	·5	·25	·5	·25	—	—	—	—	—	—	—	—	—	—	2
15—15·5	—	—	·25	·75	·5	—	—	—	—	—	—	—	—	—	—	1·5
15·5—16	·5	·5	—	·5	2·25	1	·25	·5	·5	—	—	—	—	—	—	6
16—16·5	—	·25	1·25	·75	1·75	2·5	1	·5	·5	·25	·25	—	—	—	—	9
16·5—17	·5	1·75	1·5	2·75	9·25	10·75	5·75	2·5	1·25	·5	·5		—	—	—	37
17—17·5	—	1	6·25	12·75	17	22·5	15·25	6·75	1·75	1·25	·5	—		—	—	85
17·5—18	·25	2·25	5	13·25	25·75	40·25	35	23·75	11	5·75	·75	·25	·25	—	—	163·5
18—18·5	·25	1·25	4·75	9·25	32·75	50·5	54·5	40·25	27	10·25	3	2	1·75	—	—	237·5
18·5—19	—	1	2·75	5	17·25	37	52·75	43·5	29·25	8·25	4·5	2	·75	·25	·25	204·5
19—19·5	—	1	2	2·75	9·25	22·75	35·75	34·5	31·25	11·75	4·5	1·25	·25	·25	·25	157·5
19·5—20	—	—	—	1	3·25	11·5	18·25	14·25	21	14·75	8	2	·25	·75	·5	95·5
20—20·5	—	—	—	·25	1·25	2·5	6	7·5	7	6·25	4·25	1·75	·5	·75	·5	38·5
20·5—21	—	—	—	—	·75	·75	1·25	2	3·75	2	1·5	·25	·25	·5	·5	13·5
21—21·5	—	—	—	—	·25	·75	1·75	1·25	1·25	1·5	·25	—	—	—	—	7
21·5—22		—	—	—	—	·25	1	1·75	—	1	—	—	—	—	—	4
22—22·5	—		—	—	—	—	—	—	—	—	—	—	—	—	—	—
Totals	2	9·5	24	49·5	121·5	203	229·5	179	135·5	63·5	28	9·5	4	2·5	2	1063

TABLE XXV.

Father's Stature and Daughter's Stature.

Father's Stature.

Daughter's Stature.	*58—59*	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	*74—75*	*75—76*	Totals
52—53	—	—	—	—	·25	·25	—			—		—		—	—	—	—	—	·5
53—54	—			—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	·5
54—55	—	—	—		—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
55—56	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
56—57	·25	·25		·25	1·25	·5		1	·5	·5					—	—	—	—	4·5
57—58	·25	·25	·5	1·5	4·5	1	1·5	1·5	2·5	—	·5	·5	—	—	—	—	—	—	14·5
58—59	·25	·75	·5	·75	·75	1	1·75	1·25	5	2·75	·5	·25			—			—	15·5
59—60	·5	1	2	—	6	4·75	5	6·25	11·75	3·5	3·5	2	1·75	·5	—	—	—	—	48·5
60—61	·75	·75	—	2·5	8	6·25	12·5	18·25	20·25	11	9	4·75	2·5	1·25	1·25	—	—		99
61—62	—	·5	1·75	2	9·75	11·5	13	23·75	23·75	20·25	16·5	10·25	4·25	3	1·25		—		141·5
62—63	—	1	2·25	2	4·5	12	22·75	26	33	28·25	24·75	14·25	13·75	4·75	·75	·5		—	190·5
63—64	—	—	·25	2	6	8·25	11	27·25	35·75	37·25	31·5	26·25	16·25	7·75	1·5	·75	·25	—	212
64—65	—		·25	2·5	1·75	3·25	9·25	23	18·75	28·5	33	34·25	24·5	11·75	5·5	1	·25	1	198·5
65—66	—	—	—	·5	1	·5	11	12·25	9·25	19·75	30	26·5	22·25	15	4·75	3·75	2	1	159·5
66—67	—	—	—	·5	·5	1·5	3·25	7·25	8·75	16	26·25	26·75	20·5	18·5	7·75	4·25	·25	·5	142·5
67—68	—	—	—	—	—	—	1	5·75	7	4	14·25	13·25	12	11·25	4·5	3·75	·75	—	77·5
68—69	—	—	—	—	·25	·25	·25	·25	1·5	3	5·5	4·25	5·75	5·25	3·75	2·5	1·5	2	36
69—70	—	—	—	—	·25	·25	·25	·25	·25	·25	1	2·5	6·5	2·25	2·75	2	1	—	19·5
70—71	—	—	—	—	—	—	—	—	—		1·75	·25	4·5	·75	1·25	·75	·25	—	9·5
71—72	—	—	—	—	—	—	—	—	—		·5	—	·5	·5	1·5	·75	·25	—	4
72—73	—		—	—	—	—	—	—	—	—	1	—	—	—	—	—		—	1
Totals	2	4·5	7·5	14·5	45	51·5	92·5	155	178	175	199·5	166	135	82·5	36·5	20	6·5	4·5	1376

TABLE XXVI.

Father's Span and Daughter's Span.

Father's Span.

Daughter's Span.	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	*74—75*	*75—76*	*76—77*	*77—78*	*78—79*	*79—80*	*80—81*	Totals
53—54	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
54—55	—	—	·25	·5	·25	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
55—56	—	—	·25	·75	·5	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
56—57	—	—	·75	·25	—	1	·5	2·5	·5	—	—	·5	·5	—	—	—	—	—	—	—	—	—	6·5
57—58	—	—	·75	·25	·25	·5	3·75	4·5	4	2·5	·5	—	—	—	—	—	—	—	—	—	—	—	17
58—59	·25	·75	—	2·25	4	5·25	9·5	10·5	7·5	6·25	3	·75	1	1·5	—	—	—	—	—	—	—	—	52·5
59—60	·75	1·75	·75	2·75	4·25	9	8·75	10·5	16·5	13·75	7·75	3·5	3	1·75	1·25	—	—	—	—	—	—	—	86
60—61	·75	2·25	2·25	2·5	4·25	15·75	19	18·25	17	14·75	11	10	5·5	4	1·75	1	—	—	—	—	—	—	130
61—62	·25	1·25	3·5	4·25	6·5	15	16·75	16·25	24·75	17·5	20·75	13	5	7·5	3·25	·5	·5	—	—	—	—	—	156·5
62—63	—	·5	1	3·25	7·25	11·75	11·75	18·5	23	23·75	20·5	23·5	14·5	8	7·25	2·75	1·25	1	1	—	—	—	180·5
63—64	·25	1·25	1·5	1	3·25	6	9·5	21·25	23·75	25·5	31	21·75	14·75	11	5·75	1·5	2	·5	—	—	—	—	181·5
64—65	·25	·25	1·25	·25	1·75	9	6	15·25	15·75	28·5	25·75	18	16·25	13·75	8·5	3·75	2·75	1	—	—	—	—	168
65—66	—	—	·25	1·25	—	3·25	4	6·75	13·75	20	27·25	22	14·75	9·5	6	6·25	5	1·75	·75	—	1	—	143·5
66—67	—	—	—	—	2	2	5·5	6·5	7·75	11·25	13·5	12·75	8·5	4·75	8·25	4·25	2	1·75	·25	—	·5	—	91·5
67—68	—	—	—	—	·5	1	1·5	1·25	4·5	9·75	5	12·5	13·5	6	5·5	4·25	1·75	1	—	·5	·5	·5	69·5
68—69	—	—	·25	·25	·5	·25	·75	·5	1·75	3·5	2·5	4·5	4·25	10·75	1·25	3	4·5	1·5	—	·5	—	·5	41
69—70	—	—	·25	·25	—	·25	·25	1	·5	2	1·75	1	3	5·5	1·75	2·75	2·75	·75	·25	—	—	—	24
70—71	—	—	—	—	—	—	—	—	·5	·5	1·75	·25	2·5	1	1	1	—	·25	·25	—	—	—	9
71—72	—	—	—	—	—	—	—	—	—	—	1·5	1	—	1	—	—	1	1·5	—	—	—	—	6
72—73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	·5
73—74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
74—75	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	·5	—	—	—	—	—	1
75—76	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	·5	—	—	—	—	—	1
Totals	2·5	8	13	20	35·5	81	97·5	134	161·5	179·5	173·5	145	108	86	51·5	31	25	11	2·5	1	2	1	1370

TABLE XXVII.

Father's Forearm and Daughter's Forearm.

Father's Forearm.

Daughter's Forearm.	*15—15·5*	*15·5—16*	*16—16·5*	*16·5—17*	*17—17·5*	*17·5—18*	*18—18·5*	*18·5—19*	*19—19·5*	*19·5—20*	*20—20·5*	*20·5—21*	*21—21·5*	*21·5—22*	*22—22·5*	Totals
13·5—14		—	·25	·25	·5	—	—	.25	·25			—	—	—	—	1·5
14—14·5	·5	·75	·75	·5	1	—	1	·25	·25						—	5
14·5—15	—	·25	2	5·25	3	2·5	3	1·25	·25		·25	·25			—	18
15—15·5	—	·75	1·75	9·5	10·5	11·25	18·25	6·75	1·25	—	·25	·25	—	—	—	66·5
15·5—16		2·5	5·5	15·75	31·5	43·5	36·25	19	13·5	3	1	·25	·25	·25	·25	172·5
16—16·5	—	1·25	5·25	14·75	46·5	66	81·5	45	25·25	6·25	3	1·75	1·5	·5	·5	299
16·5—17	—	1·75	2·25	8	34	59·5	75·75	58·75	34·5	14·25	6	2·25	1·25	·5	·25	299
17—17·5	—	·25	·75	4·25	22	34·25	58·75	55	37·75	17	9·75	3·5	1·5	·25	—	245
17·5—18	—	·25	·25	2·5	7·5	14·25	28·5	33	31·75	21·25	8	1·75	1	·25	·25	150·5
18—18·5	—	·25	·25	·75	1·5	4·25	12·25	16	17·75	12	6·25	2·25	·5	·25	·75	75
18·5—19	—	—	—	—	—	·75	2·75	5·25	5·75	4·75	3·25	·5	—	—	·5	23·5
19—19·5	—	—	—	—	·25	·5	·75	1·5	2·5	1·5	2	·5	—	·25	·25	10
19·5—20				—	·25	·25	·25	·5	·5	·25	·25	·25	—	·25	·25	3
20—20·5	—	—	—	—	—	—	—	—	·5	·5	—	—	—	—	—	1
20·5—21	—		—	—	—		—	—	·25	·25		—	—	—	—	·5
Totals	·5	8	22	61·5	158·5	240	319	242·5	172	81	40	13·5	6	2·5	3	1370

TABLE XXVIII.

Mother's Stature and Son's Stature.

Mother's Stature.

Son's Stature.	*52—53*	*53—54*	*54—55*	*55—56*	*56—57*	*57—58*	*58—59*	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	Totals
59—60	—	—	—	—	—	—	—	·25	·25	·25	·25	—	—	—	—	—		—	—	1
60—61	—	—	—	—	—	—	—	·75	·25	·25	·25	—	—	—	—	—	—	—	—	1·5
61—62	—	—	—	—	—	—	1	·5	—	—	—	—		—	—	—	—	—	—	1·5
62—63	—	—	—	—	—	—	—	3·5	1·5	2	·5	—	—	·5	—	—	—	—	—	8
63—64	·5	·5	—	—	·25	1·25	2·5	6·75	9	5	3·75	—	—	·5	—	—		—	—	30
64—65	—	—	·25	·25	1	3·25	5·75	13	7·75	7	6·75	1·25	1·75	—	1	—		—	—	49
65—66	—	—	·5	1	3 75	5·5	1·5	9	14·25	18·25	7·25	6·75	1·75	—	1·25	·25	—	—	—	74
66—67	—	—	·25	·75	1·75	2·75	8·5	15	19·5	20	22·25	10	7·75	2	2·75	·75	·5	—	—	114·5
67—68	—	—	—	—	·25	4·25	5	11·25	27·25	31	36	27	12·75	1	2·25	1·5	·5	—	—	163
68—69	—	—	—	—	—		6·75	12·25	27·5	27·25	31·5	26·75	23·25	12·75	5	1·75	·75	—	—	175·5
69—70	—	—	—	—	—	2	—	6	15·25	17	23·25	20·5	20	13·25	4·5	1	·75	·5	—	124
70—71	—	—	—	—	—	1	·5	2·75	9·5	22	21·5	23·5	15·75	14·25	7·5	2·75	·5	·5	—	122
71—72	—	—		—	—	—	·5	1	2·75	10·25	13·5	16	13·25	12·75	6	1	1	—	—	78
72—73	—	—	—	—	—	—	·5	1	5	4·25	6	11	7·5	4	4·75	2·5	1	—	—	47·5
73—74	—		—	—	—		—	—	1·25	4	6·75	6·25	3·75	7	1·5	1·75	1·25	1·5	1	36
74—75	—	—	—	—	—	—	—	—	—	2·5	2·5	2·5	1	3	2	·75	1·25	1·5	—	17
75—76	—	—	—	—	—	—	—	—	—	·5	—	·5	—	1	1·5	1	1·5	—	·5	6·5
76—77	—	—	—	—	—	—	—	—	—	—	—	1		1	·75	·25	—	—	·5	3·5
77—78	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	·25	·75	—	—	—	1·5
78—79	—	—	—	—	—	—	—	—	—	·5	—	—	—	·5	·5	·5	—	—	—	2
79—80	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	·5	·5	1	2	7	20	35·5	83	141	172·5	182	153	108·5	76·5	41·5	17·5	9	4	2	1057

TABLE XXIX.

Mother's Span and Son's Span.

Mother's Span.

Son's Span.	*53–54*	*54–55*	*55–56*	*56–57*	*57–58*	*58–59*	*59–60*	*60–61*	*61–62*	*62–63*	*63–64*	*64–65*	*65–66*	*66–67*	*67–68*	*68–69*	*69–70*	*70–71*	Totals
58–59			—			—			·25	·25	—	—	—	—	—	—	—	—	·5
59–60	—	—	—		—	—	—		·25	·25	—	—	—	—	—	—	—	—	·5
60–61				—		—	·5	—	—	—	—	—	—	—	—	—	—	—	·5
61–62	—	—	—	—	·25	·75	1	1	—	—	—	—	—	—	—	—	—	—	3
62–63	1	—	—	—	·25	·25	1·25	·25	1	—	—	—	—	—	—	—	—	—	4
63–64	—	·25	1·25	·5	3·25	2·25	2·25	2·25	—	·5	·5	—	—	—	—	—	—	—	13
64–65	·25	5	·25	2·5	4·75	2·5	5·75	2·5	1·75	3·25	·5	2	1	—		—	—	—	27·5
65–66	·25	·25	2·5	3·75	5·75	10·5	8·75	8	10·25	5·75	1·5	2·25	·5	—	—	—	—	—	60
66–67		—	2	8	7·25	8·75	13·25	10	7·75	9·25	5·5	5	2·75	·25	1·25	—	—	—	81
67–68		·25	4	4·5	4·5	11·25	18	14·25	18·75	8·75	9·25	1·75	·25	1·75	1	·5	·25	—	102
68–69	—	·25	1·75	3·75	4	11	13·25	17·25	20	18·25	19	9·5	3·5	1·25	·5	·5	·25	—	124
69–70	—	·25	1	2·25	6·5	8	20	23·25	21	22·75	14·25	9	4·75	2	1	1	·5	—	137·5
70–71	—	1·25	1·25	2·25	3	7·25	15·25	16·5	23	26·5	16·5	15·5	7·25	2·25	2·75	1·5	—	—	142
71–72	—	—	—	1	·5	8·25	6·75	12·25	19	15·75	13·25	12·5	6·75	5·25	2·25	4·5	1·25	·25	109·5
72–73	—	—	—	·5	·5	2·5	4·25	11	17	18·25	11·5	7·75	8·75	6	1·5	1	·25	·25	91
73–74	—		—	·5	—	2·5	2·75	6	11·75	8·75	13	5·5	9·5	8·5	4·75	—	1·5	—	75
74–75	—	—	—	—	·25	2·75	1·25	3·25	5·5	5·25	8·75	5·25	4·75	1·5	1	—	2·5	—	42
75–76	—		—	·5	·25	·5	·25	2·25	1·75	2·25	4·75	3·5	2·25	1·5	1·75	3·5	1·5	1	27·5
76–77	—	—	—	·5	—	—	—	·5	1·25	1·5	·75	1·5	1·5	·75	·75	·5	—	—	9·5
77–78	—	—	—	—	—	—	—	—	1·25	·75	1·5	1	1·25	·25	—	—	·75	·75	7·5
78–79	—	—	—	—	—	1	—	—	—	—	1·75	·25	·25	·5	1·25	—	·75	1·75	7·5
79–80			—	—	—	—	—	—	—	—	·5	·5	—	·25	·25	—	—	—	1·5
80–81	—	—	—		—	—	—	—	—	—	·25	·25	—	—	—	—	—	—	·5
81–82	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5		—	—	1
Totals	1·5	3	11	30·5	41	80	114·5	130·5	161·5	148	123	86	55	32·5	20·5	13	9·5	1	1068

TABLE XXX.

Mother's Forearm and Son's Forearm.

Mother's Forearm.

Son's Forearm.	*13–13·5*	*13·5–14*	*14–14·5*	*14·5–15*	*15–15·5*	*15·5–16*	*16–16·5*	*16·5–17*	*17–17·5*	*17·5–18*	*18–18·5*	*18·5–19*	*19–19·5*	Totals
14–14·5	—	—	—	—	—	—	—	1	—	—	—	—	—	1
14·5–15		—	—	·5	·5	—	—	—	—		—	—	—	1
15–15·5	—	—	—	·5	·5	—	—	—	—	—	—	—	—	1
15·5–16	—	—	·25	2·75		·25	·75	·25	·25		—	—	—	4·5
16–16·5	—	—	·25	·75	1	2	2·5	1	1	—	—	—	—	8·5
16·5–17	—	—	2	3·75	6	10·25	9	4·25	1	·25	—		—	36·5
17–17·5	—	·25	1·25	5·5	10·5	25·5	21·75	11	5·5	·75	—	—	—	85
17·5–18		·25	1·5	4·25	16·75	38·5	44	33·5	15·75	6·75	·5	·25	—	162
18–18·5	—	—	1·25	4·5	16·25	40	68·5	54·25	33·25	15·5	5	·5	—	239
18·5–19	—	—	·75	3·5	10	28·75	53·5	50	36·25	11·25	5·5	2·25	1·25	206
19–19·5	—	—	·25	3	4	18·5	30·5	42·5	31·75	14·75	7	3	1·25	159·5
19·5–20	·25	·25	·25	1	·75	5·25	16·5	24·5	21·75	12	8·75	2·5	·75	94·5
20–20·5	·25	·25	·25	·5	·25	1·5	8	7	8·75	5·5	1·5	1	·25	38
20·5–21	—	—	—	—	—	·25	1·75	3·25	2·5	3·5	3	·75	—	15
21–21·5			—	—	—	1·25	·75		·75	1·25	2·25	1	·25	7·5
21·5–22	—	—	—	—	—	—	1·5	1	—	—	—	·75	·75	4
Totals	·5	1	8	30·5	66·5	172	259	236·5	161·5	74·5	36·5	12	4·5	1063

TABLE XXXI.

Mother's Stature and Daughter's Stature.

Mother's Stature.

Daughter's Stature.	*52—53*	*53—54*	*54—55*	*55—56*	*56—57*	*57—58*	*58—59*	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	Totals
52—53	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	·5
53—54	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	·5
54—55	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
55—56	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
56—57	—	—	—	—	—	—	1	·25	1·5	1·25	·25	·25	—	—	—	—	—	—	—	4·5
57—58	—	—	—	1·5	1·25	1·5	1·25	1·5	3·25	1·25	1·25	·75	1	—	—	—	—	—	—	14·5
58—59	—	—	—	·5	2·25	1·5	2·25	4·75	2·5	·25	·5	—	—	·25	·75	—	—	—	—	15·5
59—60	—	—	—	—	—	3·5	5·75	12	11·25	9·25	3	1·5	·75	·25	1·25	—	—	—	—	48·5
60—61	·25	·25	—	—	2·75	4·75	9·25	15	19·5	15·75	17	7·75	3·25	1·75	1·5	·25	1	—	—	100
61—62	·25	·25	·25	·25	·75	2	8	18·5	18·25	34	27·25	15	8·75	6	3·25	·25	—	—	—	143
62—63	·5	—	·25	·25	·25	4	11·5	19·75	33·25	34·75	34·75	24	12·75	10·25	2·75	1	·5	—	—	190·5
63—64	·5	—	—	—	·75	3·75	6·25	17·5	29	41·5	46·5	33·5	18·25	10·25	3·75	·5	·5	—	—	212·5
64—65	—	—	—	—	—	2·5	4	6·25	23·75	32	38·5	40	27·75	18·75	3·5	1·5	·5	—	—	199
65—66	—	—	—	—	—	·25	1	5·75	10·5	18	27·5	35	26	16·5	12·25	3·25	—	—	·5	156·5
66—67	—	—	—	—	—	·25	1·25	3·25	10·25	14·75	23	29·25	21·75	16·25	15·75	2·75	2	·5	·5	141·5
67—68	—	—	—	—	—	—	—	1·5	5·5	7·75	7·75	15	14·25	11·25	8·25	5	1·25	·5	1	79
68—69	—	—	—	—	—	—	—	·5	—	1·5	5·75	6·5	4·75	5·75	3·25	4·75	1·25	1	—	35
69—70	—	—	—	—	—	—	—	—	—	—	1	2·5	5·25	3·25	3·25	·75	1	1	·5	18·5
70—71	—	—	—	—	—	—	—	—	—	—	1	1	1	·25	1·25	2	1	1·5	·5	9·5
71—72	—	—	—	—	—	—	—	—	—	—	—	·5	—	·75	·75	·5	—	·5	1	4
72—73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
73—74	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
Totals	1·5	·5	·5	2·5	8	24	51·5	108·5	168·5	212	235	213·5	145·5	101·5	61·5	22·5	9	5	4	1375

TABLE XXXII.

Mother's Span and Daughter's Span.

Mother's Span.

Daughter's Span.	52—53	53—54	54—55	55—56	56—57	57—58	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	Totals
53—54	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
54—55	—	—	—	—	·25	·25	—	—	1	—	—	·25	·25	—	—	—	—	—	—	—	2
55—56	—	—	—	—	·5	—	1	—	—	—	—	·25	·25	—	—	—	—	—	—	—	2
56—57	—	—	—	—	2	·5	1·75	1·75	·5	—	—	—	—	—	—	—	—	—	—	—	6·5
57—58	—	·5	—	·5	1·75	4	5	1·25	2	1	·5	·5	—	—	—	—	—	—	—	—	17
58—59	—	·75	·5	3	3	4	8·5	11·5	10·75	1	3·25	2·75	1·75	·75	—	1	—		—	—	52·5
59—60	—	·25	·5	5·5	6·25	7·75	8·5	14·5	10	5·75	7·75	9·25	4·5	3·5	1	1	—	—	·25	·25	86·5
60—61	—	1	·5	2·75	9·5	8·75	12·5	21·5	20·5	16·75	13·75	8	6·75	4·75	1·5	·5	·5		·25	·25	130
61—62	·5	—	·5	3·75	4·5	6·5	14·25	17·5	26·25	26·5	22·25	15·5	13	3·25	1·75	·5	·5	—	·25	·25	157·5
62—63	·5	—	1·5	1·5	4·75	5·75	14	31	30·75	27·5	28·5	11·75	14	6·75	1·25	1·5		—	·25	·25	184·5
63—64	—	—	—	1	2·75	3·5	6·25	23·5	33·5	27·25	30·5	20·5	16	10·75	4·5	1·5	·5	—	—	—	182
64—65	—	—	·5	·5	3	2·75	8	19·75	18·75	21·5	32	22·75	15·25	12·5	2·5	·75	3·25	1·25			168
65—66	—	—	—	—	2·5	5	7·5	11·75	14·5	17·25	22·25	18·5	15·75	13	5·75	2·75	3·75	2·25	·5	—	143
66—67	—	—	—	—	1	2	1·25	1·25	5·5	13	19	9·25	10·75	8·25	5·25	4·5	2	·5	·5	—	87
67—68	—	—	—	—	—	—	1·5	2·5	3·75	6·5	17·5	11·75	8·75	4·75	7·75	2	1	1·25	—	—	69
68—69	—	—	—	—	—	—	·5	1·5	4·5	1	3·5	3·75	5·25	7·25	3	2·5	1·5	2·75	·5	—	43·5
69—70	—	—	—	—	—	—	—	·25	1·75	1·5	·75	2	5	4	2·25	2·5	·5	1	·5	—	22
70—71	—	—	—	—	—	—	—	—	—	—	—	1·25	1·25	1	1·25	1	·25	·5	1·5	—	8
71—72	—	—	—	—	—	—	—	—	—	—	1	1·5	·5	—	1·25	1	·25	·25	·25	—	6
72—73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	—	·5
73—74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
74—75	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	·5	—	1
75—76	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	·5	—	1
Totals	1	2·5	4	18·5	42	51	90·5	162·5	181	172·5	202·5	143·5	119	80·5	39	23	17	10	6	1	1370

TABLE XXXIII.

Mother's Forearm and Daughter's Forearm.

Mother's Forearm.

Daughter's Forearm.	*13—13·5*	*13·5—14*	*14—14·5*	*14·5—15*	*15—15·5*	*15·5—16*	*16—16·5*	*16·5—17*	*17—17·5*	*17·5—18*	*18—18·5*	*18·5—19*	*19—19·5*	*19·5—20*	Totals
13·5—14	—	—	—	—	—	—	—	·5	1	—	—	—	—	—	1·5
14—14·5	—	—	1	·25	·25	·25	·75	1·5	1	—	—	—	—	—	5
14·5—15	·25	·25	·25	2	3·5	2	3·75	3·5	1·75	·25	·25	·25	—	—	18
15—15·5	·25	·25	1·25	6·25	11·5	17	14	7·25	6·75	1·5	·25	·25	—	—	66·5
15·5—16	—	—	1·5	11	23·75	47·25	42	24·75	16	4·5	1	·25	—	—	172
16—16·5	—	·75	1·75	8	27·75	61·75	86·75	63·5	35·25	11·5	3·25	·25	—	—	303·5
16·5—17	—	·25	1·5	6·25	14·75	48·5	80·5	77	43	19	6·75	1·5	1	—	300
17—17·5	—	—	·25	3·25	8·75	30	54·75	66·5	52·25	16·75	8·75	1·75	1	—	244
17·5—18	—	—	—	·25	2	10·5	27·75	39·5	30	22·5	12·5	2·25	·75	—	148
18—18·5	—	—	—	·5	1·5	4·25	4·25	14·5	22·5	12·75	8	3·5	1·75	—	73·5
18·5—19	—	—	—	·25	·25	·25	1·25	3·75	6·75	3·25	2·75	3·5	1·5	—	23·5
19—19·5	·25	·25	—	—	—	·5	·5	1·75	2·25	1·25	2·25	·5	·5	—	10
19·5—20	·25	·25	—	—	—	·25	·25	—	1	·25	·25	—	·25	·25	3
20—20·5	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	1
20·5—21	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	·5
Totals	1	2	7·5	38	94	225·5	316·5	301	219·5	93·5	46	14	7·5	1	1370

TABLE XXXIV.

Father's Stature and Son's Span.

Father's Stature.

Son's Span.	*58—59*	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	*74—75*	*75—76*	Totals
58—59	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	·5
59—60	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	·5
60—61	—	—	—	—	—	—	1	·5	—	—	—	—	—	—	—	—	—	—	1·5
61—62	—	—	—	—	—	—	—	·5	1	1	—	—	—	·25	·25	—	—	—	3
62—63	—	—	—	·5	·5	—	1	·25	·25	1	—	·25	·25	·25	·25	—	—	—	4·5
63—64	—	—	—	—	1	1	1·5	3·5	4·75	1·75	—	·25	·25	—	—	—	—	—	14
64—65	—	—	—	1·75	1·5	3·5	1·25	4·75	3	4·75	4	2	1·25	·5	·25	—	—	—	28·5
65—66	·25	·25	·5	3·75	5·25	4·75	5·75	8·5	9·75	10·5	4·25	4·75	·5	5	·25	·25	·25	—	60
66—67	·25	·5	1·75	3·25	4·75	6·25	10·25	12·25	13	9	8	5·75	3·25	1·5	·25	·25	·25	—	80·5
67—68	—	1·25	1·75	·75	4·25	6·25	11·25	16	20	11·75	12·25	5·5	4·25	2·5	2·25	—	—	—	100
68—69	—	1	—	1·25	5·5	5·75	13·5	17·25	18·25	17·25	14	9·75	12·25	5·75	1·5	·5	—	—	123·5
69—70	—	—	1	·75	2	5·5	9	24·25	19·5	20	22·5	15·25	11·25	4	1·75	·25	·5	·5	138
70—71	—	—	—	·5	2·75	3·25	8	19	16	20·75	27·25	21·5	14·25	5·5	3	1	·25	—	143
71—72	—	—	—	—	1	4	8	7·75	11·75	13·75	15	19·25	14	9·75	4·25	1·25	·75	·5	111
72—73	—	—	·5	·75	1·5	1·5	3·5	5·5	7·75	17·5	14·5	11·25	8·5	10	5	3·25	1	—	92
73—74	—	—	—	·25	1	1	3·75	3·75	6·5	9·75	9·25	9·75	13·25	8·25	3·75	·75	1·5	·5	73
74—75	—	—	—	—	—	·75	·25	2·5	3·25	4·75	8·75	5	7·75	5·25	5	1·25	—	1	45·5
75—76	—	—	—	—	—	—	1·5	·75	·25	—	6·25	2·75	3·5	3·75	4·5	2·25	—	—	25·5
76—77	—	—	—	—	—	—	·5	—	—	·5	1·5	1	3·25	1	·75	1·5	—	—	10
77—78	—	—	—	—	—	—	—	—	—	—	3	1	1	2·5	—	—	—	—	7·5
78—79	—	—	—	—	—	—	—	—	—	—	3·5	1	·25	·25	·75	·75	—	—	6·5
79—80	—	—	—	—	—	—	—	—	—	—	—	—	1	—	·75	·25	—	—	2
80—81	—	—	—	—	—	—	—	—	—	—	—	—	1	—	·5	—	—	—	1·5
Totals	·5	3	5·5	13·5	31	44·5	80	127	135	144	154	116	101	61·5	35	13·5	4·5	2·5	1072

TABLE XXXV.

Father's Span and Son's Stature.

Father's Span.

Son's Stature.	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	76—77	77—78	78—79	79—80	80—81	Totals
59—60	·25	·25	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
60—61	·25	·25	—	·25	·25	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	1·5
61—62	—	—	—	—	—	—	—	—	—	·75	·75	—	—	—	—	—	—	—	—	—	—	—	1·5
62—63	—	—	—	·5	·5	2·5	2	1·75	1	·5	·5	·25	—	—	—	—	—	—	—	—	—	—	9·5
63—64	—	·25	·75	·75	4	9·25	3	1·75	4	3	1·5	1·25	·75	·75	—	—	—	—	—	—	—	—	31
64—65	·25	1	1·25	·75	2·5	7·75	6·75	6	5·25	7	2·75	6	1	1·5	·25	—	—	—	—	—	—	—	50
65—66	·25	·75	1·5	2·25	3·75	12·5	6·5	13·75	6·25	8	7·5	4	3·25	3·5	·25	—	—	—	—	—	—	—	74
66—67	—	1	1·25	3·5	2·5	8·75	9	23	19·75	15·5	8·75	10·25	6·25	2	3	·25	·25	—	—	—	—	—	115
67—68	—	3	3	2	4·5	7·25	13	20·25	27·75	27·25	22·75	13	9·25	6	5	1·25	2·25	—	—	—	—	—	167·5
68—69	·25	·25	1·75	1·75	2·25	11·75	15·25	14·5	19·5	24	26·75	21	14·75	9·75	6·5	2	1·5	1	—	1	—	—	175·5
69—70	·25	·25	—	1·25	·25	4	7	11	15·25	17·25	13·25	13·75	17·75	9·25	5·75	3·5	1·75	1	—	—	—	—	122·5
70—71	—	—	—	2	2	4·75	5·25	10·75	10·75	18·75	17·25	18	11	10	5·25	4·5	1·25	2·5	·5	2	—	—	126·5
71—72	—	—	—	·75	·25	2·5	4·25	6	10	6·5	10·25	7·75	7·75	7	6	5·75	3	1·25	—	—	—	—	79
72—73	—	—	—	—	1	—	1·5	1·25	5	4·75	6·5	3·5	7·5	7	7	1·75	2	·25	—	—	—	—	49
73—74	—	—	—	1	—	1	—	2·25	3·75	4·25	4·75	6	4·25	4·25	1	3	1·5	—	—	—	—	—	37
74—75	—	—	—	—	—	—	1	·25	1	2	2	5·25	·5	2	·5	·75	1·25	—	—	—	—	·5	17
75—76	—	—	—	—	—	—	—	—	·75	·75	·5	·5	—	1·5	·5	·25	1	·25	—	—	—	·5	6·5
76—77	—	—	—	—	—	—	—	—	—	—	—	—	2	·25	·25	—	·75	·25	—	—	—	—	3·5
77—78	—	—	—	—	—	—	—	—	·5	—	·5	—	—	·25	·25	—	—	—	—	—	—	—	1·5
78—79	—	—	—	—	—	—	—	—	·5	—	·5	—	—	—	—	—	·5	·5	—	—	—	—	2
79—80	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
Totals	1·5	7	9·5	17	24	72	74·5	112·5	131	140·5	128	110·5	86	65	41·5	23	17	7	·5	3	—	1	1072

TABLE XXXVI.

Father's Stature and Son's Forearm.

Father's Stature.

Son's Forearm.	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	Totals
14—14·5	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
14·5—15	—	—	—	—	—	—	—	·5	—	—	·25	·25	—	—	—	—	—	—	1
15—15·5	—	—	—	—	—	—	—	·5	—	—	·25	·25	—	—	—	—	—	—	1
15·5—16	—	—	—	—	·25	·25	·25	·75	—	·5	·5	—	—	—	—	—	—	—	2·5
16—16·5	—	—	—	—	1	1·5	1	1	·75	2·5	·75	—		—	—	—	—	—	8·5
16·5—17	·5	·5	—	1	2·25	2·5	4·5	4	7·75	6·75	3	·75	1·25	·25	—	—	—	—	35
17—17·5	—	·5	·5	4·25	3·5	6·5	10·75	16·5	16	9·5	4·25	4·75	2·5	1·5	—	—	—	—	81
17·5—18	—	1	2·5	3	4·5	7·5	11·75	25·25	21	20	15·25	14·75	8·5	4·25	3·25		·5	·5	149·5
18—18·5	—	1·5	1	1·5	5	11·5	18·25	27·5	30·5	31·5	30·5	25	20·25	8·75	3	1	·75	·5	218
18·5—19	—	—	—	·5	1·5	5·75	9·25	21·5	25	24·75	30·75	28·75	23·25	15	6·5	3·25	·25	—	196
19—19·5	—	—	—	·25	1·5	2·75	8·25	9	17·75	22·75	26·75	23	20·75	12·5	7·25	4	1·5	·5	158·5
19·5—20	—	—	—	—	·75	1	5·5	6·25	7·25	13	17	9·25	9·5	8·25	6·75	3·25	·25	1	89
20—20·5	—	—	—	—	·25	·25	2	2·25	1·5	5	8·5	2·75	3·75	4	2·75	2·25	1·25	—	36·5
20·5—21	—	—	—	—	—	—	·5	·75	·25	·25	3·75	2	1·25	1·5	1·75	5	—	—	12·5
21—21·5	—	—	—	—	—	—	—	·25	·25	—	1·5	·5	1·5	1	1·75	·25	—	—	7
21·5—22	—	—	—	—	—	—	—	—	—	—		1·5	1·5	—	—	—	—	—	3
Totals	·5	3·5	4	10·5	20·5	39·5	75	116	131	136·5	141	113·5	94	57	33	14·5	4·5	2·5	1000

TABLE XXXVII.

Father's Forearm and Son's Stature.

Father's Forearm.

Son's Stature.	15—15·5	15·5—16	16—16·5	16·5—17	17—17·5	17·5—18	18—18·5	18·5—19	19—19·5	19·5—20	20—20·5	20·5—21	21—21·5	21·5—22	22—22·5	Totals
59—60	—	—	—	·25	·5	·25	·5	—	—	—	—	—	—	—	—	1·5
60—61	—	—	—	·25	·5	·25	·5	·25	·25	—	—	—	—	—	—	2
61—62	—	—	—	—	—	·5	·5	·25	·25	—	—	—	—	—	—	1·5
62—63	—	—	·5	·75	4·25	2·5	1·25	·5	·25	—	—	—	—	—	—	10
63—64	—	·75	2·75	4·5	6·25	8·5	4	2·75	·75	·25	—	—	—	—	—	30·5
64—65	·25	1·5	1·75	5·5	8·75	17·5	9·75	4·5	2·75	1	·25	—	—	—	—	53·5
65—66	·25	1·75	2·5	7	16·75	17·5	12·75	9·25	5·5	·5	·25	·25	·25	—	—	74·5
66—67	—	·5	2·5	9	18	25·75	29·5	17	6·5	1	1·75	·75	·75	—	—	113
67—68	·75	1·5	4·75	7·5	16·25	39·25	36·75	30·75	17·25	7·75	3	1·25	·25	—	—	167
68—69	·75	2	3	7·25	19·75	26	44·5	35	22	11	1·75	1·25	·75	·25	·25	175·5
69—70	—	—	1·25	1·25	12·75	19·5	23·75	29·5	20·5	12·25	2·25	1·25	·25	·25	·25	125
70—71	—	—	·75	3·75	10·75	20·25	26·5	23·25	22·75	9·25	4·25	1·5	·5	—	—	123·5
71—72	—	—	·75	2·25	3·75	11·25	17·25	13·5	19	6	4·75	1·25	·25	·25	·25	80·5
72—73		—	—	·75	2·25	7	8·5	5·5	11·75	6·75	4	·5	·25	·75	·5	48·5
73—74	—	—	—	·25	·75	5	8·25	7·75	5	5	2	—	·25	1	·75	36
74—75	—	—	—	·25	·25	3·75	5	2	1·25	1·25	2	1·5	·25	—	—	17·5
75—76	—	—	—	—	—	·25	1·25	·25	1	1·25	1·25	·5	·25	—	—	6
76—77	—	—	—	—	—	—	·5	1	1·25	·25	·5	—		—	—	3·5
77—78	—	—	—	—	—	·25	·5	·25	·5	—	—	—	—	—	—	1·5
78—79	—	—	—	—	—	·25	·5	·25	·5	·5	—	—	—	—	—	2
79—80	—	—	—	—	—	—	·5	·5	—	—	—	—	—	—	—	1
Totals	2	8	20·5	50·5	121·5	205·5	232·5	184	139	64	28	10	4	2·5	2	1074

TABLE XXXVIII.

Father's Span and Son's Forearm.

Father's Span.

Son's Forearm.	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	*74—75*	*75—76*	*76—77*	*77—78*	*78—79*	*79—80*	*80—81*	Totals
14—14·5	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
14·5—15	·25	·25	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	1
15—15·5	·25	·25	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	1
15·5—16	—	—	—	—	1	1·25	1	·25	·75	·25	·25	·25	—	—	—	—	—	—	—	—	—	—	5
16—16·5	—	—	·25	·75	·75	·5	1·75	·5	2·25	·75	1	·5	—	—	—	—	—	—	—	—	—	—	9
16·5—17	·5	·5	·25	1·25	2·75	5	5	1	9·25	4·25	2·25	·75	·25	·25	·25	—	—	—	—	—	—	—	36·5
17—17·5	—	1·25	1·75	2·25	4·25	13·75	12	16·25	7·25	12·25	5	4·5	1·75	1·5	·75	—	—	—	—	—	—	—	84·5
17·5—18	—	2·5	2·25	1·75	4·75	19	12·75	23·5	20·75	22·75	18	13·75	7·5	6·75	3·5	1·5	1	1	—	1	—	—	164
18—18·5	—	1·25	4	2·75	6	16·25	19·5	27·25	31·75	34	27·5	25·25	17	12	7·75	1·75	2·5	1·25	·25	1	—	—	239
18·5—19	·5	·75	1·75	3	2	6·25	11·25	18·75	29·5	30·5	25	28·75	19·5	14	8·25	3·75	3·5	·75	·25	—	—	—	208
19—19·5	—	·25	1·25	1·75	1·5	5	5	14·25	15·75	19·5	20·75	22·25	21	14·25	9	3·5	2·5	1·5	—	—	—	—	159
19·5—20	—	—	·5	2·25	·75	1·75	4	5·5	5·5	8	18	10	10·5	9·5	8	5	5	·75	—	1	—	—	96
20—20·5	—	—	—	·25	·25	·75	1·75	1·25	4·5	3·75	4·25	1·5	5·75	1·25	3·5	3·5	2	·25	—	—	—	·5	38
20·5—21	—	—	—	—	—	·75	·75	—	1·25	2·75	1·25	·5	1·5	2	·75	·5	·75	·25	—	—	—	·5	13·5
21—21·5	—	—	—	—	—	·25	·25	—	·5	·75	1·25	—	·75	1·5	·25	1	·75	·25	—	—	—	—	7·5
21·5—22	—	—	—	—	—	—	—	—	·5	—	2·5	—	—	—	—	—	—	1	—	—	—	—	4
Totals	1·5	7	12	16	24	70·5	75	111·5	130·5	140·5	127	108	85·5	66	12	20·5	18	7	·5	3	—	1	1067

TABLE XXXIX.

Father's Forearm and Son's Span.

Father's Forearm.

Son's Span.

	15—15·5	*15·5—16*	*16—16·5*	*16·5—17*	*17—17·5*	*17·5—18*	*18—18·5*	*18·5—19*	*19—19·5*	*19·5—20*	*20—20·5*	*20·5—21*	*21—21·5*	*21·5—22*	*22—22·5*	Totals
58—59	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	·5
59—60	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	·5
60—61	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	·5
61—62		—	—	·25	1·25	—	—	·5	·75	·25	—	—	—	—	—	3
62—63	—	—	—	—	1·5	1·5	·5	—	·5	·5	—	—	—	—	—	4·5
63—64	—	—	·5	1·5	2·75	4·75	2·25	1·25	·25	·25	—	—	—	—	—	13·5
64—65	·5	·75	1·75	1	5	7·5	6	3·75	1	1	·25	—	—	—	—	28·5
65—66	·5	3·25	5·75	5·25	10·75	13·25	10·25	6·5	2·25	1	·5	·25	—	—	—	59·5
66—67	—	1	4·5	10·25	16·75	18	14·25	7·75	4·25	·75	·25	·5	·25	—	—	78·5
67—68	·75	1·25	1·25	8·25	17·25	25·75	24	12	6·25	1·75	1	·5	·5	—	—	100·5
68—69	·25	1	1·5	6·5	15·25	27·5	27·75	22·5	14·25	5·25	1·75	·25	·25	—	—	124
69—70	—	·25	1·5	6·5	15·25	27	32	30·5	15	5·75	·75	1·5	·5	—	—	136·5
70—71	—	·25	·75	3·75	13·25	28	37·25	29·25	22	5·75	2·25	·5	—	—	—	143
71—72	—	·25	1	2·75	7·75	19·5	24·75	24·25	18·5	8·75	1·5	1·25	·25	·5	·5	111·5
72—73	—	·5	1	1·5	6·75	13·75	20·5	18	16	8·75	3·5	·5	·25	·5	·5	92
73—74	—	·25	·75	1·5	2·5	10	17	10·75	15	6·75	6·75	1·75	·75	·5	·25	74·5
74—75	—	·25	·25	·25	2·75	4·25	5·5	6·5	9·75	7·5	2·5	·5	·25	·5	·25	41
75—76	—	—	—	—	·25	1·75	1·75	4·25	8·5	7	3·5	·75	·25	—	—	28
76—77	—	—	—	—	·25	1·25	2	1·25	·5	·25	1	·75	·25	—	—	7·5
77—78	—	—	—	—	—	·25	1·25	1·75	1·5	1	·75	—	—	·5	·5	7·5
78—79	—	—	—	—	—	·5	2·25	1	1·25	1·25	1·25	·25	·25	—	—	8
79—80	—	—	—	—	—	—	·25	·25	·25	·25	—	·25	·25	—	—	1·5
80—81	—	—	—	—	—	—	—	—	·25	·25	—	—	—	—	—	·5
81—82	—	—	—	—	—	—	·5	·5	—	—	—	—	—	—	—	1
Totals	2	9	20·5	49·5	120	205	230	182·5	138	64	27·5	9·5	4	2·5	2	1066

TABLE XL.

Father's Stature and Daughter's Span.

Father's Stature.

Daughter's Span.	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	Totals
52—53	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
53—54	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	·5
54—55	—	—	—	—	1	·25	·25	·25	·25	—	—	—	—	—	—	—	—	—	2
55—56	—	—	—	—	1	—	—	·25	·75	—	—	—	—	—	—	—	—	—	2
56—57	—	—	—	·75	·25	—	—	1·5	3	·75	·25	—	—	—	—	—	—	—	6·5
57—58	—	—	—	·75	1	1·5	2·5	3·75	2·5	2·25	1·25	1·5	—	—	—	—	—	—	17
58—59	—	—	2	1·5	8·75	4	4	7·5	11·25	4	5	4	·5	—	—	—	—	—	52·5
59—60	·5	·5	1	·5	6·25	4·75	9·25	13	19·75	14·25	10	3·25	1·75	·25	·25	·75	—	—	86
60—61	·5	·5	—	1	7·75	8·5	13·25	19·75	23·75	15·25	13·25	11·75	8·25	3	·75	1·25	·5	—	129
61—62	—	1·75	2	2	5·5	10	16	23·25	29·25	20	19·75	8·5	10·75	5·75	2·5	—	—	—	157
62—63	—	·25	·5	2·25	6·75	8·75	15·25	25·25	22·25	25·75	30·25	19·25	13	7·5	4·5	·5	1	—	183
63—64	—	1·5	·5	1·25	5·25	6·25	10·5	17·5	20	30·5	29	24·5	21·25	11·5	4	·75	·25	—	184·5
64—65	—	—	·25	·75	1	2·25	10·25	17·75	17·75	21·25	25	28	20·5	12·5	6	3·75	·5	1	168·5
65—66	—	—	·25	·25	·75	2·75	5	10·75	10·75	17	25·75	27·25	17·5	11·5	5·75	3·75	·75	2·25	142
66—67	—	—	—	—	·75	1·25	4	10	7	9	15·5	11·5	12·25	11	3·5	2·5	1·5	·25	90
67—68	—	—	—	—	—	—	1·5	4·25	5·75	9·5	11·75	13·5	10	7·75	3·5	1·5	1	·5	70·5
68—69	—	—	—	—	—	·5	·5	·25	2·75	2	1·5	5·75	12	8·25	4·75	1·75	·5	·5	41
69—70	—	—	—	—	—	1·5	—	—	1·25	1·5	5	3	1·75	2·5	3	2·75	·25	—	22·5
70—71	—	—	—	—	—	—	—	·5	·5	—	2·25	1·75	3·5	—	—	·25	·25	—	9
71—72	—	—	—	—	—	—	—	—	—	—	·5	·5	2·5	1	1·5	—	—	—	6
72—73	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	·5
73—74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
74—75	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	·25	·25	—	—	1
75—76	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	·25	·25	—	—	1
Totals	1	4·5	6·5	11	16	52·5	92·5	155·5	178·5	173	200	164	136	82·5	40·5	20	6·5	4·5	1375

TABLE XLI.

Father's Span and Daughter's Stature.

Father's Span.

Daughter's Stature.	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	76—77	77—78	78—79	79—80	80—81	Totals
52—53	—	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
53—54	—	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
54—55	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
55—56	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
56—57	·25	·25	·25	·25	·25	1	·75	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3·5
57—58	·25	·25	·75	·75	1	2·75	2	2·25	2	2	—	·25	·25	—	—	—	—	—	—	—	—	—	14·5
58—59	—	·25	·75	·25	1·25	1	3·5	4·75	2·25	·5	—	·25	·25	—	·5	—	—	—	—	—	—	—	15·5
59—60	—	1·25	·25	·75	4·75	6·25	5·25	8·5	5·75	4·25	3·5	2	2	2	·5	—	—	—	—	—	—	—	47
60—61	·5	·5	·75	2·75	3	11·5	13·25	15·75	17·75	13	8·75	3·75	2·25	3·25	1·25	·5	·5	·5	·5	—	—	—	100
61—62	·5	1·5	4	5	7	10·25	12·5	20	20·5	14	18·5	9·25	6·5	7	2	1·5	·5	—	—	—	—	—	140·5
62—63	—	1	3·75	2	7·5	17·5	20	16	32·25	22	19·5	21·25	13·25	5·5	5·5	2·25	·25	—	—	—	—	—	189·5
63—64	—	1·5	1·5	4	3·75	11·25	13·25	19·75	31·25	39·5	27·5	26	11·75	9·5	4	2·25	1·25	·5	—	—	—	—	208·5
64—65	·5	1	·5	2·75	2	9·75	9·75	17	19	26	32·25	28	18·5	13·25	9·75	3·5	2·5	1·5	1	·5	—	—	199
65—66	·5	·5	—	2	2	3·25	5·75	11·5	12·5	21	27·5	23	13·25	10·25	11	6	5	3	·5	1	—	—	159·5
66—67	—	—	—	—	2	1·5	5	7·25	10·25	21	18	14	18	18·5	8·75	6·25	5	2·25	·25	1	—	—	139
67—68	—	—	—	—	1	2·75	5·25	3·25	3·75	11·5	10·25	7·25	12	6·25	5·5	2·5	3·25	·75	·25	·5	—	·5	76·5
68—69	—	—	—	—	—	·25	1·25	1·5	3·5	1·5	4·75	4·5	4·5	4	3·25	2·25	1·75	1	—	—	—	·5	34·5
69—70	—	—	—	—	—	—	—	1·5	1	·5	1	1·5	2·5	3	·25	2·25	3	·5	—	—	—	—	17
70—71	—	—	—	—	—	—	—	1	·25	·75	·5	2	—	1·75	1	1·25	—	—	—	—	—	—	8·5
71—72	—	—	—	—	—	—	—	·5	—	·5	·5	—	—	·25	·25	—	1	1	—	—	—	—	4
72—73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
73—74	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
Totals	2·5	8	12·5	20·5	35·5	80	98	131·5	162	178	172·5	143	106	84·5	53·5	30·5	24	11	2·5	3	—	1	1360

TABLE XLII.

Father's Stature and Daughter's Forearm.

Father's Stature.

Daughter's Forearm.	*58—59*	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	*74—75*	*75—76*	Totals
13·5—14	—	—	—	—	—	·25	·25	·25	·25	—	·5	—	—	—	—	—	—	—	1·5
14—14·5	—	—	—	—	·25	·5	·25	·5	2	—	1·5	—	—	—	—	—	—	—	5
14·5—15	—	—	—	·75	3	1·75	1·25	2·75	1	1·75	·75	1	—	—	—	·25	·25	—	17·5
15—15·5	·25	·25	·5	1·25	9	4·75	5	7·25	14·75	9	5·5	5	1·75	·75	—	·25	·25	—	65·5
15·5—16	·25	1	1·75	2·75	8·75	12·25	17	20·75	34·25	21·75	22·75	11·75	8·5	5·75	1·25	1·5		—	172
16—16·5	—	1	1·5	3	8·25	13·75	28	44·5	46·75	13·75	44·5	31·25	19·75	11	3·75	1·25	·5	—	302·5
16·5—17	·5	1·5	1·75	2	10·5	10·5	20·75	38·25	33	40·25	37·5	38·25	31·75	19·5	9·5	5	1	—	301·5
17—17·5	—	·75	·5	·25	1·75	5·5	12	25·25	24·25	34	38·25	33·5	29	20·75	8	3·75	2	2·5	245
17·5—18	—	—	·25	·5	·75	2·25	5	7·75	12·25	14	28	26	21·25	15·25	7·25	4	2	2	148·5
18—18·5	—	—	·25	·5	·5	·75	·5	6	6·25	7·5	11·5	12	14·5	5·5	5·75	3	—	—	74·5
18·5—19	—	—	—	—	·25	·25	—	1·5	1·25	·75	5·75	3·75	5·5	2·25	1·25	·5	—	—	23
19—19·5	—	—	—	—	—	—	—	·25	·75	·5	3	1	1·5	1·25	1·75	·75	·25	—	11
19·5—20	—	—	—	—	—	—	—	—	·25	·25	·5	—	1	·5	—	·25	·25	—	3
20—20·5	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
20·5—21	—	—	—	—	—	—	—	—	—	—	—		·5	—	—	—	—	—	·5
Totals	1	4·5	6·5	11	46	52·5	90	155	180	173·5	200	163·5	136	82·5	38·5	20·5	6·5	1·5	1372

TABLE XLIII.

Father's Forearm and Daughter's Stature.

Father's Forearm.

Daughter's Stature.	15—15·5	15·5—16	16—16·5	16·5—17	17—17·5	17·5—18	18—18·5	18·5—19	19—19·5	19·5—20	20—20·5	20·5—21	21—21·5	21·5—22	22—22·5	Totals
52—53	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	·5
53—54	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	·5
54—55	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
55—56	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
56—57	—	·5	—	1·25	2·5	·25	—	—	—	—	—	—	—	—	—	4·5
57—58	—	·5	—	3·75	4·25	3·5	2	—	—	—	—	—	—	—	—	14
58—59	—	·25	·75	1·75	2·75	5·5	3·5	·75	·25	—	—	—	—	—	—	15·5
59—60	·25	1	2·25	6	8	11·5	7·25	6·75	3	1	·5	—	—	—	—	47·5
60—61	·25	1·75	2·5	7·25	14	26·25	29	10·75	4·75	·75	1·25	·5	—	—	—	99
61—62	—	1·25	3·5	8·75	24·75	32	36	19·5	7·25	2·75	2·75	1·25	·75	—	—	140·5
62—63	—	1·25	4	12·25	30·25	42·25	40·5	28·5	24·75	6	1·75	·25	·25	—	—	192
63—64	—	·75	3·25	6·5	25	43	56	42	19·25	7·75	4·5	2	1	·5	·5	212
64—65	—	·25	2·25	5·75	21·25	28·75	49	39·25	28·25	13	4·25	2	2	1	·5	197·5
65—66	—	—	·5	3·5	10·75	21·25	38	33·75	27·5	15·25	6·75	·75	·5	—	—	158·5
66—67	—	—	·5	2	7	15·75	31·75	34	30·25	13	4·75	1·5	·5	—	—	141
67—68	—	·5	·5	·75	4·25	7·25	15·25	15·25	15	9·75	5	1·5	·5	·25	·25	76
68—69	—	—	—	2	1·75	3	5·75	7·75	4·5	4·5	4	1·25	·5	·25	·25	35·5
69—70	—	—	—	—	1	1	2	2·25	3·25	3·75	2·25	1·5	—	·5	1·5	19
70—71	—	—	—	—	—	·25	1·5	2·5	3	1·25	·75	·25	—	—	—	9·5
71—72	—	—	—	—	—	—	·5	·5	1	·25	1·5	·25	—	—	—	4
72—73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
73—74	—	—	—	—	—	—	—	·5	·5	—	—	—	—	—	—	1
Totals	·5	8	20	63	158	241·5	318	244	172·5	79	40	13	6	2·5	3	1369

TABLE XLIV.

Father's Span and Daughter's Forearm.

Father's Span.

Daughter's Forearm.	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	*74—75*	*75—76*	*76—77*	*77—78*	*78—79*	*79—80*	*80—81*	Totals
13·5—14	—	—	—	·25	·25	—	·25	·25	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	1·5
14—14·5	—	—	—	·25	·5	·25	·5	2	—	1·25	·25	—	—	—	—	—	—	—	—	—	—	—	5
14·5—15	—	—	·5	1·75	1·25	3·5	2	3·75	2·25	1·25	1·25	—	·25	·25	—	—	—	—	—	—	—	—	18
15—15·5	·25	·75	1·5	2·25	3·5	7	7·75	8	11·25	8·5	5·5	5	3·75	1·25	·25	—	—	—	—	—	—	—	66·5
15·5—16	·5	2·75	3	5	6	15·5	25·25	24·75	23	21·5	14	11·5	6	6·75	5·25	·5	·25	—	—	—	—	—	171·5
16—16·5	·25	2·25	3	4	10·75	24	29	38	48·75	37·75	31	30·75	18·5	11·5	5·75	2·25	1	·75	·75	—	—	—	300
16·5—17	·5	1	2·75	5·5	7	19·5	15·5	30	35·75	40·75	42·5	34·5	23·75	15·25	14·25	5·5	3·5	1·25	·75	·5	—	—	300
17—17·5	1	1·25	1·25	3	4·75	6·75	10·5	15·75	24	35·25	41·75	30·25	20·25	17·75	8·75	6·75	6·25	4·5	·75	1·5	—	—	242
17·5—18	—	—	·25	·25	1·5	2·25	3·75	7	11·25	21	22	16	18·25	15	11·75	9·25	6	1·75	·25	1	—	·5	149
18—18·5	—	—	·25	·25	—	1·25	1	3	4·5	8·25	11·5	10	10·5	9·5	5	3	4·75	1·25	—	—	—	·5	71·5
18·5—19	—	—	—	—	—	·75	·25	·5	1	2·25	3·25	2·75	2·75	4·75	1·5	1·75	1·25	1·75	—	—	—	—	24·5
19—19·5	—	—	—	—	—	·25	·75	·5	·25	1	·25	1·25	1·5	1·5	·5	·75	1·75	·75	—	—	—	—	11
19·5—20	—	—	—	—	—	—	·5	·5	—	·5	—	—	1	—	—	·25	·25	—	—	—	—	—	3
20—20·5	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
20·5—21	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	·5
Totals	2·5	8	12·5	22·5	35·5	81	97	131	162	179·5	173·5	142	108	83·5	53	30	25	12	2·5	3	—	1	1368

TABLE XLV.

Father's Forearm and Daughter's Span.

Father's Forearm.

Daughter's Span.	15—15·5	15·5—16	16—16·5	16·5—17	17—17·5	17·5—18	18—18·5	18·5—19	19—19·5	19·5—20	20—20·5	20·5—21	21—21·5	21·5—22	22—22·5	Totals
53—54	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	·5
54—55	—	—	·5	1·5	—	—	—	—	—	—	—	—	—	—	—	2
55—56	—	—	·25	1·25	·25	·25	—	—	—	—	—	—	—	—	—	2
56—57	—	—	—	1·5	2·5	1	1	—	—	·25	·25	—	—	—	—	6·5
57—58	—	—	—	2	3·25	4·75	3·75	1·75	—	·75	·75	—	—	—	—	17
58—59	—	·25	3·5	8	10	15·5	9·5	3·25	2	·25	·25	—	—	—	—	52·5
59—60	·5	1·75	2·75	6·25	18	17·25	21	8·75	3·75	1·75	·25	·5	·5	—	—	83
60—61	—	1·75	2·75	8·75	21	29·25	30·75	18·25	10·75	2·5	1·25	·5	—	—	—	127·5
61—62	—	2	4·25	9·5	24·5	37·75	38·25	18·75	17·75	3·25	1·5	1·5	·5	—	—	159·5
62—63	—	·25	4·25	8	21	34·5	45·5	34·5	24	8·5	3·5	·5	—	—	—	184·5
63—64	—	·5	·75	4·25	20·5	38·75	50·25	34·5	19·5	8	3·25	1·25	1	1	1	184·5
64—65	—	·75	1	4	17·25	24	43	37·75	25·75	8	3	2·5	2·25	·25	—	169·5
65—66	—	·25	·25	2	9·5	20·75	32·5	33·5	16·75	12·75	9·5	1·75	1·25	·25	—	141
66—67	—	·25	·25	2·75	6·75	11	19	18	16·5	10	2·5	·5	—	—	—	87·5
67—68	—	·25	·25	·25	2·25	3·25	14	19	17	9·5	3·25	·75	·25	·25	·25	70·5
68—69	—	—	—	·75	·75	1·75	7	7·75	10	9·75	4	1·5	·25	·25	·75	44·5
69—70	—	—	—	—	1	1·25	3·25	4	3·25	2·5	3·5	1·75	—	·5	1	22
70—71	—	—	—	—	—	—	1·5	3	1·75	1·5	1·25	—	—	—	—	9
71—72	—	—	—	—	—	—	·75	1·25	2·25	1·5	·25	—	—	—	—	6
72—73	—	—	—	—	—	—	—	—	—	·25	·25	—	—	—	—	·5
73—74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
74—75	—	—	—	—	—	—	—	·25	·25	—	·5	—	—	—	—	1
75—76	—	—	—	—	—	—	—	·25	·25	—	·5	—	—	—	—	1
Totals	·5	8	21	61	158·5	241	321	244·5	171·5	81	39·5	13	6	2·5	3	1372

TABLE XLVI.

Mother's Stature and Son's Span.

Mother's Stature.

Son's Span.	52–53	53–54	54–55	55–56	56–57	57–58	58–59	59–60	60–61	61–62	62–63	63–64	64–65	65–66	66–67	67–68	68–69	69–70	70–71	Totals
58–59	—	—	—	—	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	·5
59–60	—	—	—	—	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	·5
60–61	—	—	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	·5
61–62	—	—	—	—	—	—	—	2·25	·25	·25	·25	—	—	—	—	—	—	—	—	3
62–63	—	—	—	—	—	—	·5	1	·5	·25	·25	1	—	—	—	·5	—	—	—	4
63–64	·5	·5	·5	·5	—	·5	1·5	4·75	·75	1	2	—	—	—	—	·5	—	—	—	13
64–65	—	—	·5	·5	—	1·25	2·25	3·5	4	4	3·75	5	1·75	—	1	—	—	—	—	27·5
65–66	—	—	·25	·25	3	3·25	2·5	5·75	11	13·25	7·5	8·25	2·5	·75	·5	·25	—	—	—	59
66–67	—	—	·25	·25	2·25	2·75	4·25	10·5	14·75	15·5	10·5	7·25	5·25	3·75	1	1·25	1	—	—	80·5
67–68	—	—	—	—	1·25	4·25	7·5	18·25	22	16·5	17	6·5	2	1·75	2·5	·5	—	—	—	100
68–69	—	—	—	—	·5	3·25	6·5	12·25	15·75	20·25	24	15·5	16·5	6·5	1·5	·5	—	—	—	123
69–70	—	—	—	—	—	2	6·75	10·25	25·5	20·75	27·75	19·25	15·75	5·25	1·5	2·25	·5	—	—	137·5
70–71	—	—	—	1	—	3·5	2·75	3·5	22·25	21·5	29·75	28	15·75	9·75	3·5	1	·75	—	—	143
71–72	—	—	—	—	—	·75	·75	5	11·5	20	19·25	16·75	16·5	8·75	5·5	1·75	2	1	—	109·5
72–73	—	—	—	—	—	·5	·5	2·75	8·25	18	19	13·75	12	9·75	5·5	2·75	·25	—	—	93
73–74	—	—	—	—	—	—	·25	3	4·25	9·25	12·5	10·75	10·25	14·25	7·75	1·75	1	—	—	75
74–75	—	—	—	—	—	—	—	1	1·5	4	7·25	11·75	6·25	7·5	2·75	·5	2	—	—	44·5
75–76	—	—	—	—	—	—	—	·25	1·75	3	2·75	4·75	5	2	3·5	1	·5	2	1	27·5
76–77	—	—	—	—	—	—	—	·25	·75	1	·5	2	2	2	—	·25	·25	1	—	10
77–78	—	—	—	—	—	—	—	—	—	·5	·5	1·5	·5	3·75	·75	·25	·25	—	—	8
78–79	—	—	—	—	—	—	—	—	—	1	—	·5	·5	1·25	·25	1	·5	—	1	6
79–80	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	·5	—	—	—	1
80–81	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	—	—	—	—	·5
81–82	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
Totals	·5	·5	1·5	2·5	7	22	36	84·5	145	170·5	185	152·5	112·5	77·5	38	17·5	9	4	2	1068

TABLE XLVII.

Mother's Span and Son's Stature.

Mother's Span.

Son's Stature.	53—54	54—55	55—56	56—57	57—58	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	Totals
59—60	—	—	—	—	—	—	·5	—	·25	·25	—	—	—	—	—	—	—	—	1
60—61	—	—	—	—	—	·25	·75	—	·25	·25	—	—	—	—	—	—	—	—	1·5
61—62	—	—	—	—	1	·25	·75	—	—	—	—	—	—	—	—	—	—	—	2
62—63	—	—	—	·5	·5	—	—	3	1	1·75	2	·25	—	—	—	—	—	—	9
63—64	—	—	1	·5	3	5·5	7·75	4·75	1·5	2·5	3·25	·75	—	—	—	—	—	—	30·5
64—65	1·25	·5	1·5	5·25	6·75	4	7·5	7	7·5	3·5	1·75	4·5	—	—	—	—	—	—	51
65—66	·25	·5	4·25	7·25	4·5	9·5	15	8	9·5	7·75	2·5	3·5	2·25	·25	—	—	—	—	75
66—67	—	—	4·25	8·75	6·75	11·5	15·5	14·25	18	13·75	11·25	6·75	2	1·75	1·5	·25	·25	—	116·5
67—68	—	—	2	1·75	7·75	11·5	19	26·25	31·75	23·5	15	13·25	7·25	2·5	1·75	1	·25	—	164·5
68—69		1	1	1	4·75	12·75	17·5	19·25	30·25	31·25	18·75	13·5	10·25	3·25	3·25	2·75	1	—	174·5
69—70	—	1	—	2	3	9	10	17·25	14·5	19·5	16·5	9·5	8·5	5·25	1·5	2·5	2·5	1	123·5
70—71		—	1	2·5	2·5	1·5	10·25	15·25	17·5	18	21	14·25	5·75	7	3	1·5	1·25	·25	125·5
71—72	—	—	—	—	—	2	3·75	8·25	11·5	13	13·75	9·5	7·75	4·5	3	2	·75	·25	80
72—73		—	—	—	—	2	2	5·5	9·25	6·25	9·25	3	3·75	1	1·5	1	1·25	·25	49
73—74	—	—	—	—	·5	1·75	1	1·5	6·75	6·5	6·25	3·75	2·5	2	·75	1·25	·25	1·25	36
74—75		—	—	—	—	—	·75	1·75	3	·25	1·75	2·5	3	·5	1	1·5	·5	—	16·5
75—76	—	—	—	—	—	—	—	—	·5	—	·5	1	1·5	·5	1·25	·25	·5	·5	6·5
76—77	—	—	—	—	—	—	—	—	—	—	1	1	1	—	—	—	—	·5	3·5
77—78	—	—	—	—	—	·5	—	—	—	—	—	—	—	·25	·25	—		—	1
78—79		—	—	—	—	·5	—	—	—	—	·5	·5	—	·25	·25	—	—	—	2
79—80	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	—	—	1
Totals	1·5	3	15	29·5	41	75·5	112	132	163	151	125	87·5	55·5	32·5	19·5	14	8·5	4	1070

TABLE XLVIII.

Mother's Stature and Son's Forearm.

Mother's Stature.

Son's Forearm.	52—53	53—54	54—55	55—56	56—57	57—58	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	Totals
14—14·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
14·5—15	—	—	—	—	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	1
15—15·5	—	—	—	—	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	1
15·5—16	—	—		—		·5	—	·25	·25		·5	2	1	—	—	—	—	—	—	4·5
16—16·5	—	—	—	—	—	·5	—	2·75	·75	·75	·75	2·25	·25	—	—	—	—	—	—	8
16·5—17	·25	·25	·5	·5	1·5	1·75	3·25	7·25	8·75	4·75	3·75	1	1·5	—	·25	1	·25	—	—	36·5
17—17·5	·25	·25	·25	·25	3·25	2·25	9	11·75	14	14	14·75	6·75	3·75	1	·75	1·25	1·5	—	—	85
17·5—18	—	—	·5	·5	2·25	5·75	8·75	18	27	26	35	18·5	12·5	4·75	2·75	1	·25	—	—	163·5
18—18·5	—	—	·25	·25	—	5·25	8	19·75	38·75	42·5	45	36·5	21·5	12·25	5·5	1·75	·75	—	—	238
18·5—19	—	—	—	·5	—	2·5	4·5	11·5	28·75	39·5	40·25	31·25	21·25	13·5	8·5	3·25	·75	1	—	207
19—19·5	—	—	—	·5	—	1	2	6	15·75	23·75	22·75	27·25	24·25	20·25	8·5	1·5	1·5	·5	·5	159
19·5—20	—	—	—	—	—	—	·5	4·75	4	10·25	16·75	17·75	17·25	13·5	6	1·75	2	1·5	·5	96·5
20—20·5	—	—	—	—	—	—	·5	1·5	1·75	4·75	5	7·25	6·25	5·5	2·5	1·25	1·25	·5	—	38
20·5—21	—	—	—	—	—	—	—	—	1	1·75	·25	3·5	2	3	·5	·75	·75	·5	·5	14·5
21—21·5	—	—	—	—	—	—	—	—	·75	1·5	1·25	·5	—	1·75	·75	·5	—	—	·5	7·5
21·5—22	—	—	—	—	—	—	—	—	·5	1	—	—	1	—	—	1·5	—	—	—	4
Totals	·5	·5	1·5	2·5	7	19·5	36·5	84·5	143	170·5	186	154·5	112·5	75·5	37	18·5	9	4	2	1065

TABLE XLIX.

Mother's Forearm and Son's Stature.

Mother's Forearm.

Son's Stature.	*13—13·5*	*13·5—14*	*14—14·5*	*14·5—15*	*15—15·5*	*15·5—16*	*16—16·5*	*16·5—17*	*17—17·5*	*17·5—18*	*18—18·5*	*18·5—19*	*19—19·5*	Totals
59—60	—	—	—	·25	·25	·25	·25	—	—	—	—	—	—	1
60—61	—	—	—	·25	·5	·5	·25	—	—	—	—	—	—	1·5
61—62	—	—	—	·5	·75	·25	—	—	—	—	—	—	—	1·5
62—63	—	—	—	—	—	1·25	2·75	2·75	2·25	—	—	—	—	9
63—64	—	—	—	—	2·5	11·5	11·25	3·25	1	—	—	—	—	29·5
64—65	—	·25	·25	5	7·5	13·75	9·75	11·25	3·25	1·5	—	—	—	52·5
65—66	—	·25	2·5	5·5	12	18·75	16·75	10	5·5	2·75	·5	—	—	74·5
66—67	—	—	1·25	6·25	16	21·75	21	23	14·25	8	1·5	·5	·5	114
67—68	—	—	1·5	2·75	7·25	30·75	47·5	41·5	22·25	7·75	1·75	1·75	·75	165·5
68—69	—	—	1	2·5	10	27·75	51	38	22·5	12·5	7·25	2·5	·5	175·5
69—70	—	—	·75	3·5	2·25	16·25	31	30·5	24·25	9·25	3·75	1·25	·25	123
70—71	—	—	·25	1·75	4·25	16	30·5	30·75	25·75	8·75	5·75	1·25	·5	125·5
71—72	·5	·5	·5	·75	1·75	5	14·75	20·75	21·75	6·5	5	1·25	·5	79·5
72—73	—	—	—	·5	·5	3·25	12	11·25	7·75	7·75	4	1	—	48
73—74	—	—	—	1	1	2·75	6·25	12·25	5	3·75	4·25	·75	—	37
74—75	—	—	—	—	—	2·75	1·75	3·25	4	3·25	2·25	·5	·25	18
75—76	—	—	—	—	—	—	·75	·5	1·25	1·5	1·5	·75	·25	6·5
76—77	—	—	—	—	—	—	—	—	2·25	·75	·5	—	—	3·5
77—78	—	—	—	—	—	—	·5	—	·5	—	—	·25	·25	1·5
78—79	—	—	—	—	—	—	·5	—	·5	·5	—	·25	·25	2
79—80	—	—	—	—	—	—	—	—	—	—	—	·5	·5	1
Totals	·5	1	8	30·5	66·5	172·5	258·5	239	164	74·5	38	12·5	4·5	1070

TABLE L.

Mother's Span and Son's Forearm.

Mother's Span.

Son's Forearm.	53—54	54—55	55—56	56—57	57—58	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	Totals
14—14·5	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
14·5—15	—	—	—	—	·25	·25	·5	—	—	—	—	—	—	—	—	—	—	—	1
15—15·5	—	—	—	—	·25	·25	·5	—	—	—	—	—	—	—	—	—	—	—	1
15·5—16	—	—	—	·25	·5	·25	·5	·5	1	2	·5	—	—	—	—	—	—	—	5·5
16—16·5	—	—	—	·75	1·25	1	2	1·5	·5	1·25	·75	—	·5	—	—	—	—	—	9·5
16·5—17	·5	—	2·25	1	5·25	7·25	5	5·25	3·75	2·5	·75	1·25	1·75	—	—	—	·25	·25	37
17—17·5	·75	·25	3	4·25	8·5	9	17·5	12·5	10	8·5	3	4·75	2·5	—	1	·5	·25	·25	86·5
17·5—18	·25	·5	5·5	10	7	16·25	22	23·5	24·25	20·5	17·75	8·5	3·75	1·75	1	·5	—	—	163
18—18·5	—	·5	2·25	7·5	7·25	15	26·25	31·25	40·5	40·25	31	16·75	8·25	7	2	·5	·25	—	236·5
18·5—19	—	·25	1	5	6·75	12·5	22·75	23·75	33·5	29·5	22·75	23·75	11	4·5	2·75	3·75	1·75	·25	205·5
19—19·5	—	1	·5	·75	2·5	7·5	11·25	16·5	21·25	21·75	27·75	16	12	8·25	5	3·75	2	1·25	159
19·5—20	—	—	—	—	1	2·75	2·25	14·5	12·5	14·5	12·75	9·25	8·75	6·5	3·5	3	3·75	1	96
20—20·5	—	—	—	—	·25	1·75	2	3·75	7	4·5	5·25	2·75	4·5	2·25	2·25	1	·25	—	37·5
20·5—21	—	—	—	—	·25	·5	·5	·25	2·25	2·25	3·5	1·75	·5	·75	1	·5	—	·5	14·5
21—21·5	—	—	—	—	—	·75	·5	·25	·5	1·5	·75	1·25	1	·25	·25	—	—	·5	7·5
21·5—22	—	—	—	—	—	·5	—	—	—	—	1	—	—	1·25	1·25	—	—	—	4
Totals	1·5	2·5	14·5	29·5	41	75·5	113·5	133·5	157	149	127·5	86	55·5	32·5	20	13·5	8·5	4	1065

TABLE LI.

Mother's Forearm and Son's Span.

Mother's Forearm.

Son's Span.	13—13·5	13·5—14	14—14·5	14·5—15	15—15·5	15·5—16	16—16·5	16·5—17	17—17·5	17·5—18	18—18·5	18·5—19	19—19·5	19·5—20	Totals
58—59	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	·5
59—60	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	·5
60—61	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	·5
61—62	—	—	—	·25	1	1·25	·5	—	—	—	—	—	—	—	3
62—63	—	—	—	1	·25	·25	1·75	·75	—	·25	·25	—	—	—	4·5
63—64	—	—	—	1·25	1·25	3·25	5·25	1·5	·5	·25	·25	—	—	—	13·5
64—65	—	·25	·75	5·25	3·75	5	4·75	1·75	1	·75	·25	—	—	—	26·5
65—66	·25	·5	1·5	3·25	6	14·75	11·5	9	4·5	2·25	1	·5	·5	—	58·5
66—67	·25	·25	1·25	2·75	8·5	20	18	13·5	8·75	3	·75	1	1	—	79
67—68	—	—	2	3·75	15	23·5	24·5	17·75	11	3	—	—	—	—	100·5
68—69	—	—	1	2·75	9	23	35·5	28·5	14·25	6·75	2·5	·25	—	—	123·5
69—70	—	—	·5	1·5	8·75	22·75	39·5	32·5	20·75	6·25	2·75	1·25	—	—	136·5
70—71	—	—	—	2	5·75	21·5	37	38·25	21·75	9·5	2·5	·25	—	—	141·5
71—72	—	—	—	3·25	2·5	11·5	25·5	27	21·75	13·25	5	1·25	·5	—	111·5
72—73	—	—	—	1	2·75	12·25	24·5	23	14·75	6·5	3·75	1	1	·5	91
73—74	—	—	·5	·75	·75	5·75	14·25	19	17·25	9·75	5·25	1·75	·5	—	75·5
74—75	—	—	—	·25	·25	3·5	8	11	11	4·75	3·75	1·5	·5	—	44·5
75—76	—	—	·25	·5	·25	1	2·5	7	5·5	3·25	5	1·25	—	—	26·5
76—77	—	—	·25	·5	·25	·75	1	2	3·25	1·5	·5	—	—	—	10
77—78	—	—	—	·25	·25	·5	·5	—	2	1·75	1·25	—	—	—	6·5
78—79	—	—	—	—	—	—	1·25	2·25	1	·75	1·75	·5	—	—	7·5
79—80	—	—	—		—	—	·25	·25	·25	·25		·5	·5	—	2
80—81	—	—	—	—	—	—	—	—	·25	·25	—	—	—	—	·5
81—82	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	1
Totals	·5	1	8	30·5	66·5	171	259·5	238	162·5	71	36·5	11·5	5	·5	1065

TABLE LII.

Mother's Stature and Daughter's Span.

Mother's Stature.

Daughter's Span.	52—53	53—54	54—55	55—56	56—57	57—58	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	Totals
53—54	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	·5
54—55	—	—	—	—	—	—	—	·5	1	—	—	—	·25	·25	—	—	—	—	—	2
55—56	—	—	—	—	·25	·25	—	1	—	—	—	—	·25	·25	—	—	—	—	—	2
56—57	—	—	—	—	·5	1	1·5	1·25	1·25	1	—	—	—	—	—	—	—	—	—	6·5
57—58	—	—	—	—	1·25	1·75	1·25	2·25	5·25	2·75	2·5	—	—	—	—	—	—	—	—	17
58—59	—	—	—	2	—	·75	6	11·25	10	9	5	2·5	2·75	1·25	1·25	·25	—	—	—	52
59—60	·5	·5	—	—	3	5·75	7·5	10	14·25	9·25	16·25	8·5	2·75	5·5	3	·75	—	—	—	87·5
60—61	—	—	·25	·75	1·5	1·5	9·25	18·5	19·5	26·5	18·5	17·5	6	4·5	2·25	1·25	·75	—	—	128·5
61—62	·5	—	·25	·75	1	1	7·5	18·5	20	27·75	26·25	26	17·5	7·25	1·75	·75	·75	—	—	157·5
62—63	·5	—	—	—	1·25	4·25	7·5	17·75	30	32·75	35·75	22	19·5	11	2·25	—	—	—	—	184·5
63—64	—	—	—	—	·25	1·5	5·25	12·75	22·25	34·75	34·25	31·25	17·25	14·75	7·75	1·5	—	—	—	183·5
64—65	—	—	—	—	—	1	3·5	8	20·75	24·25	33·5	33·5	21·75	12·25	8	·5	1	—	—	168
65—66	—	—	—	—	—	2·25	1·5	5	11·5	20·25	26·75	27·25	18·5	12·75	10·5	5	·75	·5	—	142·5
66—67	—	—	—	—	—	1	·75	·75	6	9·5	15·75	18·5	15	8·25	6·75	4·25	1	·5	—	88
67—68	—	—	—	—	—	—	—	1	5	8·75	12·75	14·5	10	9·75	5	1·75	1	—	—	69·5
68—69	—	—	—	—	—	—	—	—	1·75	3·5	6·25	4·25	5·75	7·25	3·75	2·25	2	2·75	1	40·5
69—70	—	—	—	—	—	—	—	1	1	—	—	5	4·5	3·5	6·25	1·75	1·25	·25	—	24·5
70—71	—	—	—	—	—	—	—	—	—	—	—	1·75	3·75	1·5	—	·75	·25	1	—	9
71—72	—	—	—	—	—	—	—	—	—	—	1	·5	·5	1·5	1	·75	·25	—	·5	6
72—73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5	·5
73—74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
74—75	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	·5	1
75—76	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	·5	1
Totals	1·5	·5	·5	3·5	9	22	51·5	110	169·5	210	234·5	214	146	101·5	59·5	21·5	9	5	3	1372

TABLE LIII.

Mother's Span and Daughter's Stature.

Mother's Span.

Daughter's Stature.	52–53	53–54	54–55	55–56	56–57	57–58	58–59	59–60	60–61	61–62	62–63	63–64	64–65	65–66	66–67	67–68	68–69	69–70	70–71	71–72	Totals
52–53	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
53–54	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
54–55	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
55–56	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
56–57	—	—	—	—	—	—	1·5	·5	·75	·25	1	·25	·25	—	—	—	—	—	—	—	4·5
57–58	—	—	—	2·75	2·75	—	2	1	2·75	·25	·25	2	·75	—	—	—	—	—	—	—	14·5
58–59	—	—	—	1·25	2·75	2	3	2	2·5	·5	·25	·25	·25	·25	·25	·25	—	—	—	—	15·5
59–60	—	—	·25	·75	3	4·25	7	12	9	2·25	3·75	2·75	1·5	1·5	·25	·25	—	—	—	—	48·5
60–61	—	·75	·75	4·75	7	8·25	12·5	12·5	15·75	10	9	7·75	4·75	3·25	—	1	1	—	—	—	99
61–62	—	·75	1	3·75	7·25	4·25	9·75	21	22·75	22·25	19·25	11·25	10	7	1·25	1	—	—	—	—	142·5
62–63	·5	—	1·5	3·25	10	10·5	15	20·25	29·25	25·5	26	22·75	12·25	9	3·25	·5	·25	1·25	—	—	191
63–64	·5	1	—	1	2·75	9·25	14	38	36·75	26	26·75	19·5	18·25	10·25	2·5	1·5	1·25	1·25	·5	·5	211·5
64–65	—	—	—	·5	2·5	6	11·25	25·25	25·25	27·5	34·75	19·25	26	10·25	2·25	1·25	2·75	1·25	·5	·5	197
65–66	—	—	—	—	·25	2·25	4·5	13·5	15·25	23·5	32·5	24·5	19·75	10·25	7·5	3·25	1·5	2	—	—	160·5
66–67	—	—	·5	·5	2·25	2·5	5·25	8·5	14	21	20·25	19·75	14·25	15·75	8·25	4·25	2·5	1·5	·5	—	141·5
67–68	—	—	—	—	1	1·75	1	3·75	6·5	9	19	8·75	4·75	7·75	6·5	3	2	1·25	1	—	77
68–69	—	—	—	—	·5	1·5	·75	1·75	3	3	5·75	3·25	3·25	4·75	2·75	2	·75	—	1	—	34
69–70	—	—	—	—	—	—	—	·5	1·25	·75	2	1·5	3·5	2·5	1·75	1·5	1·25	1·25	·75	—	18·5
70–71	—	—	—	—	—	—	·25	·25	·25	·25	1	—	1·5	1	·75	2	1·25	·25	·25	—	9
71–72	—	—	—	—	—	—	·25	·25	—	—	—	1	—	—	·25	·25	·5	—	1	—	3·5
72–73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
73–74	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
Totals	1	2·5	4	18·5	42	52·5	89	162	185	172	201·5	145·5	121	83·5	37·5	22	15	10	5·5	1	1371

TABLE LIV.

Mother's Stature and Daughter's Forearm.

Mother's Stature.

Daughter's Forearm.	*52—53*	*53—54*	*54—55*	*55—56*	*56—57*	*57—58*	*58—59*	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	Totals
13·5—14	—	—	—	—	—	—	—	·5	—	—	—	·5	—	—	·5	—	—	—	—	1·5
14—14·5	—	—	—	—	—	—	1·25	1·25	—	·5	—	·5	—	—	1·5	—	—	—	—	5
14·5—15	—	—	—	—	·75	·75	2·75	5·25	2·5	1·25	1·5	1·75	·75	·75	—	—	—	—	—	18
15—15·5	—	—	·25	1·25	2·25	2·5	3·25	12·5	11·25	8·25	9·75	7·75	3	1·75	1·25	·25	·25	—	—	65·5
15·5—16	·5	·5	·25	1·5	2·25	4·75	10·75	23·25	32·25	29·25	27·25	19·75	11·25	5	3·25	1·25	·5	—	—	173·5
16—16·5	1	—	—	·5	2·5	7·5	14	26·75	44·25	59·5	58	37·5	24·25	14·25	8	2·25	1·25	—	—	301·5
16·5—17	—	—	—	·25	1	3	8·5	23·75	38·75	48	54·75	53·75	38·25	20·5	5·5	3·25	·75	—	—	300
17—17·5	—	—	—	—	·25	3	5·75	12·25	25	39·75	44·5	45·25	33·75	19	12	3·25	·75	—	—	244·5
17·5—18	—	—	—	—	—	—	3	3	12·25	13·75	26	27·5	19·5	22·25	13	4·5	2·5	·75	·5	148·5
18—18·5	—	—	—	—	—	—	·75	1·25	5	7	9·75	12·75	9	11·25	8·75	5·25	1·5	·75	1·5	74·5
18·5—19	—	—	—	—	—	—	—	·25	·75	·75	1·5	4	4·75	6·25	2·75	1·25	1·25	1	—	24·5
19—19·5	—	—	—	—	—	—	—	—	—	—	2	2·5	1	1·5	1·5	·25	·25	·5	1	10·5
19·5—20	—	—	—	—	—	—	—	—	—	—	—	2	—	—	·5	—	—	·5	—	3
20—20·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
20·5—21	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	·5
Totals	1·5	·5	·5	3·5	9	21·5	50	110	172	208	235	215·5	145·5	102·5	58·5	21·5	9	5	3	1372

TABLE LV.

Mother's Forearm and Daughter's Stature.

Mother's Forearm.

Daughter's Stature.	*13—13·5*	*13·5—14*	*14—14·5*	*14·5—15*	*15—15·5*	*15·5—16*	*16—16·5*	*16·5—17*	*17—17·5*	*17·5—18*	*18—18·5*	*18·5—19*	*19—19·5*	*19·5—20*	Totals
52—53	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	·5
53—54	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	·5
54—55	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
55—56	—	—	—	—	·5	·5	—	—	—	—	—	—	—	—	1
56—57	—	—	—	·5	·5	·25	2	1·25	—	—	—	—	—	—	4·5
57—58	—	—	—	1	2·5	2·75	5·5	2·75	—	—	—	—	—	—	11·5
58—59	—	—	—	1·5	5	3·25	2·25	1·25	1·25	·5	—	—	—	—	15
59—60	—	—	1	3·25	4	10·75	14·25	7·75	5	1·5	·5	—	—	—	48
60—61	—	·25	1·25	5·75	10·75	26·25	19	18	12·5	4·75	1	—	—	—	99·5
61—62	—	·25	·25	3·5	12	36·5	32·25	36	12·5	5·25	2·25	1	·75	—	142·5
62—63	—	·25	1·25	7·75	22	30·75	49·75	14·25	23	7·75	3·5	1·25	·5	—	192
63—64	—	·25	1·25	6·5	11·25	42	63·5	38	32·25	10·75	4·25	1·5	1	—	212·5
64—65	—	—	1·5	2·75	9·25	27·75	52·5	47	37·75	14·25	6·5	2	·25	—	201·5
65—66	—	—	—	1·75	4·5	18·75	35·25	40	33·5	12·5	7·75	1·5	1	—	156·5
66—67	—	—	·5	2·25	5	16·75	25·5	40·25	29	13·75	6	1·5	—	—	140·5
67—68	·25	·25	·5	·5	2·5	6·5	13·75	16·5	14·5	10·5	5	2·75	2·25	·25	76
68—69	·25	·25	—	—	1·25	3·75	5	7·5	5·5	5·25	4	1·25	·5	·5	35
69—70	—	—	—	·5	·5	—	1·5	2·75	5·75	4·25	1·5	1	·5	·25	18·5
70—71	·25	·25	—	—	—	—	—	1·25	3	1·25	1·75	1·25	·5	—	9·5
71—72	·25	·25	—	—	—	—	—	·5	·5	·25	1·5	·5	·25	—	4
72—73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
73—74	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
Totals	1	2	7·5	38·5	91·5	226·5	322	305	217	92·5	45·5	15·5	7·5	1	1373

TABLE LVI.

Mother's Span and Daughter's Forearm.

Mother's Span.

Daughter's Forearm.	*52—53*	*53—54*	*54—55*	*55—56*	*56—57*	*57—58*	*58—59*	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	Totals
13·5—14	—	—	—	—	·25	·25	—	—	—	·25	·25	—	·25	·25	—	—	—	—	—	—	1·5
14—14·5	—	—	—	—	·25	·25	·5	·75	·5	·5	·5	·25	·25	1·25	—	—	—	—	—	—	5
14·5—15	—	—	—	·5	1·75	1·25	4·25	3·25	3	·75	1·5	·5	1·25	·25	·25	—	—	—	—	—	18·5
15—15·5	—	—	·25	3·25	7·5	5·75	9·75	10	7·5	4·75	5·5	4·5	2·25	2·25	1·25	·5	·5	—	·25	·25	66
15·5—16	—	—	·5	4	10·5	13	18·75	26·5	30·5	17·75	16	15·25	8·25	5·5	1	1	·5	—	·25	·25	169·5
16—16·5	1	·25	1·75	5·5	10·25	14·75	23·25	43·5	47	38·5	44·75	30·25	20·75	13·75	2·75	2·5	1	—	·25	·25	302
16·5—17	—	1·25	·75	2·75	5·75	7·25	17·25	37·75	39·25	48·75	47·75	34·5	30·25	14·75	5·75	1·75	1·25	1·75	·25	·25	299
17—17·5	—	1	·5	1	3·25	6	11	25	33·5	34·5	39·75	28·75	27·5	17·25	9	3	2·5	·5	—	—	244
17·5—18	—	—	·25	·5	1·5	1·75	4·5	9·75	14·5	18·25	27·25	19·5	12·5	15·75	8·75	6	3·5	4·25	1	—	149·5
18—18·5	—	—	—	—	—	·75	1·25	4	6·5	5·25	14·25	7·5	9·75	8	7·5	3·25	3·25	2·25	1	—	74·5
18·5—19	—	—	—	—	—	—	—	·5	1·75	1·75	1·75	2	5	2·75	1·75	3·25	1·25	1·25	—	—	23
19—19·5	—	—	—	—	—	—	·25	·25	—	1	1·5	2·25	1	1·25	—	·75	·75	—	1	—	10
19·5—20	—	—	—	—	—	—	·25	·25	—	·5	·25	·75	—	·5	—	—	—	—	·5	—	3
20—20·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
20·5—21	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	·5
Totals	1	2·5	4	17·5	41	51	91	161·5	184	172·5	201	146	119	83·5	38	22	14·5	10	6	1	1367

TABLE LVII.

Mother's Forearm and Daughter's Span.

Mother's Forearm.

Daughter's Span.

Daughter's Span.	13—13·5	13·5—14	14—14·5	14·5—15	15—15·5	15·5—16	16—16·5	16·5—17	17—17·5	17·5—18	18—18·5	18·5—19	19—19·5	19·5—20	Totals
53—54	—	—	—	—	—	—	—	·25	·25	—	—	—	—	—	·5
54—55	—	—	—	—	—	—	—	1·5	·5	—	—	—	—	—	2
55—56	—		—	·25	·25	—	·5	·75	·25	—	—	—	—	—	2
56—57	—	—	—	1	2·75	1	1·75	—	—	—	—	—	—	—	6·5
57—58	—	—	·75	1·25	2·5	6	2·5	1·75	·75	—	—	—	—	—	15·5
58—59	—	·25	1·25	4·75	7·5	13	14	5·75	3·5	1·75	·75		—	—	52·5
59—60	—	·25	1·5	3·5	11·25	16·25	20·75	13	14	5·25	·75	·25	·25	—	87
60—61	·5	·5	·5	6·5	14·25	29·75	30	26·75	12·25	5·5	2·5	·75	·25	—	130
61—62	—	·25	1·25	7.25	10	32	40·5	38·5	15·5	8	3·5	·5	·25	—	157·5
62—63	—	·25	·75	5	15·75	37	50·5	39	24·5	7	3	·75	·5	—	184
63—64	—	—	—	3·25	10·75	27·75	53·25	43·25	29·75	9·5	2·5	·25	·25	—	180·5
64—65	·25	·25	·75	2·25	7·75	27·25	40·75	34·75	33·5	12·5	6·25	·75	·5	—	167·5
65—66	·25	·25	·75	1·75	5·75	18·25	29	42	23·75	10·25	7·75	2·25	·75	·25	143
66—67	—	—	—	·25	1·75	6·5	17	29·25	15·5	14·25	1	1·25	1	·25	88
67—68	—	—	—	—	1·25	7	11·5	16·75	18·75	10	3·5	·5	·25	—	69·5
68—69	—	—	—	—		2·75	4·5	9·75	12	5·25	5·75	2·5	1	—	43·5
69—70	—	—	—	·5	·5	1·25	1·75	2·75	7·5	3·25	1·75	2·25	·5	—	22
70—71	—	—	—	—	—	·25	·25	·25	2·75	1·75	2	1	1·25	·5	10
71—72	—		—	—	—	—	—	1·5	1	·75	·5	·75	·5	—	5
72—73	—	—	—	—			—	—	—	—	—	·25	·25	—	·5
73—74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
74—75	—	—	—	—	—	—	—	—	·5	—	·5	—	—	—	1
75—76	—	—	—	—	—	—	—	—	·5	—	·5	—	—	—	1
Totals	1	2	7·5	37·5	92	226	318·5	307·5	217	92	45·5	14	7·5	1	1369

TABLE LVIII.

Brother's Stature.

Elder Brother.

Younger Brother.

Younger Brother.	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	76—77	77—78	78—79	Totals
60—61	—	—	—	—	·5	—		—	—	—	—	—	—	—	—	—	—	·5
61—62	—	—	—	—	1·5	—	—	—	—	—	—	—	—	—	—	—	—	1·5
62—63	—	—	—	2	—	—	—			1·5	—	—	—	—	—	—	—	3·5
63—64	1	—	1·25	1·25	—	1·25	·25	—	—	·5		—	—	—	—	—	—	5·5
64—65	—	1	1·25	3·75	3·75	3	1·75	—	—	—	—	—	—	—	—	—	—	14·5
65—66	—	1	·25	·75	4	4	1·25	·25	1	—	—	—	—	—	—	—	—	12·5
66—67	·5	1·5	5·25	4·25	6·25	7·5	6	2·25	2·75	1·25	1	—	—	—	—	—	—	38·5
67—68	·5	—	1	5	2·25	9·5	10·75	6·5	5	2·25	1·75		—	—	—	—	—	44·5
68—69	—	1·5	2·5	3	7	12·25	9·25	6·5	11·5	5·75	1·25	2	—	—	—	—	—	62·5
69—70	·25	·25	1·5	·5	·75	4·5	7·25	6·25	6·5	4·25	1·5	2·75	·25	—		—		36·5
70—71	·25	·75	·5	1	3	6·5	12·75	5·5	3·25	6	1·25	2·5	1·25	—	—	—	—	47·5
71—72	—	—	—	1·5	1	1	3·25	3	7·75	3	1·75	·25		·25	·25		—	23
72—73	—	—	—	·5	—	·5	·75	4·5	1·75	3	2	1·5	—	·25	·25	—	—	15
73—74	—	—	—	·5	1·5		·75	2·75	1·5	—	·5	1·5	1·5	—	—	—	—	10·5
74—75	—	—	—	—	—	—	·25	·25	·5	1	·5	·5	1·25	·25	—	—	—	4·5
75—76	—	—	—	—	—	—	·25	·25	—	·5	—	1	·75	·25	—	—	—	3
76—77	—	—	—	—	—	—	—	—	—	1	—	1·5	—	—	—	—	—	2·5
77—78	—	—	—	—	—	·25	·25		—	—	—	—	—	—	—	—	—	·5
78—79	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	·5
79—80	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	1
Totals	2·5	6	13·5	24	31·5	50·5	55	38	41·5	30	11·5	13·5	5	1	·5	·5	·5	328

TABLE LIX.

Brothers' Span.

Elder Brother.

Younger Brother.	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	76—77	77—78	78—79	79—80	Totals
61—62	—	—	—	—	—	—	1	—	—	·25	·25	—	—	—	—	—	—	—	—	1·5
62—63	—	—	—	—	—	—	—	—	—	·25	·25	—	—	—	—	—	—	—	—	·5
63—64	1	—	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	—	2
64—65	—	—	1·25	·25	2·5	2·75	1·25	1	·5	—	·25	·25		—	—	—	—	—	—	10
65—66	—	—	·75	1·75	·75	6·5	·75	2·5	1	·5	·75	·25	—	—	—	—	—	—	—	15·5
66—67	—		·75	1·75	2·5	1·75	2·5	1	2·25	2	1·5	·75	1·25	—	—	—	—	—	—	18
67—68	—	1	·25	·75	1·25	3·75	4·25	1·5	2·5	2·75	1·25	3·5	1·25	—	—	—	—	—	—	24
68—69	—	—	—	1·75	2·25	1·25	3·5	4	7·75	8·25	7	4·5	1·75	—		—	—	—	—	42
69—70	—	—	·25	1	2	2·75	3·75	6·5	9·5	7·75	10·25	4	2·25	—	—	—	—	—	—	50
70—71	—		·25	·25	·25	1·25	1·75	5	7·5	5·75	4·25	3·75	4	2·5		—	—	—	—	36·5
71—72	—	—	—	—	·5	1	3	4·75	2·5	5·75	3·5	3·5	4·5	2	2·5	·75	·25	—	—	34·5
72—73	—	—	—	—	·5	1	1·75	1·25	6	1·25	5	3·25	2·75	1·25	1	·25	·25	—	—	28·5
73—74	—	—	—	—		—	·5	1	1	4	4·25	3·75	1·25	3·25	·5	—	1	—	—	23·5
74—75	—	—	—	—		—	1	·5	2·25	1·25	1·5	3·5	3	1·5	1	—	2	—	—	17·5
75—76	—	—	—				—	·5	2·25	·75	2·25	1·25	·5	·5	—	1	—	—	—	9
76—77	—	—	—	—	—	—	—	·5	—	·5	1·75	·5	·25	—	—	—	—	—	—	3·5
77—78	—	—	—		—	—	—	—	—	—	—	·25	·25	—	—	—	·75	·25	—	1·5
78—79	—	—	—	—	—	—	1	—	—	—	—	—	—	·5	2·5	—	·75	·25	—	5
79—80	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
80—81	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
81—82	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	1
Totals	1	1	3·5	7·5	12·5	22·5	26·5	30	45	44	44	33	26	11·5	7·5	2	5	1	·5	324

TABLE LX.

Brothers' Forearm.

Elder Brother.

Younger Brother.	15·5—16	16—16·5	16·5—17	17—17·5	17·5—18	18—18·5	18·5—19	19—19·5	19·5—20	20—20·5	20·5—21	21—21·5	21·5—22	Totals
15·5—16	1	—	·5	—	—	—	—	—	—	—	—	—	—	1·5
16—16·5	—	—	1·75	·25	·25	·25	—	—	—	—	—	—	—	2·5
16·5—17	—	—	1·25	·75	3·25	·75	1·25	·25	—	—	—	—	—	7·5
17—17·5	—	·5	1	5·25	6·25	5·75	4	1·25	·5	—	—	—	—	24·5
17·5—18	·5	1	2	5	8·25	13·25	13	4·25	·5	·25	—	—	—	48
18—18·5	—	·5	2	7	15	20·75	13·5	8·75	3·5	2·25	1·25	·5	—	75
18·5—19	·25	·75	·5	3	8	15·5	15·75	12	7·25	3·5	1	—	—	67·5
19—19·5	·5	·5	—	1·25	3·25	8·25	10	15·75	8·75	1·5	·75	—	—	50·5
19·5—20	·25	·25	—	—	1·25	5·25	4·25	6	5·75	3·5	1	—	—	27·5
20—20·5	—	—	—	—	—	1·5	·5	1·5	3·5	·5	—	—	—	7·5
20·5—21	—	—	—	—	—	·25	1	1·75	1	1·5	—	—	—	5·5
21—21·5	—	—	—	—	·5	—	·25	·5	·25	·5	—	—	—	2
21·5—22	—	—	—	—	·5	—	—	—	—	·5	·5	·5	·5	2·5
Totals	2·5	3·5	9	22·5	46·5	71·5	63·5	52	31	14	4·5	1	·5	322

TABLE LXI.

Sisters' Stature.

Elder Sister.

Younger Sister.	*52—53*	*53—54*	*54—55*	*55—56*	*56—57*	*57—58*	*58—59*	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	Totals
55—56	—	—	—	—	—	—	—	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	1
56—57	—	—	—	—	—	·5	—	·5	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	1·5
57—58	—	—	—	—	—	1	1	·75	1·5	1·25	—	—	—	—	—	—	—	—	—	—	—	—	5·5
58—59	—	—	—	—	—	·5	—	·25	2·25	1	—	—	—	—	·5	—	—	—	—	—	—	—	4·5
59—60	—	—	—	—	—	1·75	1·75	·5	·5	3·75	2·25	3	·5	1	·5	·25	·25	—	—	—	—	—	16
60—61	—	—	—	—	·5	·75	1·25	2·75	4·75	2·5	6·5	3·25	5	1·5	1·25	·25	·25	—	—	—	—	—	30·5
61—62	—	—	—	—	·5	—	1	4	6·25	6·75	6	7	3·75	2	2·25	·5	—	—	—	—	—	—	40
62—63	—	—	—	—	·5	—	1	4·5	4·75	8·5	13·5	9·25	5·5	5·5	5·25	·75	1	—	—	—	—	—	60
63—64	·5	·5	—	—	—	—	·5	4	5	8·25	9	14·5	18	6·25	4·25	3·25	—	—	—	—	—	—	74
64—65	—	—	—	—	—	—	—	·75	1·75	10	11	11	13·75	4·75	6·5	2·5	1·75	·25	—	—	—	—	64
65—66	—	—	—	—	—	—	—	·25	2·75	7·25	5·5	8·5	16·25	9	9·75	2·75	4·25	1	·25	·5	—	—	68
66—67	—	—	—	—	—	—	—	1·25	1·5	2·75	6·75	5	8·75	9·75	6·5	4	4·25	3·75	·75	·5	—	—	55·5
67—68	—	—	—	—	—	—	—	—	·25	·25	·5	1·75	4·5	4	8·25	2·75	3·5	1·75	1·25	·25	—	1	30
68—69	—	—	—	—	—	—	—	—	—	—	·5	·75	2·5	·75	2·25	2·25	1·75	·25	·25	·25	—	—	11·5
69—70	—	—	—	—	—	—	—	—	—	—	—	—	1	1	1·5	2·5	·5	·5	—	—	—	—	7
70—71	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·75	2	·5	—	—	—	—	3·5
71—72	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	·5
Totals	·5	·5	—	—	1·5	4·5	6·5	19·5	31·5	53	62	64	79·5	45·5	49	23	19·5	8	2·5	1·5	—	1	473

TABLE LXII.

Sisters' Span.

Elder Sister.

Younger Sister.	55—56	56—57	57—58	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	Totals
53—54	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	·5
54—55	—	—	·5	1	·5	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	—	2·5
55—56	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
56—57	·25	·25	—	—	·5	1·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2·5
57—58	·25	·75	·5	·5	1	·5	—	—	·5	·5	—	—	—	—	—	—	—	—	—	—	—	4·5
58—59	1	—	·75	2	·75	1·5	2	2·5	—	1	—	—	—	—	—	—	—	—	—	—	—	11·5
59—60	—	—	1·25	1·75	3·75	4·25	2·75	3·25	3·5	1	3·75	·75	—	—	—	—	—	—	—	—	—	26
60—61	—	—	·5	5	5·75	5·25	8·5	5·5	3	5	2·75	1·25	—	—	—	—	—	—	—	—	—	42·5
61—62	—	—	1·25	4·5	3·5	6·5	7·75	6·75	8·5	5·25	4	·75	·25	—	1	—	—	—	—	—	—	50
62—63	—	—	·75	4·75	5·25	3·5	8·5	11	10·75	8·5	2·5	3	2	—	—	—	—	—	—	—	—	60·5
63—64	—	—	·5	1·5	2·75	6·5	10·5	8·25	9·75	10·25	3·5	8	2·5	·25	·25	·5	—	—	—	—	—	65
64—65	—	—	—	1·5	2·75	7	7·25	9	5·75	9·25	5	4·5	3·25	2	·25	·5	·5	—	—	—	—	58·5
65—66	—	—	—	·5	4·5	3	6·75	4·75	8	9	7·25	7	4·25	4	·75	·25	·5	—	—	—	—	60·5
66—67	—	—	—	—	2	1	3	2·5	3·75	4·5	3	1	1·5	1·25	1·75	·25	—	—	—	—	—	25·5
67—68	—	—	—	—	—	—	1	·75	1·75	5·25	7	8·25	4·75	2·25	1·5	—	—	—	—	·5	·5	33·5
68—69	—	—	—	—	—	—	—	·25	2·25	1	2·75	3	1·5	5	2·25	—	·5	·5	—	—	—	19
69—70	—	—	—	—	—	—	—	—	—	2·5	·5	2·5	1·5	1·75	1·5	·25	—	—	—	—	—	10·5
70—71	—	—	—	—	—	—	—	—	—	·5	·75	1·25	—	—	1·75	·25	1	—	—	—	—	5·5
71—72	—	—	—	—	—	—	—	—	—	—	·25	·75	—	1	·5	—	—	—	—	—	—	2·5
Totals	1·5	1	6	23	33	40·5	58	55·5	57·5	63·5	43	42	21·5	17·5	11·5	2	2·5	·5	—	·5	·5	481

TABLE LXIII.

Sisters' Forearm.

Elder Sister.

Younger Sister.	14—14·5	14·5—15	15—15·5	15·5—16	16—16·5	16·5—17	17—17·5	17·5—18	18—18·5	18·5—19	19—19·5	19·5—20	20—20·5	Totals
13·5—14	—	·25	·25	—	·5	—	·25	·25	—	—	—	—	—	1·5
14—14·5	—	·75	·75	—	·5	—	·25	·25	—	—	—	—	—	2·5
14·5—15	—	·75	·75	·25	1·25	·75	·25	—	—	—	·25	·25	—	4·5
15—15·5	—	1·25	3·75	6	4·5	3·25	·75	—	—	—	·25	·25	—	20
15·5—16	·5	1	6·5	14·5	13·75	11·75	6·5	·25	·25	—	—	—	—	55
16—16·5	—	·5	7·25	20·5	26·25	22	17·25	5·25	·5	—	·5	·5	—	100·5
16·5—17	—	1	4·75	11	25	29·5	24·25	7	2	—	—	—	—	104·5
17—17·5	—	·75	1·5	7	17·25	17	20·25	12·75	4	1	—	—	—	81·5
17·5—18	—	·25	·5	3·5	7·25	7·5	15·25	10·75	5·5	2·25	1	·25	—	54
18—18·5	—	—	—	1·25	4·75	5	5·75	7·25	5·5	3	1·75	·25	—	34·5
18·5—19	—	—	—	—	·5	1·25	1	3·5	2·75	·75	·25	—	—	10
19—19·5	—	—	—	—	—	—	·25	1·25	1	—	—	—	—	2·5
19·5—20	—	—	—	—	—	—	—	—	—	—	—	—	—	—
20—20·5	—	—	—	—	—	—	—	—	—	—	·25	·5	·25	1
20·5—21	—	—	—	—	—	—	—	—	—	—	·25	·5	·25	1
Totals	·5	6·5	26	64	101·5	98	92	48·5	21·5	7	4·5	2·5	·5	473

TABLE LXIV.

Brother's Stature and Sister's Stature.

Brother's Stature.

Sister's Stature.	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	*74—75*	*75—76*	*76—77*	*77—78*	*78—79*	*79—80*	Totals
52—53	—	—	—	—	—	—	·25	·25	—	·5	—	—	—	—	—	—	—	—	—	—	—	1
53—54	—	—	—	—	—	—	·25	·25	—	·5	—	—	—	—	—	—	—	—	—	—	—	1
54—55	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
55—56	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
56—57	—	—	—	·25	1	·75	1·5	1	·5	—	—	—	—	—	—	—	—	—	—	—	—	5
57—58	—	—	—	1·25	3·75	2·5	3·25	4·25	·5	—	—	—	—	—	—	—	—	—	—	—	—	15·5
58—59	—	—	—	1	·5	3·25	4	5·25	3	—	—	—	—	—	—	—	—	—	—	—	—	17
59—60	—	—	—	3·25	5·5	8	6	7·75	11·5	5·25	2·75	2	·25	·25	—	—	—	—	—	—	—	52·5
60—61	—	—	1	1	5·75	11·25	16	19	16	12	5·75	3	3·25	1·5	1	—	—	—	—	—	—	96·5
61—62	·5	·5	—	2·25	6·5	10·25	15	21·5	31·5	24·25	9·75	11·5	8·25	1·25	4·5	·5	—	—	—	—	—	148
62—63	—	—	—	1	8·5	12·25	16	37·25	40	34·25	25·75	20·75	7·75	2	2	—	—	—	·25	·25	—	208
63—64	—	—	—	1	1·5	6·25	12·25	20·25	39·5	40·5	21·5	22·5	10·5	4·5	4·25	1	—	—	·25	·25	—	186
64—65	—	—	—	—	—	3·5	7	24·75	30·75	36·5	30	39·25	18·5	8	6·75	2·25	·25	1	·5	·5	—	209·5
65—66	—	—	—	—	1	4	2·5	13·25	23	34·75	23·25	27·75	16·25	10·5	7·25	1·5	·5	·5	—	—	—	166
66—67	—	—	—	·25	1·25	·5	3	4·25	11·5	21·75	25	26·25	19	20·5	7·5	3	1·25	1	—	—	—	146
67—68	—	—	—	·25	·25	—	1	·5	4·75	13	12·5	13	8	8·25	6·25	4·75	2	1·5	·5	·5	1	78
68—69	—	—	—	—	—	—	—	1·5	·5	3·25	5	7·25	7·25	3·25	4·5	1·75	2·75	—	—	—	—	37
69—70	—	—	—	—	—	—	—	—	—	·5	3·75	2·25	·5	3	·5	1·25	1·25	—	—	1	—	14
70—71	—	—	—	—	—	—	—	1	—	—	—	·25	1·75	—	1	2	1·5	·5	·75	·25	·5	9·5
71—72	—	—	—	—	—	—	—	—	—	—	—	1·25	1·75	—	1	·5	·5	·5	·25	·25	·5	6·5
72—73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
73—74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1	—	—	—	2
Totals	·5	·5	1	11·5	36·5	63·5	88	162	213	227	165	177	103	63	47·5	18·5	10	6	2·5	3	2	1401

TABLE LXV.

Brother's Span and Sister's Span.

Brother's Span.

Sister's Span.	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	76—77	77—78	78—79	79—80	80—81	81—82	Totals
53—54	—	—	—	—	—	·5	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
54—55	—	—	—	—	—	·5	—	1·25	·25	·75	2·75	—	—	—	—	—	—	—	—	—	—	—	—	—	5·5
55—56	—	—	—	—	—	1	—	—	·25	·5	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	2
56—57	—	—	—	—	—	—	·25	1	1·75	2	·5	1	·5	—	—	—	—	—	—	—	—	—	—	—	7
57—58	—	—	—	—	·5	·5	1·25	2	3·5	1·5	2·5	2·75	—	1	—	—	—	—	—	—	—	—	—	—	15·5
58—59	—	—	—	1	·5	3·25	4·75	7	7·25	6·75	7·75	8·75	4·75	·75	·5	—	—	·5	·75	·25	—	—	—	—	54·5
59—60	—	—	—	—	·25	4	4·25	11	7·25	14	15·75	12·25	5·25	4	1·5	1·75	1·25	—	·25	·25	—	—	—	—	83
60—61	·25	·25	—	1·5	2	2·75	7·75	12	16·75	20·5	15·75	19·5	14·75	9·25	2·75	3·5	1·75	—	—	1	1	—	—	—	133
61—62	·25	·25	—	—	1·25	2·5	4	11·5	20	24·5	20·75	24·25	19	15	6·25	4·5	2·75	1·75	—	—	—	—	—	—	158·5
62—63	—	—	—	—	1·5	2	2·5	12	17·75	21·75	24·25	23·25	25·25	13·75	17·5	9·75	9	2·75	1	—	—	—	—	—	184
63—64	—	—	—	—	·5	—	3	5·75	10·75	14	17·75	23	32	23·75	20·25	13	6·5	2	1·75	2	—	—	—	—	176
64—65	—	—	—	—	—	·5	1·75	6	10·75	15	17·25	26·5	27·25	19·75	21·25	15·25	6	2	1·75	1	1	—	—	—	173
65—66	—	—	—	—	—	—	1	3·75	3·5	6·5	14	25·25	25·25	20·75	21·75	12·25	7	2·5	1	1	·5	—	—	—	146
66—67	—	—	—	—	—	—	1·5	·5	1·75	2·25	5·75	9·25	9·25	11·25	14·75	11·75	6·25	6	1·75	1·5	1·5	1	—	1	87
67—68	—	—	—	—	—	—	·5	—	·25	1·5	3·5	6	13·25	11·75	12·5	11·5	7·75	5·5	1	2·5	·75	·25	—	—	78·5
68—69	—	—	—	—	—	—	—	—	1	·5	1·5	5·25	3·25	7·5	8·75	4·25	5	5·75	1·75	1·5	·25	·25	—	—	46·5
69—70	—	—	—	—	—	—	—	—	—	—	—	·5	·75	2·5	2	3·25	3·75	4·25	·5	1	2	—	—	—	20·5
70—71	—	—	—	—	—	—	—	—	—	—	·5	·5	—	·75	1	·25	—	2	·5	1	1·5	·5	—	·5	9
71—72	—	—	—	—	—	—	—	—	—	—	—	1	—	·75	1·25	1·5	—	1	·5	—	·5	1	·5	·5	8·5
72—73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
73—74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
74—75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	·75	·75	—	—	—	2
75—76	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	·75	·75	—	—	—	2
Totals	·5	·5	—	2·5	6·5	17·5	32·5	74	103	132	150·5	189	180·5	142·5	132	92·5	57	37	12·5	14·5	10·5	3	·5	2	1393

TABLE LXVI.

Brother's Forearm and Sister's Forearm.

Brother's Forearm.

Sister's Forearm.

	14—14·5	*14·5—15*	*15—15·5*	*15·5—16*	*16—16·5*	*16·5—17*	*17—17·5*	*17·5—18*	*18—18·5*	*18·5—19*	*19—19·5*	*19·5—20*	*20—20·5*	*20·5—21*	*21—21·5*	*21·5—22*	*22—22·5*	*22·5—23*	*23—23·5*	Totals
13·5—14	—	—	—	—	·5	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	1
14—14·5	1	—	—	—	·5	1	—	1·75	2	·75	·5	—	—	—	—	—	—	—	—	7·5
14·5—15	—	—	—	·25	·75	1·25	4·25	5·25	6·25	3	1	1·25	·25	—	—	—	—	—	—	23·5
15—15·5	—	·25	·25	·75	1·75	5·75	10·5	16·75	15·5	7·5	2	1	·5	·25	·75	—	—	—	—	63·5
15·5—16	—	·25	·25	·75	2·75	16·75	28·5	36·75	41·5	23·5	14·5	5·5	1·75	1	·75	—	—	—	—	174·5
16—16·5	—	—	—	·5	2	14·5	36·25	59·75	88	59·75	33·25	15·75	3·75	2·25	1	·25	—	—	—	317
16·5—17	—	—	—	·25	2·75	6·75	20	50·5	79	65·5	40·25	19·25	7·25	3·5	1·25	1·75	—	—	—	298
17—17·5	—	—	—	—	—	1	5·5	30·5	57	58·25	41	28·25	10·25	3·5	1·75	1·5	—	—	·5	239
17·5—18	—	—	—	—	—	1·5	4·25	8	23	31·5	38·25	30·25	10·25	4	3	2	—	—	1	157
18—18·5	—	—	—	·25	·25	·5	·75	2·25	8·75	18·75	23	16	6·75	3·75	2·25	·75	—	—	·5	84·5
18·5—19	—	—	—	·25	·25	—	—	·25	1·25	2·5	5·5	4	2·5	1·75	·5	1·75	—	—	—	20·5
19—19·5	—	—	—	—	—	—	—	·25	2	·75	1·25	2	2·5	1·75	1	—	—	—	—	11·5
19·5—20	—	—	—	—	—	—	—	—	·5	—	—	·75	1·75	·25	·25	—	—	—	—	3·5
Totals	1	·5	·5	3	11·5	49	110	212	325	272	200·5	124	47·5	22	12·5	8	—	—	2	1401

TABLE LXVII.

Brother's Stature and Brother's Span.

First Brother's Stature.

Second Brother's Span.	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	76—77	77—78	78—79	79—80	Totals
61—62	—	—	—	1	—	—	1	—	—	—	—	·5	—	—	—	—	—	—	—	—	2·5
62—63	—	—	—	—	—	—	—	1	·5	—	—	·5	—	—	—	—	—	—	—	—	2
63—64	—	—	1	—	—	—	3	1·5	—	—	·5	—	—	—	—	—	—	—	—	—	6
64—65	—	—	—	—	4·25	1·25	3·5	4·75	2·5	·75	1	—	—	—	—	—	—	—	—	—	18
65—66	—	—	·5	1	1·75	6·5	6	5·75	4·25	·75	·5	1	·5	—	—	—	—	—	—	—	28·5
66—67	—	—	2	2·75	6·5	5	6·5	6·5	7·75	1·5	1·25	1	·25	1	—	—	—	—	—	—	42
67—68	·5	1·5	·5	2·25	4	2·25	10	8·25	9·25	1·25	3·75	2·5	2·5	1	—	—	—	·5	·5	—	50·5
68—69	—	—	—	1	2·25	3·5	8·25	17	16	8·75	10·5	4	1·75	·5	—	—	—	—	—	—	73·5
69—70	—	—	·5	1·25	4	8·5	8·25	13	17·5	11·5	18·5	7	2	3	—	—	—	—	—	—	95
70—71	—	—	1·25	2	1·5	2·5	7	9·5	16	9·75	10·5	10·5	5	2·25	2·25	·5	—	—	—	—	80·5
71—72	—	—	·25	·25	2·25	1·75	6·25	10·5	18·25	7·5	13·5	5·25	6	4	1·75	1	1	—	—	—	79·5
72—73	—	—	—	—	1·5	1·25	6·25	10	10·25	8	7	6·75	3·25	3	1·75	·5	—	—	—	—	59·5
73—74	—	—	—	—	—	·5	3	5·25	7	10·5	9·5	5	5·75	1·75	1·25	—	—	—	—	—	49·5
74—75	—	—	—	—	—	1	1	1	4·75	5·5	6·25	3·5	1	2·25	1·75	·5	·5	—	—	—	29
75—76	—	—	—	—	—	·5	—	1	1·75	2·5	3·25	3	1	·25	·25	1·5	·5	—	—	—	15·5
76—77	—	—	—	—	—	—	—	—	·25	2·25	·75	2·25	—	—	—	—	—	—	—	—	5·5
77—78	—	—	—	—	—	—	—	—	·5	3	·25	·25	—	1·5	—	—	1	—	—	—	6·5
78—79	—	—	—	—	—	—	—	·5	·5	1	—	—	—	3	·5	—	—	—	—	·5	6
79—80	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5
80—81	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
81—82	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	1
Totals	·5	1·5	6	11·5	28	34·5	70	95·5	117	74·5	87	53	29	23·5	9·5	4	3	1	1	1	651

TABLE LXVIII.

Brother's Stature and Brother's Forearm.

First Brother's Stature.

Second Brother's Forearm.	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	*74—75*	*75—76*	*76—77*	*77—78*	*78—79*	*79—80*	Totals
15·5—16	—	—	—	—	·5	·5	1	1	1	—	—	—	—	—	—	—	—	—	—	—	4
16—16·5	—	—	·5	—	·25	·5	2·5	1·75	1	—	—	—	—	—	—	—	—	—	—	—	6·5
16·5—17	—	—	·5	2	·75	1·5	4	2	4·25	—	1	·5	·25	·25	—	—	—	—	—	—	17
17—17·5	—	—	—	1·75	5·5	6	8·75	9·25	9·25	1·75	2	1·25	·25	·75	—	—	—	—	—	—	46·5
17·5—18	·25	·75	·5	3·25	5·75	7·5	15	18·25	17·5	4·25	14·25	4	1·75	1·5	—	—	—	·5	·5	—	95·5
18—18·5	·25	·75	1·5	2·5	7·25	8·25	18·5	21	27	17	18·25	10	8	6	·5	·25	—	—	—	—	147
18·5—19	—	—	1·75	1·75	3·25	3·25	9·5	20	23·75	18·75	17·5	14·25	6·25	4·75	3·25	1	—	—	—	—	129
19—19·5	—	—	·75	·25	3	2·75	5	12	16·5	15	18·25	14·25	6	3·5	2	1	1·75	—	—	—	102
19·5—20	—	—	·5	—	·75	2·5	1·75	6·75	8·75	10·75	12·25	4·75	5·25	2·25	1·75	·75	·25	—	—	—	59
20—20·5		—	—	—	—	·75	1	2·5	3·75	3·75	2·5	2·5	1·25	1·25	·25	1	1	—	—	—	21·5
20·5—21	—	—	—	—	—	·5	—		1·75	2·75	·5	·5	1	2·25	·75	—	—	—	—	—	10
21—21·5	—	—	—	—	—	·5	—	·25	·75	—	—	—		1	—	—	—	—	—	·5	3
21·5—22	—	—	—	—	—	—	—	·25	·25	—	—	—	·5	·5	—	—	—	·5	·5	·5	3
Totals	·5	1·5	6	11·5	27	34·5	67	95	115·5	74	86·5	52	30·5	24	8·5	4	3	1	1	1	644

TABLE LXIX.

Brother's Span and Brother's Forearm.

First Brother's Span.

Second Brother's Forearm.	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	*74—75*	*75—76*	*76—77*	*77—78*	*78—79*	*79—80*	*80—81*	*81—82*	Totals
15·5—16	—	—	—	1·5	2	·25	·25	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	4·5
16—16·5	·5	—	·25	1	1·75	·75	1·75	·25	·25	—	—	·25	·25	—	—	—	—	—	—	—	—	7
16·5—17	·5	—	1·25	3	1	·75	2·75	3·25	1·25	·5	·25	1·5	·5	—	—	·25	·25	—	—	—	—	17
17—17·5	—	·5	·5	5	6·25	6·75	5	4·25	6·75	4·25	3·25	3·25	·5	·25	—	·25	·25	—	—	—	—	47
17·5—18	·5	·5	1·25	2·25	5·5	13·25	11	9·75	15	13·5	10·5	6·5	4·25	1	·25	—	—	1	—	—	—	96
18—18·5	·5	—	1	2	9·25	11	11	23·75	25·75	18·75	14·75	11·5	9·5	4	2·5	·75	·5	—	—	—	—	146·5
18·5—19	·5	·5	·75	2·75	3·5	4·5	10·75	14·75	22·25	19·25	16·5	13	10·75	4·25	5·5	·5	1	—	—	—	—	131
19—19·5	—	—	·25	·25	1·25	3	4·75	11·5	10	14	17·5	15	10·75	7·25	2·25	1·75	2	1	—	—	—	102·5
19·5—20	—	—	·25	·25	—	·5	1·5	2·5	8	5·75	10	8·25	6·25	10·25	2	2	1	1	—	—	—	59·5
20—20·5	—	—	—	—	—	·25	1·25	·75	3	1·75	3·5	2·75	3	1·25	1	—	·5	2	—	—	—	21
20·5—21	—	—	—	·25	·25	—	—	1	·75	1	1·75	—	1·75	·75	1·5	—	·5	·5	—	—	—	10
21—21·5	—	—	—	·25	·25	—	·5	—	·25	·25	—	—	—	—	·5	—	·5	—	—	—	·5	3
21·5—22	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	1	—	—	·5	·5	—	·5	3
Totals	2·5	1·5	5·5	18·5	31	41	51	72	93·5	79	78	62	47·5	29	16·5	5·5	6·5	6	·5	—	1	648

TABLE LXX.

Sister's Stature and Sister's Span.

First Sister's Stature.

Second Sister's Span.	*52—53*	*53—54*	*54—55*	*55—56*	*56—57*	*57—58*	*58—59*	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	Totals
53—54	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	—	—	·5
54—55	—	—	—	—	—	—	—	1	·5	1	—	—	—	—	—	—	—	—	—	—	—	—	2·5
55—56	—	—	—	—	—	·5	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1·5
56—57	—	—	—	—	—	·5	·5	·5	·75	·75	·5	—	—	—	—	—	—	—	—	—	—	—	3·5
57—58	—	—	—	—	·25	·75	1·5	1·25	1	3·25	·75	1·75	—	—	—	—	—	—	—	—	—	—	10·5
58—59	—	—	—	—	·5	3·5	1·75	3·25	4·25	1·25	2·75	8·25	2	2·75	1·75	—	—	—	—	—	—	—	35
59—60	—	—	—	—	·5	1·25	1·5	1·5	8	6	10·75	5·5	8	6	5	·5	·5	—	—	—	—	—	55
60—61	—	—	—	1	1·25	2	·75	4	11·25	10	15·25	14·5	10·5	6·75	3·75	1	·5	—	·5	·5	—	—	83·5
61—62	·25	·25	—	—	·5	·5	1·75	6·5	7·25	8·75	21·75	17·75	17·5	11·75	8	2	·5	—	—	—	—	—	108
62—63	·25	·25	—	—	—	1	1·75	5	7	15·25	16	23	23	9·25	9	2·75	—	—	—	—	—	—	113·5
63—64	—	—	—	—	—	—	1	5·25	8·25	13	15·25	21	21·5	12	13·5	5	3	·25	—	—	—	—	119
64—65	—	—	—	—	—	—	—	2·25	5·75	14	17·75	14·5	19·5	17·5	12·25	7	4	3·5	·5	·5	—	—	119
65—66	—	—	—	—	—	—	—	·5	4·5	10·25	8·25	16	14·75	12·75	15·75	7	8	1·25	—	·5	—	—	99·5
66—67	—	—	—	—	—	—	·5	2	3	4·5	5·5	8·75	9	11	8·75	9·75	4·25	1·25	1·25	·5	—	—	70
67—68	—	—	—	—	—	—	—	—	—	1·5	5	5·5	9	11·75	9	4·75	3·25	1·5	·25	—	—	1	52·5
68—69	—	—	—	—	—	—	—	—	1	—	·5	1·5	4·25	6	11·75	3·75	2	4·5	2·25	—	—	—	37·5
69—70	—	—	—	—	—	—	—	·5	·5	—	—	1	2·5	3	5	3·5	1·5	2·25	1·25	—	—	—	21
70—71	—	—	—	—	—	—	—	—	—	—	—	·5	·5	·5	2	2·5	1	·5	—	—	—	—	7·5
71—72	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	·5	1	1·5	1	—	—	—	—	5
72—73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	·5
73—74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
74—75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	·5
75—76	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	—	—	—	·5
Totals	·5	·5	—	1	3	10	11	31·5	63	93	123	139·5	142·5	111·5	106	52	30	16	6	2	—	1	916

TABLE LXXI.

Sister's Stature and Sister's Forearm.

First Sister's Stature.

Second Sister's Forearm.	*52—53*	*53—54*	*54—55*	*55—56*	*56—57*	*57—58*	*58—59*	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	Totals
13·5—14	—	—	—	—	—	—	—	—	—	·5	—	·5	·25	·25	—	—	—	—	—	—	—	—	1·5
14—14·5	—	—	—	—	—	—	—	—	·5	1	—	·75	·5	·25	—	—	—	—	—	—	—	—	3
14·5—15	—	—	—	—	·5	1	1·25	2·25	—	1·5	1·25	·75	1·5	·5	—	—	—	—	·25	·25	—	—	11
15—15·5	·25	·25	—	—	·75	3·75	2·25	·75	6	7·5	5·25	10·25	3·5	2·75	1·25	·5	—	—	·25	·25	—	—	45·5
15·5—16	·25	·25	—	—	1·25	2·25	2	7·75	13·5	13·5	23·5	16·5	15	10·75	8·75	2·75	1	—	—	—	—	—	119
16—16·5	—	—	—	1	—	2	2·5	8·75	17·5	24·25	33·75	33·25	33·5	19	18·75	4·5	3·25	·5	·5	—	—	—	203
16·5—17	—	—	—	—	·5	1	1·75	7	15·25	22·75	26·25	35·5	37	20	17·75	9·5	3·75	2·75	·75	·5	—	—	202
17—17·5	—	—	—	—	—	—	1·25	6·75	6·75	18·75	18·75	21	26	30	23·75	11·5	6·25	3	·25	—	—	—	174
17·5—18	—	—	—	—	—	—	—	·75	3·5	3·75	9	11	13	21	21	8·75	6·25	3·75	1·25	—	—	·5	103·5
18—18·5	—	—	—	—	—	—	—	·5	—	·5	4·5	5·5	12	5·5	6	8·25	5·5	5·25	1·5	1	—	·5	56·5
18·5—19	—	—	—	—	—	—	—	—	—	—	·75	2·25	·5	2·25	5	3·75	·5	1·25	·75	—	—	—	17
19—19·5	—	—	—	—	—	—	—	—	—	—	—	·25	·25	1	2	1·25	1·75	—	·5	—	—	—	7
19·5—20	—	—	—	—	—	—	—	—	—	—	—	—	—	·25	·25	1	1	—	—	—	—	—	2·5
20—20·5	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	·25	·5	·25	—	—	—	—	1·5
20·5—21	—	—	—	—	—	—	—	—	—	—	—	—	·5	—	—	—	·25	·25	—	—	—	—	1
Totals	·5	·5	—	1	3	10	11	34·5	63	94	123	137·5	144	113·5	104·5	52	30	17	6	2	—	1	948

TABLE LXXII.

Sister's Span and Sister's Forearm.

First Sister's Span.

Second Sister's Forearm.	*53—54*	*54—55*	*55—56*	*56—57*	*57—58*	*58—59*	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	*74—75*	*75—76*	Totals
13·5—14	—	—	—	—	—	—	—	·25	·25	·5	—	—	·5	—	—	—	—	—	—	—	—	—	—	1·5
14—14·5	—	—	—	—	·5	·5	—	·25	·75	·5	—	—	·5	—	—	—	—	—	—	—	—	—	—	3
14·5—15	·25	1·25	·25	·5	·25	2	·5	1·5	1·25	1·25	·5	1·25	·25	—	—	—	—	—	—	—	—	—	—	11
15—15·5	·25	1·25	1·25	1	·25	4	5·25	6·25	7·5	8·25	4·5	4·25	1·5	—	·25	·25	—	—	—	—	—	—	—	46
15·5—16	—	—	—	·75	3·5	9·25	11	14·25	18	20·25	13	13·5	9·25	4	1·5	·75	—	—	—	—	—	—	—	119
16—16·5	—	—	—	1·25	3·5	9	15	21·5	26	29	29	24·75	17·75	17·5	5·25	2·5	—	—	—	—	—	—	—	202
16·5—17	—	—	—	—	2	3·5	12·25	18·75	29·25	25·5	29	30·75	20·75	15·25	7·5	2·75	3·75	1	·5	—	—	—	—	202·5
17—17·5	—	—	—	—	·5	4·75	7·75	13·5	17·25	18·75	25·5	21·25	24·75	11·25	12·5	9·5	6·25	·5	—	—	—	—	—	174
17·5—18	—	—	—	—	—	1	2·5	4·75	5	5·75	10·75	12·25	17·25	13	11·5	9·5	3·75	2·5	1·25	·25	—	·25	·25	101·5
18—18·5	—	—	—	—	—	—	·75	·5	2·75	2·75	6	6·75	5	6·25	8·25	6·25	6·25	2	2·25	·25	—	·25	·25	56·5
18·5—19	—	—	—	—	—	—	—	—	·5	1	1	3·25	1·25	2·25	·75	4·25	1·75	·25	·75	—	—	—	—	17
19—19·5	—	—	—	—	—	—	—	·5	—	—	·25	1·5	·25	—	1·25	2·5	·25	·25	·25	—	—	—	—	7
19·5—20	—	—	—	—	—	—	—	·5	—	—	—	·5	—	—	·75	·25	—	·5	—	—	—	—	—	2·5
20—20·5	—	—	—	—	—	—	—	—	—	—	—	—	·75	·25	—	—	—	·5	—	—	—	—	—	1·5
20·5—21	—	—	—	—	—	—	—	—	—	—	—	—	·75	·25	—	—	—	—	—	—	—	—	—	1
Totals	·5	2·5	1·5	3·5	10·5	34	55	82·5	108·5	113·5	119·5	120	100·5	70	49·5	38·5	22	7·5	5	·5	—	·5	·5	946

TABLE LXXIII.

Brother's Stature and Sister's Span.

Brother's Stature.

Sister's Span.	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	76—77	77—78	78—79	79—80	Totals
53—54	—	—	—	·5	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	1
54—55	—	—	—	·5	·5	·5	—	2·25	1·25	·5	—	—	—	—	—	—	—	—	—	—	—	5·5
55—56	—	—	—	—	·5	·5	—	·5	—	·5	—	—	—	—	—	—	—	—	—	—	—	2
56—57	—	—	—	—	2·25	1·5	1·25	3	—	—	1	—	—	—	—	—	—	—	—	—	—	9
57—58	—	—	—	—	1·25	2·75	1·25	3·25	4	1	1·5	·5	—	—	—	—	—	—	—	—	—	15·5
58—59	—	—	—	2·25	4	10·25	10	9·25	10·75	2·75	1·75	1·75	1·25	—	·5	—	—	—	—	—	—	54·5
59—60	—	—	1·5	1·5	3·75	4·75	13·25	18·25	16·25	8·5	6·25	4·25	2	·5	·75	—	—	—	—	—	—	81·5
60—61	·25	·75	·5	·25	8	12·75	16·75	16·25	23·25	21·75	10·75	13	3·75	1·25	3·25	·5	—	—	·5	·5	—	134
61—62	·25	·25	—	1·75	4	11·5	13·75	26·5	32·25	24·5	15·5	13·75	9·25	2·25	3	·5	—	—	—	—	—	159
62—63	—	—	—	3·5	4	5·75	10·25	31·5	30·5	29·5	23	20·5	9·25	7·5	6·75	2	—	·5	—	—	—	184·5
63—64	—	—	—	1·5	3	6	7·75	13·75	33·25	33·75	21·75	29·5	14·5	5·75	5·5	1	·5	1·5	—	—	—	179
64—65	—	—	·5	1·75	2·25	1·75	7·75	12·5	26·75	49·5	20	27	12·25	8	6	·5	—	—	·25	·25	—	177
65—66	—	—	—	—	1	1·5	3·75	11·75	19·75	23·75	25·75	26·75	16	8·75	5·5	2	·25	—	·25	·25	—	147
66—67	—	—	—	—	·5	1·5	2·25	3·5	11	12·75	11·75	10	12	9·25	5·75	3	2·25	—	·5	·5	1	87·5
67—68	—	—	—	—	·5	1	·5	2·75	3·25	10·5	12·25	16·75	11	9·5	5	3	1·5	1	—	—	—	78·5
68—69	—	—	—	—	—	·5	—	3·25	1·5	5·25	6·75	9·25	7·25	5·5	1·75	4	1·5	·75	·25	—	—	47·5
69—70	—	—	—	—	—	—	—	1·5	·5	1	4	5·25	2·25	2·75	1·25	1·5	2·5	·75	·25	—	—	23·5
70—71	—	—	—	—	—	—	—	—	—	1·5	3·5	·75	·25	2	—	—	—	—	·25	·25	·5	9
71—72	—	—	—	—	—	—	—	1	—	·5	1	1	1	1	·5	·5	1	—	·25	1·25	·5	9·5
72—73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
73—74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
74—75	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	·25	·75	—	—	—	2
75—76	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	·25	·75	—	—	—	2
Totals	·5	1	2·5	13·5	35·5	62·5	88·5	161	214·5	227·5	166·5	180	102	64	47·5	18·5	10	6	2·5	3	2	1409

TABLE LXXIV.

Brother's Span and Sister's Stature.

Brother's Span.

Sister's Stature.	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	76—77	77—78	78—79	79—80	80—81	81—82	Totals
52—53	—	—	—	—	—	—	—	—	—	·5	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	1
53—54	—	—	—	—	—	—	—	—	—	·5	—	—	—	·5	—	—	—	—	—	—	—	—	—	—	1
54—55	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
55—56	—	—	—	—	—	—	—	—	·5	1·5	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
56—57	—	—	—	—	—	—	·25	1	1·5	·75	1·5	—	—	—	—	—	—	—	—	—	—	—	—	—	5
57—58	—	—	—	—	—	1	·25	1·75	3·25	3·75	3·25	·5	1·75	—	—	—	—	—	—	—	—	—	—	—	15·5
58—59	—	—	—	—	—	·5	-	2·75	2·5	4·75	2·25	3·75	1	—	—	—	—	—	—	—	—	—	—	—	17·5
59—60	—	—	—	·5	—	4·75	4·75	7	5·5	7·25	10·75	7·5	3·25	3·5	·25	·5	—	—	—	—	—	—	—	—	55·5
60—61	—	—	—	1	·5	3·25	6·25	12·5	11·25	12	15	15	9	4·75	4·25	1·25	1	·75	1·25	·5	—	—	—	—	96·5
61—62	·5	·5	—	·5	·5	4·5	4·25	12·25	15	22·5	19	17·5	15·75	14·25	9·25	6·5	5·25	·25	·25	1	—	—	—	—	149·5
62—63	—	—	—	·25	·25	2	5·5	12·5	21·5	28	29·25	32	21·25	17·75	19·75	10·5	4·5	·5	1	1	·5	—	—	—	208
63—64	—	—	—	·25	3·5	·5	3·75	7·75	16·25	18	20	28·5	31·5	19·25	17·75	7·5	6	2·5	·5	1	·5	—	—	—	185
64—65	—	—	—	—	1·25	·5	2·5	10·5	12·5	18·25	18·75	29·75	35	26·75	20·25	14·75	12·75	5	2·25	1·25	1	—	—	—	213
65—66	—	—	—	—	·5	1	2·75	3	6·5	9·75	11·5	24·25	22·75	23·5	18·75	16·75	7·5	6·5	2	2	·5	—	—	—	162·5
66—67	—	—	—	—	—	—	1·75	1	2·75	2·75	10	14·5	21·75	15·75	21	20	12·5	7·75	2·75	2·75	1·5	—	—	—	141·5
67—68	—	—	—	—	—	—	1	1·25	1·75	1·75	6·25	6·25	9·5	9·25	12·25	9·25	6·75	4·75	1·75	2·5	1·25	1	—	1	77·5
68—69	—	—	—	—	—	·25	·25	·25	1·25	·5	1·5	6·25	5	4·5	7	2	1·75	4·5	·25	·5	1·75	·5	—	—	38
69—70	—	—	—	—	—	·25	·25	—	—	—	·5	·75	1	3·25	3·5	—	1	2	·5	—	—	·5	·5	—	14
70—71	—	—	—	—	—	—	—	·25	·25	—	—	1	·5	·75	·25	—	—	2·5	1	—	1	·5	—	·5	8·5
71—72	—	—	—	—	—	—	—	·25	·25	—	—	—	—	·25	·75	1·5	—	1·5	—	—	1	·5	—	·5	6·5
72—73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
73—74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	·5	·5	—	1	—	—	—	—	2
Totals	·5	·5	—	2·5	6·5	18·5	33·5	74	102·5	132·5	152·5	187·5	179	144·5	135	90·5	59·5	39	13·5	13·5	9	3	·5	2	1400

TABLE LXXV.

Brother's Stature and Sister's Forearm.

Brother's Stature.

Sister's Forearm.	*59—60*	*60—61*	*61—62*	*62—63*	*63—64*	*64—65*	*65—66*	*66—67*	*67—68*	*68—69*	*69—70*	*70—71*	*71—72*	*72—73*	*73—74*	*74—75*	*75—76*	*76—77*	*77—78*	*78—79*	*79—80*	Totals
13·5—14	—	—	—	·5	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	1
14—14·5	—	—	—	·5	—	—	—	1·25	3·25	2	·25	·25	—	—	—	—	—	—	—	—	—	7·5
14·5—15	—	—	—	·5	2·75	3·25	3	4·25	3·25	2	1·5	1·25	·75	—	—	—	—	—	—	—	—	22·5
15—15·5	—	—	—	1	3·75	5·75	11·25	11·5	13·5	6·75	4·25	3·25	3	—	·5	—	—	—	—	—	—	64·5
15·5—16	·25	·5	·75	2·5	10	18	18·5	26·75	26	23·75	17	17	9	2·5	2·5	·5	—	—	—	—	—	175·5
16—16·5	·25	·5	·75	3	11	18	24·25	49	53·75	57·5	31·25	34·75	16·25	7·75	8·25	3·25	·5	—	·25	·25	—	320·5
16·5—17	—	—	—	3·25	5·25	10·75	18·25	34·25	61·25	48·25	33·5	38·5	16·75	8·75	12·25	1·5	—	2	·75	·75	1	297
17—17·5	—	—	—	1·75	1·75	5·75	7·75	18	35·75	44·75	37	41·5	19	12·25	8·25	2·75	1·5	·25	·5	·5	—	239
17·5—18	—	—	—	·25	1·25	1·5	1·5	7·25	12·5	27·75	29·5	22·5	20·5	17	10·25	4·75	2	1	—	—	—	159·5
18—18·5	—	—	—	·25	·75	1	1·75	4·25	6·5	11·5	11·75	14·5	12	9·5	4	3·25	3·25	1·25	·75	·25	·5	87
18·5—19	—	—	—	—	—	—	·25	·25	1	3·5	·75	6·25	2	1·25	1·25	1·75	1·75	—	·25	·75	·5	21·5
19—19·5	—	—	—	—	—	—	—	1·25	·25	·75	·25	1·75	1·5	·75	1·75	·75	1	1	—	·5	—	11·5
19·5—20	—	—	—	—	—	—	—	·25	·25	—	—	—	·75	·75	·5	—	—	·5	—	—	—	3
Totals	·5	1	1·5	13·5	36·5	64	86·5	158·5	217·5	228·5	167	181·5	101·5	60·5	49·5	18·5	10	6	2·5	3	2	1410

TABLE LXXVI.

Brother's Forearm and Sister's Stature.

Brother's Forearm.

Sister's Stature.	14—14·5	14·5—15	15—15·5	15·5—16	16—16·5	16·5—17	17—17·5	17·5—18	18—18·5	18·5—19	19—19·5	19·5—20	20—20·5	20·5—21	21—21·5	21·5—22	22—22·5	22·5—23	23—23·5	Totals
52—53	—	—	—	—	—	—	·25	·5	·25	—	—	—	—	—	—	—	—	—	—	1
53—54	—	—	—	—	—	—	·25	·5	·25	—	—	—	—	—	—	—	—	—	—	1
54—55	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
55—56	—	—	—	—	—	—	1·5	·5	—	—	—	—	—	—	—	—	—	—	—	2
56—57	—	—	—	—	—	·5	2	1	1·5	—	—	—	—	—	—	—	—	—	—	5
57—58	—	—	—	—	—	1·5	2·25	6·25	4	·75	·75	—	—	—	—	—	—	—	—	15·5
58—59	—	—	—	—	·5	1·5	3	4·5	4·5	2	1	·5	—	—	—	—	—	—	—	17·5
59—60	—	·25	·25	·5	2·25	7	8·5	10	15·75	6·75	3·25	·75	·25	—	—	—	—	—	—	55·5
60—61	—	·25	·25	—	1·5	10·75	15·25	20·5	21·75	16·25	4	2·75	·25	—	1	—	—	—	—	94·5
61—62	—	—	—	1	2·75	5·25	16·25	34	37·75	26·75	13·75	7·75	2·25	1	·5	—	—	—	—	149
62—63	—	—	—	·5	1·25	9·75	22·25	40·25	57	33·5	28·5	11	3·5	·5	·25	·25	—	—	—	208·5
63—64	—	—	—	1	2·25	5·5	13·5	28·25	50	37·75	24	14·5	4·5	2·5	1·5	·25	—	—	—	185·5
64—65	—	—	—	—	1	2	10·25	27·75	52·25	48·25	34·75	22·5	7	5·25	1·5	1·5	—	—	·5	214·5
65—66	1	—	—	—	—	2·25	6	17	38·25	36·75	27	21·5	9	2	·75	1	—	—	·5	163
66—67	—	—	—	—	—	1	5	13	25·25	33·75	32·75	20·5	6·75	2	2	1	—	—	—	143
67—68	—	—	—	—	—	·5	2·25	4	12	16·25	18·5	11	5	3·5	2	1·5	—	—	·5	77
68—69	—	—	—	—	—	1·25	1·25	2·5	4·5	7·75	6·25	4·75	3·5	2·25	1	1	—	—	·5	36·5
69—70	—	—	—	—	—	·25	·25	—	1·75	1·5	4·25	3·5	1·5	·5	·5	—	—	—	—	14
70—71	—	—	—	—	—	—	—	—	1·25	·75	1	1·75	1·5	1·25	·25	·75	—	—	—	8·5
71—72	—	—	—	—	—	—	—	—	—	·75	1·75	1·25	·5	·75	·75	·75	—	—	—	6·5
72—73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
73—74	—	—	—	—	—	—	—	—	—	—	—	—	1	·5	·5	—	—	—	—	2
Totals	1	·5	·5	3	11·5	49	110	210·5	328	269·5	201·5	124	46·5	22	12·5	8	—	—	2	1400

TABLE LXXVII.

Brother's Span and Sister's Forearm.

Brother's Span.

Sister's Forearm.	58—59	59—60	60—61	61—62	62—63	63—64	64—65	65—66	66—67	67—68	68—69	69—70	70—71	71—72	72—73	73—74	74—75	75—76	76—77	77—78	78—79	79—80	80—81	81—82	Totals
13·5—14	—	—	—	—	—	·5	—	·25	·25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
14—14·5	—	—	—	—	—	·5	1	·25	·25	2	2	—	·75	·25	—	·5	—	—	—	—	—	—	—	—	7·5
14·5—15	—	—	—	—	—	1·5	·5	3·25	2·25	4·5	3·5	3	1·25	·75	1	·5	—	—	—	—	—	—	—	—	22
15—15·5	—	—	—	—	—	2·5	2	8·5	7	10	10·25	11	4·75	6·25	1·25	—	—	·25	·5	·25	—	—	—	—	64·5
15·5—16	·25	·25	—	1·5	—	6·75	8·75	17·5	22	21·25	24·25	22·75	15·75	13·25	9·75	6·75	1·25	1	1·25	·75	—	—	—	—	175
16—16·5	·25	·25	—	·5	·75	4·75	12·5	19·75	31·5	38·25	41·5	44·5	43·75	29·25	22·5	14·75	10·25	2·75	·75	1	·5	—	—	—	320
16·5—17	—	—	—	·5	4·5	1·25	4	15	25	34	27·75	42	43	33	30·25	15·5	11	4	3	1·25	1	·5	—	1	297·5
17—17·5	—	—	—	—	1·25	·25	2·75	6·5	10	14·25	22·5	44·25	39·25	31·25	23·5	23·25	10·25	5	3·5	1·25	2	—	—	—	241
17·5—18	—	—	—	—	—	·5	1·5	1	2·5	4·5	13·25	14·75	22·5	18·5	22·5	18	19·5	7·5	2·25	5·25	2·75	·25	—	—	157
18—18·5	—	—	—	—	—	—	·5	1	1·25	2·5	5·5	6	9	12	16·25	10·25	6·25	7·25	1·5	1·75	3·5	·5	—	·5	85·5
18·5—19	—	—	—	—	—	—	—	·5	—	·25	1·25	·5	·75	2·75	3·5	2·5	1·5	4·25	·25	—	1·25	·5	·25	·5	20·5
19—19·5	—	—	—	—	—	—	—	·25	·25	·5	·75	1·25	·25	1	1·25	1	—	2	—	·75	1·75	·25	·25	—	11·5
19·5—20	—	—	—	—	—	—	—	·25	·25	·5	—	—	—	·25	·75	—	—	—	—	·75	·25	—	—	—	3
Totals	·5	·5	—	2·5	6·5	18·5	33·5	74	102·5	132·5	152·5	190	181	148·5	132·5	93	60	34	13	13	13	2	·5	2	1406

TABLE LXXVIII.

Brother's Forearm and Sister's Span.

Brother's Forearm.

Sister's Span.	*14—14·5*	*14·5—15*	*15—15·5*	*15·5—16*	*16—16·5*	*16·5—17*	*17—17·5*	*17·5—18*	*18—18·5*	*18·5—19*	*19—19·5*	*19·5—20*	*20—20·5*	*20·5—21*	*21—21·5*	*21·5—22*	*22—22·5*	*22·5—23*	*23—23·5*	Totals
53—54	—	—	—	—	·5	—	—	—	·25	·25	—	—	—	—	—	—	—	—	—	1
54—55	—	—	—	—	·5	—	1·25	1·25	2·25	·25	—	—	—	—	—	—	—	—	—	5·5
55—56	—	—	—	—	—	1	·25	·75	—	—	—	—	—	—	—	—	—	—	—	2
56—57	—	—	—	—	—	·5	2·25	3·5	2·75	—	—	—	—	—	—	—	—	—	—	9
57—58	—	—	—	·25	·25	1·25	3·25	4	3·5	3	—	1	—	—	—	—	—	—	—	16·5
58—59	—	·5	·5	·5	1·75	5·75	9·75	10	12	6·5	3	·5	·25	—	·5	—	—	—	—	51·5
59—60	—	—	—	·75	1	11	9·25	17·25	22	11·5	6·25	3·25	1·25	—	·5	—	—	—	—	84
60—61	—	—	—	—	1·25	9·25	20·5	29·5	32	18·5	10·75	4	2	·75	·5	·5	—	—	—	129·5
61—62	—	—	—	·5	2·25	6·75	19·75	34·75	41	28·5	15·75	8·5	2·5	1·5	·25	—	—	—	—	162
62—63	—	—	—	·25	2·25	6·5	13·5	32·25	49·5	35·25	22·75	15	3·5	1	·75	—	—	—	—	182·5
63—64	—	—	—	·25	·75	·75	8·75	27·5	46·75	37·75	33·5	14	4·25	4	·25	—	—	—	—	178·5
64—65	—	—	—	·5	·5	2	8·5	25·25	46·75	41·5	24·5	13·75	6	2·75	2·75	·25	—	—	—	175
65—66	1	—	—	—	·25	2·25	5·5	14·25	34·5	36	26·75	19·25	5·25	·75	·5	·25	—	—	—	146·5
66—67	—	—	—	—	·25	1	3·25	6·25	12·5	20·75	15·25	12·75	6·75	2·75	·5	3·5	—	—	1	86·5
67—68	—	—	—	—	—	1	2·75	3·25	9·5	19·75	17·75	11·5	5·75	3	1·75	2	—	—	1	79
68—69	—	—	—	—	—	1	1	2·25	4·25	7·75	14·5	9·5	3·25	1·75	2·25	—	—	—	—	47·5
69—70	—	—	—	—	—	—	·5	·5	1	2	5·5	6·5	3·75	1·25	—	—	—	—	—	21
70—71	—	—	—	—	—	—	—	·5	1·5	·25	2	2·75	·5	·5	·25	·75	—	—	—	9
71—72	—	—	—	—	—	—	—	—	2	·5	1·75	1·25	·5	1	·75	·75	—	—	—	8·5
72—73	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
73—74	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
74—75	—	—	—	—	—	—	—	—	—	—	·25	·25	·5	·5	·5	—	—	—	—	2
75—76	—	—	—	—	—	—	—	—	—	—	·25	·25	·5	·5	·5	—	—	—	—	2
Totals	1	·5	·5	3	11·5	50	110	213	324	270	200·5	124	46·5	22	12·5	8	—	—	2	1399

VARIATION IN "OPHIOCOMA NIGRA" (O. F. MÜLLER).

By D. C. McINTOSH, M.A., F.R.S.E.

For two months during the autumn of 1902 I was permitted to occupy the "Aberdeen University Table" at the West of Scotland Marine Biological Station at Millport. This privilege I owe to the Professor of Natural History at the University of Aberdeen, at whose suggestion the following piece of work was primarily undertaken as an exercise in connection with the B.Sc. examination in Zoology.

The main purpose of the present communication is to give the results of my examination of certain external features of the Brittle-star *Ophiocoma nigra*. My first observations were made on specimens dredged in a single haul from near the Tan Buoy between the Cumbraes, on September 2nd, at a depth of from five to seven fathoms, from gravelly ground well known to be frequented by brittle-stars. They were gathered promiscuously, and on being brought to the laboratory were at once placed in tanks through which flowed a constant stream of sea water. Although other things were noted, I merely give here the results of my attempt to find:

1. The most general shape of the disc.
2. The shape and size of the coloured patch (if any) on the disc.
3. The correlation between arm-length and disc-breadth.
4. The percentage of specimens with an abnormal number of rays.
5. The number of madreporic plates.

In order that all the above points might be investigated when the animals were newly killed, the method I adopted was as follows. Specimens were selected at random from one of the tanks and immediately placed in fresh water. I found that in this way they died in from three to three and three-quarter minutes, and could be handled without exhibiting the brittleness characteristic of them when killed by immersion in a solution of formaline. Then each animal was examined and a note taken of

1. The shape of the disc.
2. The colour.
3. The breadth of the disc.
4. The length of the longest ray.
5. The number of rays.
6. The number and position of the madreporic plates.
7. Any unusual character.

At first I confined my observations to a thousand specimens whose discs were 10 mm. and upwards in breadth. As the results of this preliminary examination were interesting, I next endeavoured to determine whether they extended to specimens of all sizes, even the most minute obtainable.

Accordingly on the 27th of September the same ground near the Tan Buoy was again dredged, with the result that we gathered over two thousand specimens, which on being brought to the Station were killed by immersion in fresh water. They were then preserved in a fifty per cent. solution of alcohol for future examination: and I shall now proceed to give the results I have arrived at from my examination of these and the other thousand specimens.

1. *As to shape of disc.*

One of the specific characters of this species, according to Forbes*, and Jeffrey Bell†, is that the disc is round, but an examination of the accompanying table (Table I.) points rather to the disc being in general pentagonal.

TABLE I.

Showing the Number of Specimens with Circular or Pentagonal Disc.

	Circular disc as in Fig. 1	Pentagonal disc as in Fig. 2	Pentagonal disc as in Fig. 3	Percentage with Pentagonal disc	Percentage with Circular disc
T_1	248	658	94	75·2	24·8
T_2	160	781	59	84	16
T_3	124	800	76	87·6	12·4
Average of T_2 and T_3	142	790·5	67·5	85·8	14·2

For convenience, let T_1, T_2, and T_3 denote the first, second, and third thousands respectively. No trouble was experienced in classifying the shapes of disc, for there was scarcely a single specimen which did not readily fall into one of three groups, namely, distinctly circular as in Fig. 1, or distinctly pentagonal (*a*) with the rays at the angles of the pentagon, as in Fig. 2, or (*b*) with the rays from the centre of the sides of the pentagon, as in Fig. 3. The last two columns of the table give the percentages of specimens with pentagonal and with circular discs, and it will be remarked that the numbers for T_1 are somewhat different from those for T_2 and T_3, which are as consistent with each other as might fairly be expected. But it must be remembered that the shape of the disc necessarily depends largely on the quantity of food and water in the stomach of the animal at any particular time, and that T_1 were examined immediately after being killed, whereas the others had been lying for some time in alcohol. Further, T_1 consisted of "adults" only; and further still, one effect of the alcohol used for T_2 and T_3 was to slightly shrivel up the outer coating of the disc and thus better reveal its natural shape. It is clear, then, that if these three thousand specimens taken at random be really typical of *Ophiocoma nigra*, (1) the shape of the disc is in general pentagonal, and (2) the disc tends to become circular in the more fully developed animals.

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* *A History of British Starfishes* (London, 1841), by Edward Forbes, p. 50.

† *Catalogue of the British Echinoderms in the British Museum* (London, 1892), p. 129.

2. *As to colour-markings.*

There can be no doubt as to the prevailing colour of the specimens I handled: it is dark brown. Whatever colour-variation occurred consisted mainly in the disc having a central yellowish sand-coloured patch, and in the rays being occasionally of a lighter brown than the disc. Jeffrey Bell says the colour of this species is "black or dark brown, the arms lighter than the disc: sometimes lighter and sometimes spotted." Forbes says: "Müller's name* well applies to the usual colour of this Brittle-star: the disc and rays are commonly black, or brownish black; the ray-spines dusky white or bluish. Sometimes the disc is prettily variegated, and there is a variety of an orange colour not uncommon in the Irish Sea. Mr Goodsir and I found some specimens in Shetland, of a most beautiful, delicate rose colour." Gosse, who merely describes the single specimen he chanced to find, remarks† of the species that "its hues are said to be various."

Paying no attention to the shape of the disc, but noting only the shape of the coloured patch (if any), I attempted to tabulate the different shape of coloured patches—see Table II. If any colour-marking occurred at all it was found to be

TABLE II.

Showing Relative Size and Shape of Colour-Markings on Disc.

	A	*B*	*C*	*D*	*E*	*F*	*G*
T_1	238	116	145	159	103	135	104
T_2	265	114	170	172	100	78	101
T_3	220	125	191	120	130	105	109
Average percentage	24·1	11·8	16·9	15·0	11·1	10·6	10·5
			31·9		21·7		

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* The name O. F. Müller uses for this species is *Asterias nigra*, while Edward Forbes calls it *Ophiocoma granulata*.

† *A Year at the Shore* (London, 1870), P. H. Gosse, p. 227.

almost invariably regular in shape, either circular or pentagonal, but it did not follow that a circular disc had a circular colour-patch and a pentagonal disc a pentagonal one.

The shape, but not the relative size, of the central coloured part of the disc is shown in Figures 4, 5, and 6. Column *A* gives the number of specimens which were entirely of one colour, that colour being as stated, nearly always dark brown or black. (All were viewed with aboral surface upwards.) There was very little colour-variation in the rays, although generally they were not so dark as the disc, but in 2·8 per cent. of T_1 the rays were ringed with grey or white at every fifth set of plates for about half the length of the ray, beginning at the tip. In column *B* are given the numbers of those whose disc had a small coloured patch, often a mere speck, nearly always circular (as in Fig. 4) and never occupying more than one-fourth of the whole disc. Column *C* gives the numbers in which the coloured part was greater than one-quarter, but less than one-half of the disc. 16·9 per cent. had this marking, which was nearly always pentagonal (as shown in Fig. 5), although a few had the patch circular. Column *D* gives the numbers with a coloured part greater than one-half, but less than three-fourths of the area of the disc. This was always pentagonal (Fig. 5), and was found in 15 per cent. of the specimens. The numbers in columns *E* and *F* are those in which the size of the coloured part of the disc was the same as that in columns *C* and *D* respectively, while the shape was the same as that represented in Fig. 6. It is noted in column *G* that in 10·5 per cent. of the specimens, at least three-fourths—sometimes almost the whole—of the disc was sandy-coloured, although the rays still retained their characteristic brown appearance. The shape of the colour-marking in this case was usually of the pentagonal form shown in Fig. 6, the part of the disc at the point where each ray leaves the disc being usually of the colour of the ray. The conclusion then is that as regards the disc and its colour-markings,

1. 24·1 per cent. were entirely of one colour.
2. 11·8 per cent. had a small circular (Fig. 4) coloured patch.
3. 31·9 per cent. had a medium pentagonal (Fig. 5) coloured patch.
4. 21·7 per cent. had a medium pentagonal (Fig. 6) coloured patch.
5. 10·5 per cent. had a large pentagonal (Fig. 6) coloured patch.

It is thus seen (1) that in over 75 per cent. of the specimens the disc had a coloured patch, (2) that the shape of this coloured patch seemed to vary with its size (being circular when small and pentagonal when large), and (3) that the shape of the patch, like the shape of the disc itself, was in general pentagonal.

3. *Correlation between Arm-length and Disc-breadth.*

To measure the width of the disc I used a pair of finely pointed dividers, care being taken in the operation that the points touched, but did not press against the opposite sides of the disc. When the disc was pentagonal in shape, one point of

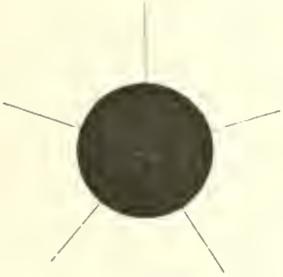

Fig. 1.

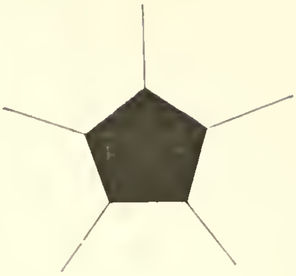

Fig. 2.

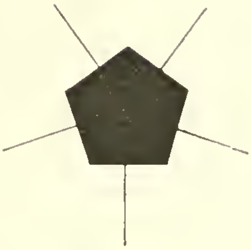

Fig. 3.

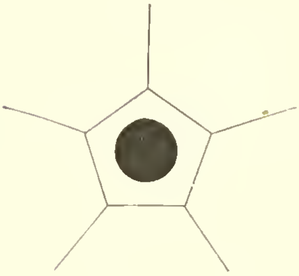

Fig. 4.

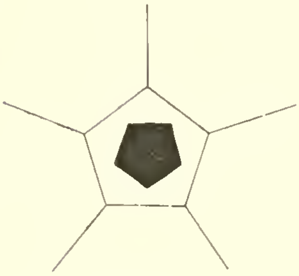

Fig. 5.

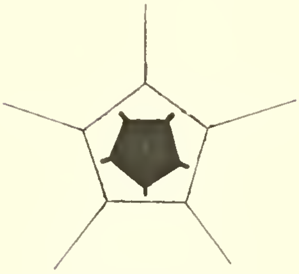

Fig. 6.

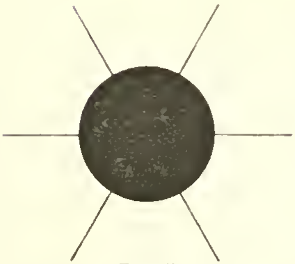

Fig. 7.

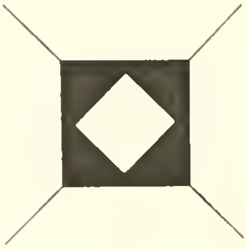

Fig. 8.

the dividers was placed at an angle of the pentagon and the other at the middle of the opposite side whereas in circular discs the measurement was made along a diameter. I next measured the length of the rays, which in a perfect specimen taper gradually to a very fine thread. This measurement was made with the ventral surface upwards, for when a brittle-star is viewed dorsally each ray seems to come from a side of the disc whereas when viewed ventrally it is seen that the rays begin nearer the mouth. Accordingly, the length of the ray was measured from the very tip of the ray to the innermost edge of the arm-plate nearest the mouth.

Forbes says* that "the rays taper gradually, and vary in their proportions as compared with the disc, but are generally from three to three and a half times as long as the disc is broad....The disc of this species generally measures half or three-fourths of an inch across. It sometimes grows much larger. Mr Ball has a specimen six inches in diameter; the disc half an inch broad: and I have one before me at present which measures eight inches across the rays." On the other hand, Jeffrey Bell says† that this species of ophiuroid is "moderately sized," and that the arms are "seven or eight times the radius of the disc," instancing five specimens whose discs varied from 4·5 to 11 millimetres and rays from 40 to 95 millimetres, the ray being therefore on the average about seven and a half times as long as the disc is broad.

Table III. gives the calculations on this correlation, the figures given being those only for the second thousand specimens examined. This table shows not only the various disc-breadths and arm-lengths, but each square gives the number of animals having a certain disc-breadth associated with a certain arm-length. From this table, by the methods and formulæ now familiar to readers of *Biometrika*, there have been deduced the following results:

Mean of disc-breadth $= M_x = 10{\cdot}106$ mm.
Standard deviation of disc-breadth $= \sigma_x = 2{\cdot}1449$ mm.
Mean of arm-length $= M_y = 50{\cdot}656$ mm.
Standard deviation of arm-length $= \sigma_y = 11{\cdot}2796$ mm.
Coefficient of correlation $= r = 0{\cdot}9311$.

Knowing these results, we are now able to find the lines of regression. The line giving mean arm-length for known disc-breadth is found from the formula $y = \frac{r\sigma_y}{\sigma_x} x$ and is $y = 4{\cdot}8962x$. From the formula $x = \frac{r\sigma_x}{\sigma_y} y$ we find the line giving mean disc-breadth for known arm-length is $x = 0{\cdot}17705y$. The equation to the line of mean values of arm-lengths referred to the axes, arm-length = 0, disc-breadth = 0, is

$$(y - M_y) = \frac{r\sigma_y}{\sigma_x}(x - M_x),$$

which is

$$y = 4{\cdot}8962x + 1{\cdot}1747.$$

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* *Op. cit.* Forbes, pp. 51, 52.

† *Op. cit.* Jeffrey Bell, p. 129.

TABLE III.

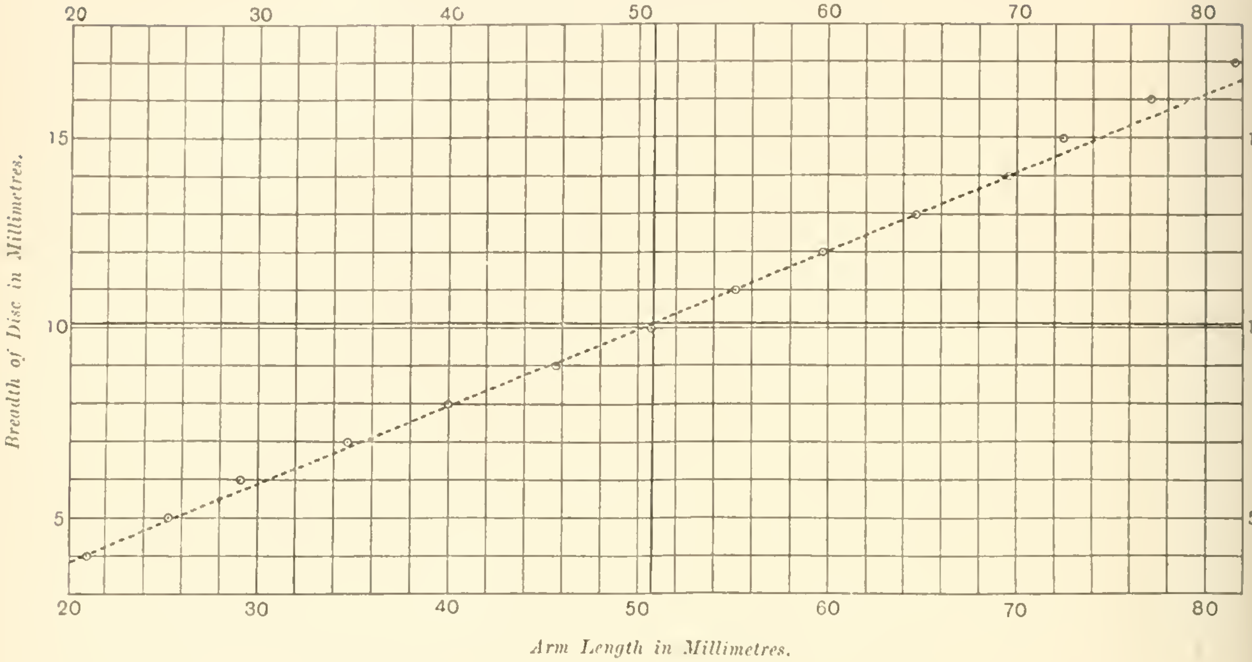
Disc Breadth in Millimetres.

Arm Length in Millimetres.

	4	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*	*17*	Totals
20	2	—	—	—	—	—	—	—	—	—	—	—	—	—	2
21	2	—	—	—	—	—	—	—	—	—	—	—	—	—	2
22	2	1	—	—	—	—	—	—	—	—	—	—	—	—	3
23	—	3	—	—	—	—	—	—	—	—	—	—	—	—	3
24	—	3	—	—	—	—	—	—	—	—	—	—	—	—	3
25	—	3	1	—	—	—	—	—	—	—	—	—	—	—	4
26	—	3	4	—	—	—	—	—	—	—	—	—	—	—	7
27	—	1	1	—	—	—	—	—	—	—	—	—	—	—	2
28	—	2	8	3	—	—	—	—	—	—	—	—	—	—	13
29	—	—	3	1	—	—	—	—	—	—	—	—	—	—	4
30	—	1	2	—	—	—	—	—	—	—	—	—	—	—	3
31	—	—	3	6	—	—	—	—	—	—	—	—	—	—	9
32	—	—	6	5	—	—	—	—	—	—	—	—	—	—	11
33	—	—	3	9	—	—	—	—	—	—	—	—	—	—	12
34	—	—	2	10	4	—	—	—	—	—	—	—	—	—	16
35	—	—	—	2	3	—	—	—	—	—	—	—	—	—	5
36	—	—	—	5	8	—	—	—	—	—	—	—	—	—	13
37	—	—	—	3	10	4	—	—	—	—	—	—	—	—	17
38	—	—	1	7	16	5	—	—	—	—	—	—	—	—	29
39	—	—	—	3	9	2	—	—	—	—	—	—	—	—	14
40	—	—	—	2	12	4	—	—	—	—	—	—	—	—	18
41	—	—	—	—	7	6	1	—	—	—	—	—	—	—	14
42	—	—	—	3	17	6	—	—	—	—	—	—	—	—	26
43	—	—	—	—	11	11	—	—	—	—	—	—	—	—	22
44	—	—	—	—	2	13	5	—	—	—	—	—	—	—	20
45	—	—	—	—	4	11	11	—	—	—	—	—	—	—	26
46	—	—	—	—	4	12	5	2	—	—	—	—	—	—	23
47	—	—	—	—	1	14	11	4	1	—	—	—	—	—	31
48	—	—	—	—	1	20	34	8	1	—	—	—	—	—	64
49	—	—	—	—	—	11	8	2	—	—	—	—	—	—	21
50	—	—	—	—	—	4	17	10	1	—	—	—	—	—	32
51	—	—	—	—	—	5	23	13	6	1	—	—	—	—	48
52	—	—	—	—	—	5	19	6	2	—	—	—	—	—	32
53	—	—	—	—	—	3	21	17	2	—	—	—	—	—	43
54	—	—	—	—	—	1	11	20	9	—	—	—	—	—	41
55	—	—	—	—	—	—	8	10	10	—	—	—	—	—	28
56	—	—	—	—	—	—	8	18	4	1	—	—	—	—	31
57	—	—	—	—	—	—	5	14	9	3	—	—	—	—	31
58	—	—	—	—	—	—	2	14	11	4	1	—	—	—	32
59	—	—	—	—	—	—	3	8	11	4	—	—	—	—	26
60	—	—	—	—	—	—	1	8	12	6	1	—	—	—	28
61	—	—	—	—	—	—	1	5	16	4	1	—	—	—	27
62	—	—	—	—	—	—	1	5	13	6	1	1	—	—	27
63	—	—	—	—	—	—	—	2	3	10	—	—	—	—	15
64	—	—	—	—	—	—	—	4	3	5	—	—	—	—	12
65	—	—	—	—	—	—	—	1	3	7	1	—	—	—	12
66	—	—	—	—	—	—	—	1	9	5	—	—	—	—	15
67	—	—	—	—	—	—	—	2	5	9	1	—	—	—	17
68	—	—	—	—	—	—	—	—	4	9	5	1	—	—	19
69	—	—	—	—	—	—	—	—	4	1	—	1	—	—	6
70	—	—	—	—	—	—	—	—	—	6	3	1	—	—	10
71	—	—	—	—	—	—	—	—	1	2	3	—	—	—	6
72	—	—	—	—	—	—	—	—	—	2	1	—	—	—	3
73	—	—	—	—	—	—	—	—	—	2	2	—	—	—	4
74	—	—	—	—	—	—	—	—	1	2	3	1	—	—	7
75	—	—	—	—	—	—	—	—	—	1	1	2	—	—	4
76	—	—	—	—	—	—	—	—	—	—	1	1	—	—	2
77	—	—	—	—	—	—	—	—	—	—	—	—	1	—	1
78	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
79	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
80	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
81	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
82	—	—	—	—	—	—	—	—	—	—	—	1	—	1	2
Totals	6	17	34	59	109	137	195	174	141	90	26	9	1	2	1000

This line is shown in Diagram I., and is seen to represent very closely the observed facts.

Diagram I. Regression of Arm Length on Breadth of Disc.



This result, that the arm-length is five times the width of the disc, is almost a mean between Jeffrey Bell's corresponding figure 7½ and Forbes' 3 to 3½. Their calculations were in all probability made from a few specimens, while mine are from one thousand. It must, however, be remembered that my measurements are from specimens which had been lying in alcohol for some weeks; but even although there may have been some contraction, it is not likely that the ratio of the breadth of disc to the length of ray would thereby be sensibly affected.

As it appears from Table III. that the entire range of variation of the disc-breadth is from 4 to 17 mm., it might readily be supposed that the size of the mesh of the net used in dredging had much to do with the fact that no specimens were found whose diameter measured less than 4 mm. Not much importance, however, can be attached to this objection, because nearly all the animals collected were brought to the surface clinging to blades of oarweed.

From the totals in Table III. we may form a polygon of frequencies. Diagram II. gives the curve which best fits this. The constants of this curve are:

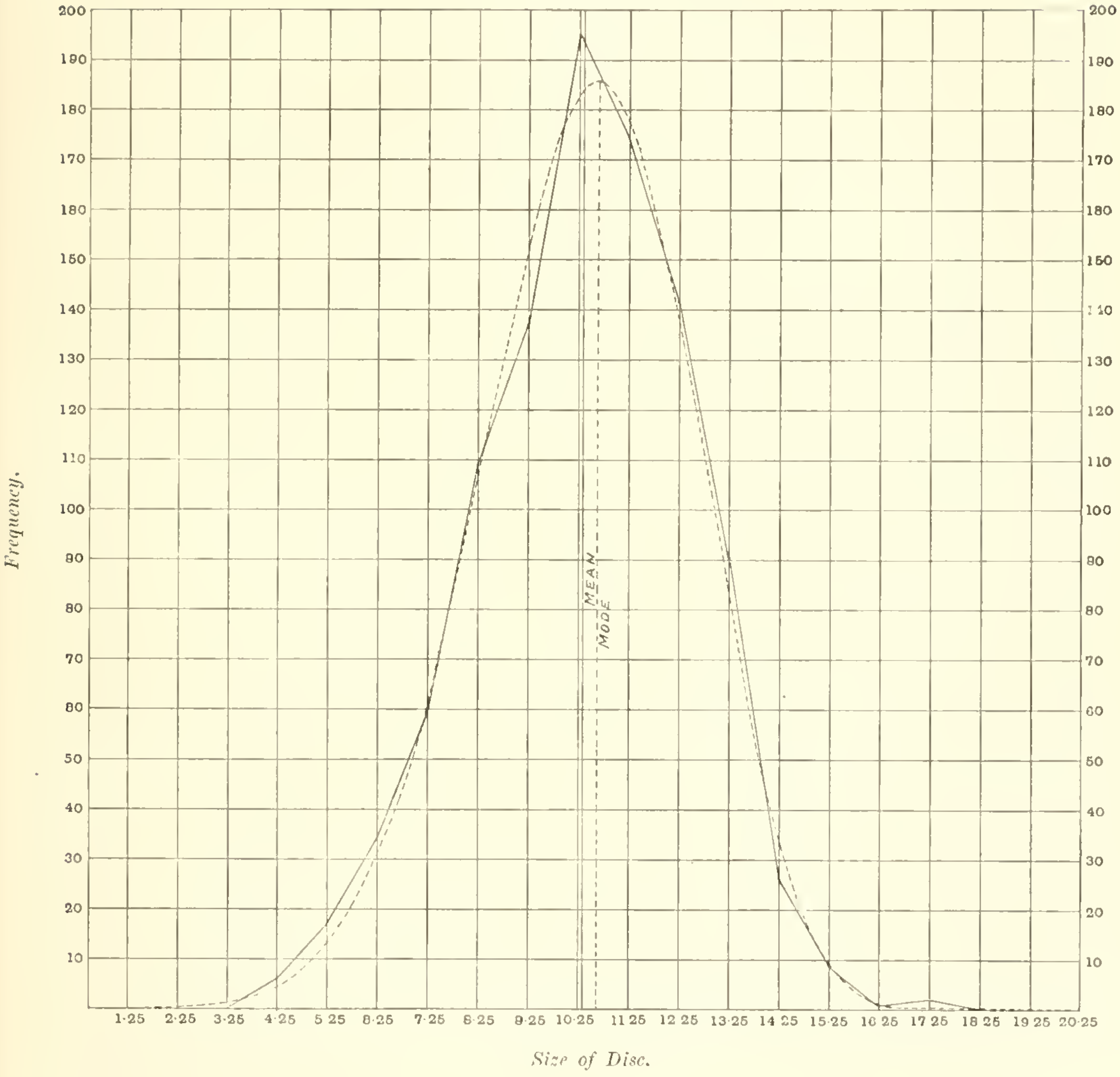
mean 10·357 mm. $\mu_2 = 4{\cdot}5222$
mode 10·628 mm. $\mu_3 = -2{\cdot}2366$
$\sigma = 2{\cdot}1266$ mm. $\mu_4 = 60{\cdot}2962$

Equation to the curve:

$$y = 185{\cdot}57 \left(1 - \frac{x}{9{\cdot}3264}\right)^{12{\cdot}6050} \times \left(1 + \frac{x}{20{\cdot}9913}\right)^{28{\cdot}3705}.$$

The origin of the curve is the mode, and the fit is a fairly good one.

Diagram II. Polygon of Frequency.



4. *As to the number of rays.*

Forbes, in discussing the arrangement of the parts of Echinoderms* in general, says: "The reigning number is five. The name of 'five-fingers,' commonly applied by mariners to the star-fishes, is founded on a popular recognition of the number regnant. It has long been noticed. Among the problems proposed by that true-spirited but eccentric philosopher, Sir Thomas Browne, is one, 'Why among sea-stars, Nature chiefly delighteth in five points?' And in his *Garden of Cyrus* he observes: 'By the same number (five) doth Nature divide the circle of the sea-star.'" Any deviation from this characteristic number is, he continues, "Always either in consequence of the abortion of certain organs, or it is a *variation by representation*, that is to say, by the assumption of the regnant number of another class"; and "in ophiuroidea† the number *five* is absolute. Colour and proportions are the subject of variation." It is now well known, however, that *five* is not by any means absolute, even as regards the Ophiuroidea, for there are many species whose rays habitually number more than five. Mr Bateson indicates the same thing in his *Materials*. Jeffrey Bell says‡: "At a very early stage in the evolution of the Echinoderma, the adult tended to acquire a marked radial symmetry; but, at first, the number of rays was by no means constantly five; now the symmetry is so generally quinqueradial that wonder is nearly always expressed when the number five is diminished or increased." In my investigations I found in the first thousand five specimens with six rays. (See Table IV. and Fig 7.) In the second thousand I found four specimens with six

TABLE IV.

Showing Correlation of Parts of Six-Rayed Specimens.

		Shape of Disc	Breadth of Disc	Length of Ray	Madreporic Plates
First 1000	*1*	Hexagonal	14	60	1
	2	Hexagonal	13	56	1
	3	Hexagonal	13·5	68	1
	4	Circular	15	70	2 adjacent
	5	Circular	13·5	61	1
Second 1000	*1*	Hexagonal	11·5	50	3 adjacent
	2	Hexagonal	9	38	1
	3	Hexagonal	10	All damaged	1
	4	Circular	11	All damaged	1
	5§	Square	8·5	46	2 adjacent
Third 1000	*1*	Hexagonal	13·5	Damaged	1
	2	Hexagonal	13	Damaged	1
	3	Hexagonal	10	Damaged	1

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* *A History of British Starfishes*, Introduction, pp. xv., xvi.

† *Op. cit.*, p. 20.

‡ *Catalogue of British Echinoderms*, Introductory Remarks.

§ Only four-rayed specimen found.

rays, and there was also a single specimen with four rays (see Fig. 8), perfect in every way. In the third thousand there were three specimens with six rays—all of which were, however, unfortunately considerably damaged. There is thus then less than one-half per cent. of deviation from the normal five rays. Are these deviations mentioned to be looked upon as* "abnormal variations," or merely as "very infrequent normal variations"?

5. *As to the number of madreporic plates.*

The madreporite, even in ophiuroids fresh from water or from alcohol, but especially in dry specimens, is very easily seen. Its position, shape, size, and colour readily assist one in finding it. Careful observation, even without a microscope, shows that it is perforated, but I do not think the perforations are nearly so numerous as seems to be generally supposed. Its colour, a delicate yellow, is distinctly lighter than that of the other plates in the angle between the rays. It is also larger and its heart-shape is characteristic. In my observations I found in all thirty-nine specimens which had two or more of the plates so like the madreporite that one could not tell the real madreporic plate. This peculiarity, I find, has been noted by Jeffrey Bell, who says that "in ophiuroids the stone-canal ends on one or several of the mouth shields." While it is true that there is normally but one madreporic plate, and it may be presumed there can be only one stone-canal, yet to all appearance more than one mouth-shield may be capable of performing the function of a madreporic plate. Table V. gives not only the number of specimens in the three thousand with more than one madreporic plate, but shows how the plates were situated in relation to each other. The relative percentages in the first, second, and third thousands are worthy of notice, there being on the average 0·13 specimens abnormal.

TABLE V.

Showing the Number and Relative Position of Madreporites.

Normal	Two adjacent	Two not adjacent	Three adjacent	Three not adjacent	Four	Five
988	4	6	—	—	1	1
987	4	4	—	5	—	—
986	8	2	1	2	1	—

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* *The Grammar of Science*, p. 390.

TABLES OF POWERS OF NATURAL NUMBERS AND OF THE SUMS OF POWERS OF THE NATURAL NUMBERS FROM 1—100.

By W. PALIN ELDERTON, *Actuary.*

In many mathematical investigations it is of great advantage to have a table of the lower powers of the natural numbers. This is provided in a handy form, as far as the second and third powers of the first 10,000 numbers are concerned, in *Barlow's Tables* (E. and F. N. Spon, London, *Stereotype Edition*). But for many purposes we require to go beyond the cubes. Accordingly a table has been prepared of the first seven powers of the natural numbers from 1 to 250, and the first instalment of it 1–100 is given in Table I. below. Only after the work was completed did I discover that in the *first edition* of Barlow, such a table was provided. My numbers were checked against his and found to agree. It was still considered, however, desirable to publish Table I. below, because the first edition of Barlow published in 1814 is a most scarce work, and copies can rarely be procured even at much enhanced price, after several years' hunting in second-hand catalogues.

With regard to the sums of powers table (Table II.) Professor Pearson has already published such a table from 1 to 20 with powers up to the 7th*. My table ranges from 1 to 100 for the same powers. The final values of the sums of powers table were checked against the sums of the powers of the first 100 natural numbers as obtained by the usual finite difference method and this was also a check on the whole of the powers table. Miss E. M. Elderton helped me in some of the work, which was done by the use of Crelle's Tables checked by an arithmometer. As is well known the sums of the powers of natural numbers can be of great service in least square investigations†, and it is hoped that the present tables will be found useful, not only to biometricians but to others who need to represent observations by parabolic curves.

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* *Biometrika*, Vol. II. p. 10.

† *Biometrika*, Vol. II. p. 9 *et seq.*

The following biometric illustrations are, of course, not exhaustive, but they may serve to indicate some cases wherein we have found a practical need for such tables as are now published.

Illustration I.

As example of the use of the Table I. of powers, we will find the coefficient of parental correlation between the sire and 2392 ♂ offspring in greyhounds. The categories selected were some black in coat colour and no black in coat colour. The following table resulted:

Sire.

Offspring.	Some Black	No Black	Totals
Some Black ...	1329	367	1696
No Black ...	316	380	696
Totals	1645	747	2392

The method of dealing with characters not quantitatively measurable was adopted* and by using Sheppard's tables† the following equation for the correlation coefficient r was found:

$$\begin{aligned}\phi(r) = {}& \cdot 002{,}726r^7 + \cdot 057{,}149r^6 + \cdot 017{,}192r^5 \\ & + \cdot 083{,}578r^4 + \cdot 088{,}331r^3 + \cdot 134{,}717r^2 \\ & + r - \cdot 560{,}386 = 0 \ldots\ldots\ldots\ldots\text{(i).}\end{aligned}$$

The derived function is:

$$\begin{aligned}\phi'(r) = {}& \cdot 019{,}082r^6 + \cdot 342{,}894r^5 + \cdot 085{,}960r^4 + \cdot 334{,}312r^3 \\ & + \cdot 264{,}993r^2 + \cdot 269{,}434r + 1 \ldots\ldots\ldots\text{(ii).}\end{aligned}$$

We can now find r by Newton's rule. We see from (i) that r is less than ·56 and the r^2 term being plus shows us that it must be less than ·52. Now take out the first seven powers of ·52 from Table I. and evaluate (i) by multiplication on an arithmometer, we find without clearing product figures from machine:

$$\phi(\cdot 52) = \cdot 016{,}384.$$

Similarly,

$$\phi'(\cdot 52) = 1 \cdot 278{,}466.$$

Hence

$$\phi(\cdot 52) / \phi'(\cdot 52) = \cdot 0128$$

and the next approximation to the root is

$$\cdot 52 - \cdot 0128 = \cdot 5072.$$

If we had kept powers of r up to the eighth instead of the seventh, the actual value of the root would be ·5070. We thus see that the above process gives the root true to three figures, amply sufficient for biometric purposes.

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* *Phil. Trans.*, Vol. 195 A, pp. 1—47.

† *Biometrika*, Vol. II. p. 182 *et seq.*

Illustration II.

To show the use of the second table we will find, by the method of least squares, a parabola of the third order fitting the following expectations of life which have been taken from the recent experience of female annuitants collected by the Institute and Faculty of Actuaries (C. and E. Layton, 1899).

Age	*20*	*25*	*30*	*35*	*40*	*45*	*50*	*55*	*60*	*65*	*70*	*75*	*80*	*85*	*90*	*95*	*100*
Expectation of Life	45·3	41·1	38·0	34·3	31·2	27·9	24·2	20·6	17·0	13·5	10·5	7·8	5·6	4·1	3·0	2·5	2·0

I took age 15 as origin and calculated the products of the expectations into the first three powers of the distances from this age ÷ 5 continuously and checked the work by multiplying the expectation by the appropriate value of n^3 from Table I.

Then since the type-equation for a_r is

$$a_0S_1^{17}x^r + a_1S_1^{17}x^{r+1} + a_2S_1^{17}x^{r+2} + a_3S_1^{17}x^{r+3} = S_1^{17}N_x x^r$$

we can write down the equations for finding the a's immediately by using Table II. with $n = 17$ and we have

$$\begin{aligned}
17a_0 + {} & 153a_1 + 1785a_2 + 23409a_3 = 328{\cdot}6\\
153a_0 + {} & 1785a_1 + 23409a_2 + 327369a_3 = 1782{\cdot}6\\
1785a_0 + {} & 23409a_1 + 327369a_2 + 4767633a_3 = 14152{\cdot}8\\
23409a_0 + {} & 327369a_1 + 4767633a_2 + 71397705a_3 = 139461{\cdot}0
\end{aligned}$$

The solution of these equations gives

$$\text{expectation of life} = 47{\cdot}46 - 2{\cdot}6013x - {\cdot}18435x^2 + {\cdot}01063x^3$$

and we obtain the following values agreeing very closely with the unadjusted expectations:

Age	*20*	*25*	*30*	*35*	*40*	*45*	*50*	*55*	*60*	*65*	*70*	*75*	*80*	*85*	*90*	*95*	*100*
Expectation	44·7	41·6	38·3	34·8	31·2	27·5	23·9	20·3	16·9	13·6	10·7	8·0	5·8	4·0	2·8	2·1	2·1

TABLE I.

Powers of Natural Numbers.

n	n^2	n^3	n^4	n^5	n^6	n^7	n
1	1	1	1	1	1	1	*1*
2	4	8	16	32	64	128	*2*
3	9	27	81	243	729	2187	*3*
4	16	64	256	1024	4096	16384	*4*
5	25	125	625	3125	15625	78125	*5*
6	36	216	1296	7776	46656	279936	*6*
7	49	343	2401	16807	117649	823543	*7*
8	64	512	4096	32768	262144	2097152	*8*
9	81	729	6561	59049	531441	4782969	*9*
10	100	1000	10000	100000	1000000	10000000	*10*
11	121	1331	14641	161051	1771561	19487171	*11*
12	144	1728	20736	248832	2985984	35831808	*12*
13	169	2197	28561	371293	4826809	62748517	*13*
14	196	2744	38416	537824	7529536	105413504	*14*
15	225	3375	50625	759375	11390625	170859375	*15*
16	256	4096	65536	1048576	16777216	268435456	*16*
17	289	4913	83521	1419857	24137569	410338673	*17*
18	324	5832	104976	1889568	34012224	612220032	*18*
19	361	6859	130321	2476099	47045881	893871739	*19*
20	400	8000	160000	3200000	64000000	1280000000	*20*
21	441	9261	194481	4084101	85766121	1801088541	*21*
22	484	10648	234256	5153632	113379904	2494357888	*22*
23	529	12167	279841	6436343	148035889	3404825447	*23*
24	576	13824	331776	7962624	191102976	4586471424	*24*
25	625	15625	390625	9765625	244140625	6103515625	*25*
26	676	17576	456976	11881376	308915776	8031810176	*26*
27	729	19683	531441	14348907	387420489	10460353203	*27*
28	784	21952	614656	17210368	481890304	13492928512	*28*
29	841	24389	707281	20511149	594823321	17249876309	*29*
30	900	27000	810000	24300000	729000000	21870000000	*30*
31	961	29791	923521	28629151	887503681	27512614111	*31*
32	1024	32768	1048576	33554432	1073741824	34359738368	*32*
33	1089	35937	1185921	39135393	1291467969	42618442977	*33*
34	1156	39304	1336336	45435424	1544804416	52523350144	*34*
35	1225	42875	1500625	52521875	1838265625	64339296875	*35*
36	1296	46656	1679616	60466176	2176782336	78364164096	*36*
37	1369	50653	1874161	69343957	2565726409	94931877133	*37*
38	1444	54872	2085136	79235168	3010936384	114415582592	*38*
39	1521	59319	2313441	90224199	3518743761	137231006679	*39*
40	1600	64000	2560000	102400000	4096000000	163840000000	*40*
41	1681	68921	2825761	115856201	4750104241	194754273881	*41*
42	1764	74088	3111696	130691232	5489031744	230539333248	*42*
43	1849	79507	3418801	147008443	6321363049	271818611107	*43*
44	1936	85184	3748096	164916224	7256313856	319277809664	*44*
45	2025	91125	4100625	184528125	8303765625	373669453125	*45*
46	2116	97336	4477456	205962976	9474296896	435817657216	*46*
47	2209	103823	4879681	229345007	10779215329	506623120463	*47*
48	2304	110592	5308416	254803968	12230590464	587068342272	*48*
49	2401	117649	5764801	282475249	13841287201	678223072849	*49*
50	2500	125000	6250000	312500000	15625000000	781250000000	*50*

TABLE I.—(*continued*).

Powers of Natural Numbers.

n	n^2	n^3	n^4	n^5	n^6	n^7	n
51	2601	132651	6765201	345025251	17596287801	897410677851	*51*
52	2704	140608	7311616	380204032	19770609664	1028071702528	*52*
53	2809	148877	7890481	418195493	22164361129	1174711139837	*53*
54	2916	157464	8503056	459165024	24794911296	1338925209984	*54*
55	3025	166375	9150625	503284375	27680640625	1522435234375	*55*
56	3136	175616	9834496	550731776	30840979456	1727094849536	*56*
57	3249	185193	10556001	601692057	34296447249	1954897493193	*57*
58	3364	195112	11316496	656356768	38068692544	2207984167552	*58*
59	3481	205379	12117361	714924299	42180533641	2488651484819	*59*
60	3600	216000	12960000	777600000	46656000000	2799360000000	*60*
61	3721	226981	13845841	844596301	51520374361	3142742836021	*61*
62	3844	238328	14776336	916132832	56800235584	3521614606208	*62*
63	3969	250047	15752961	992436543	62523502209	3938980639167	*63*
64	4096	262144	16777216	1073741824	68719476736	4398046511104	*64*
65	4225	274625	17850625	1160290625	75418890625	4902227890625	*65*
66	4356	287496	18974736	1252332576	82653950016	5455160701056	*66*
67	4489	300763	20151121	1350125107	90458382169	6060711605323	*67*
68	4624	314432	21381376	1453933568	98867482624	6722988818432	*68*
69	4761	328509	22667121	1564031349	107918163081	7446353252589	*69*
70	4900	343000	24010000	1680700000	117649000000	8235430000000	*70*
71	5041	357911	25411681	1804229351	128100283921	9095120158391	*71*
72	5184	373248	26873856	1934917632	139314069504	10030613004288	*72*
73	5329	389017	28398241	2073071593	151334226289	11047398519097	*73*
74	5476	405224	29986576	2219006624	164206490176	12151280273024	*74*
75	5625	421875	31640625	2373046875	177978515625	13348388671875	*75*
76	5776	438976	33362176	2535525376	192699928576	14645194571776	*76*
77	5929	456533	35153041	2706784157	208422380089	16048523266853	*77*
78	6084	474552	37015056	2887174368	225199600704	17565568854912	*78*
79	6241	493039	38950081	3077056399	243087455521	19203908986159	*79*
80	6400	512000	40960000	3276800000	262144000000	20971520000000	*80*
81	6561	531441	43046721	3486784401	282429536481	22876792454961	*81*
82	6724	551368	45212176	3707398432	304006671424	24928547056768	*82*
83	6889	571787	47458321	3939040643	326940373369	27136050989627	*83*
84	7056	592704	49787136	4182119424	351298031616	29509034655744	*84*
85	7225	614125	52200625	4437053125	377149515625	32057708828125	*85*
86	7396	636056	54700816	4704270176	404567235136	34792782221696	*86*
87	7569	658503	57289761	4984209207	433626201009	37725479487783	*87*
88	7744	681472	59969536	5277319168	464404086784	40867559636992	*88*
89	7921	704969	62742241	5584059449	496981290961	44231334895529	*89*
90	8100	729000	65610000	5904900000	531441000000	47829690000000	*90*
91	8281	753571	68574961	6240321451	567869252041	51676101935731	*91*
92	8464	778688	71639296	6590815232	606355001344	55784660123648	*92*
93	8649	804357	74805201	6956883693	646990183449	60170087060757	*93*
94	8836	830584	78074896	7339040224	689869781056	64847759419264	*94*
95	9025	857375	81450625	7737809375	735091890625	69833729609375	*95*
96	9216	884736	84934656	8153726976	782757789696	75144747810816	*96*
97	9409	912673	88529281	8587340257	832972004929	80798284478113	*97*
98	9604	941192	92236816	9039207968	885842380864	86812553324672	*98*
99	9801	970299	96059601	9509900499	941480149401	93206534790699	*99*
100	10000	1000000	100000000	10000000000	1000000000000	100000000000000	*100*

TABLE II.

Sums of Powers of Natural Numbers.

n	$S(n)$	$S(n^2)$	$S(n^3)$	$S(n^4)$	$S(n^5)$	$S(n^6)$	$S(n^7)$	n
1	1	1	1	1	1	1	1	*1*
2	3	5	9	17	33	65	129	*2*
3	6	14	36	98	276	794	2316	*3*
4	10	30	100	354	1300	4890	18700	*4*
5	15	55	225	979	4425	20515	96825	*5*
6	21	91	441	2275	12201	67171	376761	*6*
7	28	140	784	4676	29008	184820	1200304	*7*
8	36	204	1296	8772	61776	446964	3297456	*8*
9	45	285	2025	15333	120825	978405	8080425	*9*
10	55	385	3025	25333	220825	1978405	18080425	*10*
11	66	506	4356	39974	381876	3749966	37567596	*11*
12	78	650	6084	60710	630708	6735950	73399404	*12*
13	91	819	8281	89271	1002001	11562759	136147921	*13*
14	105	1015	11025	127687	1539825	19092295	241561425	*14*
15	120	1240	14400	178312	2299200	30482920	412420800	*15*
16	136	1496	18496	243848	3347776	47260136	680856256	*16*
17	153	1785	23409	327369	4767633	71397705	1091194929	*17*
18	171	2109	29241	432345	6657201	105409929	1703414961	*18*
19	190	2470	36100	562666	9133300	152455810	2597286700	*19*
20	210	2870	44100	722666	12333300	216455810	3877286700	*20*
21	231	3311	53361	917147	16417401	302221931	5678375241	*21*
22	253	3795	64009	1151403	21571033	415601835	8172733129	*22*
23	276	4324	76176	1431244	28007376	563637724	11577558576	*23*
24	300	4900	90000	1763020	35970000	754740700	16164030000	*24*
25	325	5525	105625	2153645	45735625	998881325	22267545625	*25*
26	351	6201	123201	2610621	57617001	1307797101	30299355801	*26*
27	378	6930	142884	3142062	71965908	1695217590	40759709004	*27*
28	406	7714	164836	3756718	89176276	2177107894	54252637516	*28*
29	435	8555	189225	4463999	109687425	2771931215	71502513825	*29*
30	465	9455	216225	5273999	133987425	3500931215	93372513825	*30*
31	496	10416	246016	6197520	162616576	4388434896	120885127936	*31*
32	528	11440	278784	7246096	196171008	5462176720	155244866304	*32*
33	561	12529	314721	8432017	135306401	6753644689	197863309281	*33*
34	595	13685	354025	9768353	180741825	8298449105	250386659425	*34*
35	630	14910	396900	11268978	333263700	10136714730	314725956300	*35*
36	666	16206	443556	12948594	393729876	12313497066	393090120396	*36*
37	703	17575	494209	14822755	463073833	14879223475	488021997529	*37*
38	741	19019	549081	16907891	542309001	17890159859	602437580121	*38*
39	780	20540	608400	19221332	632533200	21408903620	739668586800	*39*
40	820	22140	672400	21781332	734933200	25504903620	903508586800	*40*
41	861	23821	741321	24607093	850789401	30255007861	1098262860681	*41*
42	903	25585	815409	27718789	981480633	35744039605	1328802193929	*42*
43	946	27434	894916	31137590	1128489076	42065402654	1600620805036	*43*
44	990	29370	980100	34885686	1293405300	49324716510	1919898614700	*44*
45	1035	31395	1071225	38986311	1477933425	57625482135	2293568067825	*45*
46	1081	33511	1168561	43463767	1683896401	67099779031	2729385725041	*46*
47	1128	35720	1272384	48343148	1913241408	77878994360	3236008845504	*47*
48	1176	38024	1382976	53651864	2168045376	90109584824	3823077187776	*48*
49	1225	40425	1500625	59416665	2450520625	103950872025	4501300260625	*49*
50	1275	42925	1625625	65666665	2763020625	119575872025	5282550260625	*50*

TABLE II.—(*continued*).

Sums of Powers of Natural Numbers.

n	$S(n)$	$S(n^2)$	$S(n^3)$	$S(n^4)$	$S(n^5)$	$S(n^6)$	$S(n^7)$	*n*
51	1326	45526	1758276	72431866	3108045876	137172159826	6179960938476	*51*
52	1378	48230	1898884	79743482	3488249908	156942769490	7208032641004	*52*
53	1431	51039	2047761	87633963	3906445401	179107130619	8382743780841	*53*
54	1485	53955	2205225	96137019	4365610425	203902041915	9721668990825	*54*
55	1540	56980	2371600	105287644	4868894800	231582682540	11244104225200	*55*
56	1596	60116	2547216	115122140	5419626576	262423661996	12971199074736	*56*
57	1653	63365	2732409	125678141	6021318633	296720109245	14926096567929	*57*
58	1711	66729	2927521	136994637	6677675401	334788801789	17134080735481	*58*
59	1770	70210	3132900	149114998	7392599700	376969335430	19622732220300	*59*
60	1830	73810	3348900	162074998	8170199700	423625335430	22422092220300	*60*
61	1891	77531	3575881	175917839	9014796001	475145709791	25564835056321	*61*
62	1953	81375	3814209	190694175	9930928833	531945945375	29086449662529	*62*
63	2016	85344	4064256	206447136	10923365376	594469447584	33025430301696	*63*
64	2080	89440	4326400	223224352	11997107200	663188924320	37423476812800	*64*
65	2145	93665	4601025	241074977	13157397825	738607814945	42325794703425	*65*
66	2211	98021	4888521	260049713	14409730401	821261764961	47780865404481	*66*
67	2278	102510	5189284	280200834	15759855508	911720147130	53841577009804	*67*
68	2346	107134	5503716	301582210	17213789076	1010587629751	60564565828236	*68*
69	2415	111895	5832225	324249331	18777820425	1118505792835	68010919080825	*69*
70	2485	116795	6175225	348259331	20458520425	1236154792835	76246349080825	*70*
71	2556	121836	6533136	373671012	22262749776	1364255076756	85341469239216	*71*
72	2628	127020	6906384	400544868	24197667408	1503569146260	95372082243504	*72*
73	2701	132349	7295401	428943109	26270739001	1654903372549	106419480762601	*73*
74	2775	137825	7700625	458929685	28489745625	1819109862725	118570761035625	*74*
75	2850	143450	8122500	490570310	30862792500	1997088378350	131919149707500	*75*
76	2926	149226	8561476	523932486	33398317876	2189788306926	146564344279276	*76*
77	3003	155155	9018009	559085527	36105102033	2398210687015	162612867546129	*77*
78	3081	161239	9492561	596100583	38992276401	2623410287719	180178436401041	*78*
79	3160	167480	9985600	635050664	42069332800	2866497743240	199382345387200	*79*
80	3240	173880	10497600	676010664	45346132800	3128641743240	220353865387200	*80*
81	3321	180441	11029041	719057385	48832917201	3411071279721	243230657842161	*81*
82	3403	187165	11580409	764269561	52540315633	3715077951145	268159204898929	*82*
83	3486	194054	12152196	811727882	56479356276	4042018324514	295295255888556	*83*
84	3570	201110	12744900	861515018	60661475700	4393316356130	324804290544300	*84*
85	3655	208335	13359025	913715643	65098528825	4770465871755	356861999372425	*85*
86	3741	215731	13995081	968416459	69802799001	5175033106891	391654781594121	*86*
87	3828	223300	14653584	1025706220	74787008208	5608659307900	429380261081904	*87*
88	3916	231044	15335056	1085675756	80064327376	6073063394684	470247820718896	*88*
89	4005	238965	16040025	1148417997	85648386825	6570014685645	514179155614425	*89*
90	4095	247065	16769025	1214027997	91553286825	7101485685645	562308845614425	*90*
91	4186	255346	17522596	1282602958	97793608276	7669354937686	613984947550156	*91*
92	4278	263810	18301284	1354242254	104384423508	8275709939030	669769607673804	*92*
93	4371	272459	19105641	1429017455	111341307201	8922700122479	729939694734561	*93*
94	4465	281295	19936225	1507122351	118680347425	9612569903535	794787454153825	*94*
95	4560	290320	20793600	1588572976	126418156800	10347661794160	864621183763200	*95*
96	4656	299536	21678336	1673507632	134571883776	11130419583856	939765931574016	*96*
97	4753	308945	22591009	1762036913	143159224033	11963391588785	1020564216052129	*97*
98	4851	318549	23532201	1854273729	152198432001	12849233969649	1107376769376801	*98*
99	4950	328350	24502500	1950333330	161708332500	13790714119050	1200583304167500	*99*
100	5050	338350	25502500	2050333330	171708332500	14790714119050	1300583304167500	*100*

ASSORTATIVE MATING IN MAN.

A COOPERATIVE STUDY.

1. In considering the reproduction and progressive change of a population the part played by any form of differential mating within the population must be given its due weight. Darwin has given the name of sexual selection to the general conception of differential mating, but it seems necessary now to distinguish between various types of differential mating. As opposed to pure random mating within the population*, we have first *preferential mating*, in which male or female classes with certain values of a character find it less easy to mate than other classes with different values. Secondly we have *assortative mating* in which, while all classes of males and females find mates, certain classes of males appear to be attracted to certain classes of females. If the male class of a given character tends to mate with a female class with generally like character, we have a tendency to *homogamy*. Homogamy as one type of assortative mating is simply measured by the correlation between the two characters in the male and female of the pair. The influence of homogamy on the character of successive generations of a population may be very great indeed, and the whole range of effect from pure random matings to perfectly homogamous unions within a population is almost but not quite as important as the difference between self and cross fertilisation in plants. It has the distinctive features as compared with self-fertilisation, that (i) it may have any degree of intensity, (ii) it may be confined to special characters, and (iii) it is not complicated by any of the supposed harmful effects of in-breeding.

Theoretically the effect of assortative mating on offspring—especially its influence on the segregation of types—is very great, and it becomes of fundamental importance to ascertain how far it exists in actual populations, which are often *à priori* assumed to mate at random. The present cooperative study forms an attempt to measure the extent of assortative mating in man, by, not so much a single physical character, as the resulting effect of a complex of physical characters. It has been suggested that the duration of life is to some extent a rough measure of the general physical fitness to environment of an individual.

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* By *random mating* we understand theoretically a state of affairs in which every type of male would be mated with every type of female, the random mating being with regard to one, two, three, many or all characters.

In the present study we endeavour to measure how far there is an assortative mating between husband and wife with regard to their durations of life, or rather with regard to the general physical characters on which this duration depends. It has been shewn in *Biometrika*[*] that duration of life is an inherited character, and hence assortative mating in respect to this character may probably be looked upon as the mating of like stocks.

2. Our knowledge of assortative mating in man may be summed up as follows. It was first noted when dealing with Francis Galton's Family Records. K. Pearson here found a correlation of ·09 for stature[†] and ·10 for eye colour[‡]. In Pearson's more extensive series of Family Measurements, the following values were found[§]:

Assortative Mating in Man: 1000 *Cases.*

Stature	·2804
Span	·1989
Left Forearm	·1977
Mean Value	·2257

Further, values between ·14 and ·20 were found for cross assortative mating, *e.g.* between stature in husband and forearm in wife.

Now this very sensible value of ·226 is either due to, (i) a real conscious or unconscious assortative mating in man, or (ii) to individual men and women mating within sub-races, local races or other limited classes.

Now the influence of (ii) can be more or less effectively determined in our present investigation by taking data within (*a*) a fairly limited class and (*b*) within limited districts. If any small local differences produce the observed result then we ought to find correlation sensibly as large as the assortative mating correlation, when we make random couples of men and women from the same locality and put several localities into the same table.

In collecting our material we were accordingly guided by the desire to get a fairly narrow district, and one if possible where there would be no unintentional selection of husbands and wives living approximately to the same age. By using pedigree tables we can avoid any selection of this kind, if both husband and wife are given, but in many pedigrees the ages of the men only are given. The records of the Society of Friends are an exception to this rule, and that Society forms a fairly homogeneous sub-class of the community: indeed so compact that attention has more than once been drawn to the differentiation of its actuarial constants from those of the community at large.

Data from pedigrees being limited owing to the too frequent absence of the dates of birth and death of the wives, we not unnaturally turned to the records provided by tombstones. In collecting such material, it is essential to work on limited districts with a very stable population. For in a large urban population, such as is represented by the modern urban cemeteries, not only will a large mixture of local races due to immigration be found, but the population being very fluctuating, it is extremely likely that husband and wife will not be interred in the same cemetery, unless they die within a comparatively short period of each other, *i.e.* unless their ages at death are, in the modal cases of small difference of age, much alike. In most rural districts on the other hand with a stable population, there is a very strong feeling—amounting in the case of the Yorkshire Dales almost to a superstition—that husband and wife must share the same grave. To test the amount of this spurious assortative mating introduced by special urban conditions, we have collected a series of data from the cemeteries and graveyards on the north of London. Whereas the Records of the Society of Friends,—where every pair is given *without selection*—agree admirably with the two rural districts under investigation, the London results practically double the value of the apparent assortative mating. The agreement between the pedigree and rural graveyard results seems to indicate that the rural data are little influenced by any transitory character in the population. It is just possible that the slight increase of correlation as we pass from the Society of Friends Records to the very stable Yorkshire Dale population* and again to rural Oxfordshire may be really due to the source we find so markedly at work in the London series. The results given on p. 488 should be compared from this standpoint. All cases of second and third wives or husbands have been omitted from the tabulation. The proportion of cases in which a woman marries more than once is shewn by the records of the Society of Friends to be small, actually less than one per cent., so that the small fraction of these cases in the churchyard records, which may have escaped notice owing to the woman's change of name, cannot sensibly have affected our results. In Wensleydale at least it is very usual to find the names of both husbands in such cases given on the woman's tombstone.

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[*] Vol. I. p. 50. M. Beeton and K. Pearson: *On the Inheritance of the Duration of Life*, etc.

[†] *Phil. Trans.*, Vol. 187 A, p. 273.

[‡] *Phil. Trans.*, Vol. 195 A, p. 113.

[§] *Biometrika*, Vol. II., p. 373.

3. The following is the material on which our results are based:

(*a*) *Yorkshire Dale Records.* These comprise tombstone records from Wensleydale, Wharfedale and adjacent dales. Among the places included were Hawes, Askrigg, Aysgarth, West Witton, Wensley, Middleham, Redmire, Muker, Arncliffe in Littondale, Kettlewell, Hubberholme, and Bolton Abbey. The great bulk of the tombstone entries in these parish churches refer to the farming population, the labouring class more rarely having stones, and the commercial class being a small element and in itself largely recruited from the farming class. The data were collected by C. D. Fawcett, A. Lee, W. F. R. Weldon and K. Pearson. The work of tabulation and calculation is due to F. E. Lutz and A. Lee.

(*b*) *Oxford Rural Records.* The country immediately round Oxford is studded with small villages, each with its own churchyard; the records are drawn from these, and the size of most such churchyards is sufficiently indicated by the fact that 37 of them yielded only 890 legible inscriptions of the kind required. The only town of any size included is Bicester; in this place the churchyard is sharply divided into an older portion, containing graves made before 1850, and a newer portion: the older portion only, as unlikely to be influenced by modern urban conditions, was tabulated.

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* In many of the Yorkshire Dales it is possible to trace even the majority of the families back—many in the same houses—for several hundred years. A study of the church records shows a similar persistence in the labouring as well as in the farming or yeoman classes.

The mass of the records relates undoubtedly to the farming population, including a considerable proportion of the labouring class, who obtain stone tombstones more easily in Oxfordshire, where building stone is cheap and easily cut, than in many places. The proportion of resident gentry, of clergy, and of professional men, is greater than in the Yorkshire records. The data were collected by F. Buchanan and W. F. R. Weldon, tabulated and reduced by K. Pearson, E. H. J. Schuster, and W. F. R. Weldon.

(*c*) *Records of the Society of Friends.* Collected by Mary Beeton from pedigrees, chiefly printed books at Devonshire House*. The material was tabulated and reduced by M. Beeton herself. While locally far more scattered than the material considered in (*a*) and (*b*), the Society of Friends forms a remarkably homogeneous body. Special features differentiating these data from the above rural district data are the far lower average age at death of both husband and wife, and, what does not appear in the results communicated in this paper, the occurrence with considerable frequency of remarkably early marriages.

(*d*) *London Cemeteries.* Data collected by Mary Beeton from St Mary-le-bone Cemetery, Highgate Cemetery, Hampstead Cemetery, and St John's Graveyard, Hampstead. The tabulation and reduction are due to M. Beeton.

(4) The above data cover a long range of years, both for husbands and wives, the lowest age at death being 18 years, the highest 107 years; these ages are subject to some uncertainty, because a man recorded as dying "in the 90th year of his age" may have died at any time between his 89th and 90th birthdays, and a man dying "aged 90" may have died at any time between his 90th and his 91st birthdays; further, it is by no means certain that the distinction between these two forms of statement is rigidly observed. In order to employ a unit of age which should not be affected by this uncertainty, and at the same time to lessen the work of tabulation, a five year period was adopted as the unit of age, both for husbands and for wives. The first question which arises is the effect, which may have been produced on the data, by the use of so large a unit; this was tested by F. E. Lutz, who determined the mean age at death and the standard deviation for husbands and wives from the Wensleydale district and the correlation between them, using first five years and then ten years as the unit age; using Sheppard's corrections for the moment formulae† he obtained the results given below; these

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* See *R. S. Proc.* Vol. 67, p. 160. We desire again to express our thanks to Mr Isaac Sharpe for is courteous assistance.

† See *Biometrika*, Vol. I. p. 273.

show so close an agreement as to justify the belief that the system of grouping adopted is within wide limits immaterial:

	Five Year Unit		Ten Year Unit	
	Husbands	Wives	Husbands	Wives
Mean Age at Death ...	68·339	66·844	68·322	66·900
σ	13·965	16·396	13·887	16·222
r	0·2200 ± 0·0217		0·2241 ± 0·0216	

(5) Admitting that the units of age adopted have no sensible effect on the result, the correlation observed may be due to small environmental influences, affecting the group of persons from one parish in a district differently from those in other parishes. If such an effect existed, the mixture of records from different parishes would exhibit a spurious correlation, due to the heterogeneity of the material used. Such a spurious correlation would be independent of any real correlation between actual husbands and wives, and if it existed it would be equally apparent if the men and women from each parish were arranged in couples at random, the couples so formed for every parish in the district being then added together; accordingly F. E. Lutz took the records from each graveyard in the Wensleydale district, and arranged them in a series, after which a random couple was formed by taking with each man the wife of the man ten places removed from him in the series; this process being gone through for every graveyard in the district, the random couples formed were added together and arranged in Table II, from which it is found that the correlation between the age at death of one member of such a random couple and that of the other is 0·0086 ± 0·0228, a value differing from zero by less than half the probable error of the determination. This is sufficient proof that the observed correlation between actual husbands and wives is not due to environmental effects which act differently upon the married population of the different villages in the district.

(6) A further environmental effect remains to be considered: a husband and a wife live to a large extent in a similar environment, peculiar to themselves; they have on the whole the same nourishment, they live under similar sanitary conditions, and are probably subject to more nearly identical risk of zymotic disease than any random couple of individuals; lastly, the death of husband or wife is a shock affecting the survivor more intensely than anyone else, and often causing a sudden change in financial and other environmental conditions. All these circumstances, resulting directly from marriage, may be considered likely to produce a correlation between the age at death of husband and wife. This possibility was

tested by examination of the Oxford records. For every married pair the following data were calculated: (1) the excess of husband's age over wife's age at marriage, "the husband's seniority"; this quantity, called Δ, was tabulated in one year units; it is called positive when the husband is older than the wife, negative when the wife is the older; (2) the number of years during which the husband survives the wife; this quantity, called Δ', is clearly positive if the wife dies first, negative if the husband dies first. Calling H the length of husband's life, W that of the wife's life, we have as necessary relation between these four quantities

$$H = \Delta + W + \Delta'.$$

The mean value of Δ' was found to be $-1{\cdot}986$ years, showing that the wife in the Oxford district dies on an average 1·986 years after the death of her husband*. The environmental effect to be measured must, if it exists, be measured by its effect upon the coefficient of correlation between Δ' and the length of the wife's (or husband's) life. Taking the correlation between the age of the wife at death and the magnitude of Δ', the value found was

$$r_{\Delta' W} = -0{\cdot}6215 \pm 0{\cdot}0139:$$

the negative sign indicating that Δ' increases negatively with increased age of wife, or the older wives survive their husbands for a longer period than the younger, as it is, of course, obviously likely that they should do. The chance that a wife will survive her husband depends upon many things too complex for analysis: but one factor, which the data enables us to measure, is the difference between her age and that of her husband at marriage, or the value of Δ; if we consider two women of the same age, one marrying a man a year older than herself, while the other marries a man twenty years her senior, clearly one will be much more likely to survive her husband than the other. If, however, we consider the array of husbands and wives, including all those pairs for which Δ has a constant value, one factor, i.e. the expectation of the wife's survival, will be constant; and if within each such array the shock or the change in environment due to the death of husband or wife produce any sensible effect on the duration of life of the survivor, this is something peculiar to each pair within the array; accordingly, if we take all the husbands and wives in each such array, and form couples of men and women by random selection from among them, the relation between the death of one member of such a couple, and that of the other, will be independent of any environmental influences resulting from actual married life. If the Oxford records be treated in this way, and the value of Δ' computed for every random couple, the correlation between values of Δ' and the length of life of the woman placed in the couple may be determined; the difference between the value of r so obtained, and that obtained from a study of actual husbands and wives, will be a measure of the environmental effects referred to. The result of such a determination shows, however, hardly any significant difference between the two correlations; we have for $r_{\Delta W}$

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* The standard deviation of Δ' is 16·550 years. For Wensleydale it is 17·583, or there is greater variability in the survival period.

$$\text{In married pairs} \quad r_{\Delta W} = -0{\cdot}6215 \pm 0{\cdot}0139$$

$$\text{In random couples} \quad r_{\Delta W} = -0{\cdot}6576 \pm 0{\cdot}0128$$

the difference between the two being practically negligible.

It appears, then, that the correlation between the length of life in husband and in wife is not sensibly due to any of the possible environmental effects considered. It must therefore be either a real phenomenon, or a result of some selection in the records from which the data were obtained, a possibility which has already been dealt with and dismissed.

(7) We are thus forced to conclude that the correlation actually observed between the length of life in two married persons is a measure of a real tendency towards homogamy, comparable with other cases of assortative mating. Collecting the results obtained, we find the following values.

Correlation between Length of Life in Husband and Wife.

Wensleydale and District ...	0·2200 ± 0·0244
Oxfordshire	0·2500 ± 0·0211
Society of Friends	0·1999 ± 0·0212
Mean	0·2233

These values never differ by twice the probable error of their differences, and they are in remarkably good agreement with Pearson's value for the correlation of other physical characters in husband and in wife, the mean in which is 0·2257, thus affording further evidence for the propositions (1) that a very sensible amount of homogamy does occur in man, and (2) that length of life is a character which is subject to selection, as it has already been shown to be capable of hereditary transmission*. Both these propositions are clearly of great importance to the practical student of human evolution.

We thus reach the conception that husband and wife are as much alike as uncle and niece, and probably as much alike as, if not more alike than, first cousins; this is not only true for definite physical organs like stature and forearm, but also for the general physical constitution. Such a degree of resemblance is one which could certainly not have been anticipated, and which may even appear paradoxical to many.

The London cemeteries present us with an even higher degree of correlation between the durations of life of husband and wife, namely 0·4204 ± ·0176. But we believe, as we have indicated above, that this apparent doubling of the assortative mating is solely due to the transitory nature of the population,—a spurious correlation produced by husband and wife being less frequently interred in the same grave when they die at a long interval apart.

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* M. Beeton and K. Pearson: *Roy. Soc. Proc.*, Vol. 65, pp. 290—305.

(8) For only a certain portion of the Yorkshire material 662 out of the 876 pairs had Δ, the interval between the births of husband and wife, been recorded.

Comparing the Yorkshire and Oxfordshire data we have in years:

	Yorkshire		Oxfordshire	
	Mean	S.D.	Mean	S.D.
Δ	3·680	8·272	2·812	6·930
Δ'	− 2·230	17·583	− 1·986	16·550

Thus the north countryman has on the average about ·9 years more seniority over his wife than the Oxfordshire man. His wife might therefore be expected to survive him by a period longer by ·9 years than in Oxfordshire, but the difference in this respect is only ·24 years.

We place together here our whole data for length of life and its variability in the four series, the year being unit:

	Yorkshire				Oxfordshire		Friends' Records		London	
	876 Cases		662 Cases		890 Cases		935 Cases		1000 Cases	
	♂	♀	♂	♀	♂	♀	♂	♀	♂	♀
Mean ...	68·329	66·844	68·716	67·266	68·461	67·635	66·508	62·578	67·090	66·250
Standard Deviation	12·965	16·396	13·903	15·639	12·698	14·668	15·262	19·378	13·136	14·057

We see from this table that:

(*a*) The rural married population has sensibly longer duration of life than the London series, or than the Friends' series, which undoubtedly contains many urban dwellers.

(*b*) The male is in every series longer lived than the female.

This is a result of very considerable interest, for in the country at large, and even in selected series, the duration of life of the female is greater than that of the male*. We are therefore compelled to conclude that the advantage of the woman lies in the unmarried members of the community. The married woman lives longer than the unmarried woman as the married man lives longer than the unmarried man. But the life of the single woman is so much better than that of the single man, that taking the whole population it more than compensates for the fact that the married man lives longer than the married woman.

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* *Biometrika*, Vol. I. pp. 53 and 60.

(*c*) In all cases in the above table, and in fact in all the material we have yet examined for adults, duration of life is a more variable character in the woman than in the man.

(*d*) The raising of the mean lengths of life and the lowering of the variabilities when we take 662 out of the 876 Yorkshire cases, although slight is significant, and of some little interest, for it corresponds practically to a separation of the Wensleydale and Wharfedale data. It would seem to indicate that in the two adjacent dales there may really exist a slight differentiation in this character due either to a difference of race or to a difference of environment*.

(9) We have compared the Yorkshire and Oxfordshire series, namely in the relationship of the lengths of life of husband and wife (H and W) with the husband's seniority (Δ) at marriage and his survival period (Δ').

Correlation	Wensleydale	Oxfordshire
$r_{H\Delta}$	$0·2029 \pm 0·0251$	$0·1246 \pm 0·0223$
$r_{W\Delta}$	$-0·1450 \pm 0·0257$	$-0·1743 \pm 0·0219$
$r_{W\Delta'}$	$-0·6158 \pm 0·0163$	$-0·6215 \pm 0·0139$
$\rho_{H\Delta}$	$0·2518$	$0·1764$
$\rho_{W\Delta}$	$-0·2091$	$-0·2139$
$r_{\Delta\Delta'}$	$-0·1810 \pm 0·0254$	$-0·1687 \pm 0·0220$

This table shows at once a remarkably close agreement between the Oxfordshire and Wensleydale series for the correlations first between wife's length of life and husband's years of survival ($r_{W\Delta'}$) and secondly between husband's seniority and his years of survival ($r_{\Delta\Delta'}$). Within the limits of the numbers dealt with these correlations are identical for the two series, and are probably significant for the whole country. The most noteworthy feature is the comparatively small correlationship between the husband's seniority and the wife's survival period ($-\Delta'$). Looked at from the standpoint of regression we may say that in Wensleydale for every year that the husband exceeds the mean seniority of 3·680, the wife exceeds her mean survival period of 2·230 by ·385 years: while in Oxfordshire for every year that the husband exceeds the mean seniority of 2·812, the wife exceeds her mean survival period of 1·986 by ·403 years. This regression, sensibly the same for the two districts, is perhaps less than we might have anticipated; an increase of a year's seniority in the husband being accompanied by less than half a year's longer survival on the part of the wife.

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* Wensleydale runs roughly east and west, Wharfedale north and south for the part dealt with. If the sanitary and nurture conditions have changed much in the last two hundred years, which anyone well acquainted with the history of the Yorkshire dales will hardly admit, a third source of differentiation is possible. The *stone* of Wharfedale may be more lasting than that of Wensleydale, and thus in the former case a larger proportion of grave data from the 18th century may be included in the series.

To throw some light on this the *partial* correlations $\rho_{H\Delta}$ and $\rho_{W\Delta}$ are given. These measure the correlation between seniority of husband and the length of life of husband or wife for a *given length of life* of the other.

Thus $\rho_{W\Delta}$ is very substantial, or we see that among the array of wives who marry husbands of a given duration of life, that wife lives shortest whose husband has the greatest seniority. In other words great disparity in age between husband and wife tends to shorten the life of the younger of the pair. This must not be confused with the notion of 'shock' referred to on p. 486. For by the formula for partial correlation:

$$\rho_{W\Delta} = \frac{r_{W\Delta} - r_{WH}\, r_{H\Delta}}{\sqrt{1 - r^2_{WH}}\,\sqrt{1 - r^2_{H\Delta}}}$$

and this would have a finite value even if $r_{WH} = 0$, in fact the actual value of r_{WH} only influences $\rho_{W\Delta}$ in the second order of approximation. The essential part of the value of $\rho_{W\Delta}$ is due to the value of $r_{W\Delta}$, and the same remark applies to $\rho_{H\Delta}$ and $r_{H\Delta}$.

It would thus appear, that while men and women tend to select mates of physical constitutions similar to their own and leading to correlated durations of life, there is within this general tendency a rather more subtle factor at work, namely that when there is disparity of age, the older member of the pair either mates with an individual relatively less fitter to the environment than the older individual,—for example, if older men marry the less robust younger women —or else the disparity in age has a physically bad effect on the duration of life of the younger member of the pair. Whichever way we interpret the result this factor is one of very considerable interest and deserves investigation beyond the limits of a paper devoted to assortative mating. The above table suffices to show the sensible existence of this factor in very different sub-groups of the present English population. It also indicates how close the statistical constants we have dealt with remain when we pass from Yorkshire dales to Oxfordshire flats. We believe that similar results obtained from the graveyards in different parts of Great Britain would be of great value as throwing light on the influence of locality and local race on vital statistics.

(10) The main result of this paper is to demonstrate a very sensible degree of assortative mating in man, and we have already suggested the importance of this for the problem of evolution. But the field opened up leads us directly to a number of actuarial problems, which seem deserving of special consideration. They form another link in the chain which must ultimately draw together the actuary and the biologist. The actuary has been studying evolution without knowing it for generations: and the biologist to grasp evolution has got to do actuarial work not only for man, but for all types of life.

TABLE I.

Yorkshire Dales.

Age of Husbands.

Age of Wife.

	25—29	*30—34*	*35—39*	*40—44*	*45—49*	*50—54*	*55—59*	*60—64*	*65—69*	*70—74*	*75—79*	*80—84*	*85—89*	*90—94*	*95—99*	Totals
20—24	—	2	—		—		—	—	1		3	—	—		—	6
25—29	—	2	1	—	–	1	1	—	2	1	2	–	—	—	—	10
30—34	—	2	2	4	2	1	1	4	6	2	3	1	1	—	—	29
35—39	—	3	—	1	2	2	2	4	5	5	2	3	3	—	—	32
40—44		1	-	1	2	3	2	5	7	6	8	1				39
45—49	—	3	3	—	2	1	2	5	7	3	2	2	1	—	1	32
50—54	—	2		1	2	2	3	7	3	5	9	4	3	3	-	44
55—59		—	1	2	3	2	4	3	10	10	9	4	2	2	2	54
60—64	—	—	4	4	2	4	11	15	13	12	9	5	5			84
65—69	2	2	3	3	4	5	5	9	19	19	23	12	3	2	-	111
70—74	—	1	2	4	5	3	7	14	16	20	22	14	8	-	—	116
75—79	—	—	3	1	8	7	6	19	10	17	21	20	9	2	1	124
80—84	1	1	1	2	8	5	1	6	16	21	21	19	6	4	1	113
85—89	-	—	—	1	3	3	1	5	6	8	8	14	7	3	-	59
90—94	—	—	—	–	—	—	3	—	3	2	4	2	1	1	-	16
95—99	—	—	—	1	—	-	—	1	-	3	1	—	—	-	1	7
Totals	3	19	20	28	43	39	49	97	124	134	147	101	49	17	6	876

TABLE II.

Yorkshire Dales. Correlation of random Pairs from same Graveyard.

Age of Man.

Age of Woman.

	25—29	*30—34*	*35—39*	*40—44*	*45—49*	*50—54*	*55—59*	*60—64*	*65—69*	*70—74*	*75—79*	*80—84*	*85—89*	*90—94*	*95—99*	Totals
20—24		—	—	—	—	1	—	1	2			2				6
25—29			–	—	1	—	—	2	2	1	—	4	—	—	—	10
30—34	—	–	1	1	1	1	-	5	4	3	8	1	4	—	—	29
35—39	1	1	1	—	1	1	1	8	5	2	7	3	-	1		32
40—44	—	1	1	—	4	—	3	2	3	7	8	5	4	—	1	39
45—49	—	—	2	—	1	2	3	5	2	7	6	3	1	—	—	32
50—54	—	1	2	1	2	3	1	7	7	4	11	1	—	4	—	44
55—59	-	2	2	3	3	-	2	6	6	8	11	5	4	1	1	54
60—64	—	1	1	2	5	4	8	13	12	9	13	9	6	1	—	84
65—69	—	4	–	4	5	11	4	9	22	21	17	8	6	—	—	111
70—74	—	2	5	8	4	4	8	6	16	17	19	16	8	2	1	116
75—79	1	2	—	5	6	5	9	10	16	25	21	15	6	3		124
80—84	—	4	3	1	4	6	3	14	15	16	17	17	7	3	3	113
85—89	1	1	1	3	1	1	6	8	8	11	5	9	3	1		59
90—94	—	—	1	—	3	—	1	—	3	3	4	—	—	1	—	16
95—99	—	-	—	—	2	-	—	1	1	—	—	3	—	—	—	7
Totals	3	19	20	28	43	39	49	97	124	131	147	101	49	17	6	876

TABLE III.

Ages at Death of Husbands and Wives. Yorkshire Dales, Wensleydale and district only.

Husband's Age at Death.

Wife's Age at Death.	*25–29*	*30–34*	*35–39*	*40–44*	*45–49*	*50–54*	*55–59*	*60–64*	*65–69*	*70–74*	*75–79*	*80–84*	*85–89*	*90–94*	*95–99*	Totals
20–24	—	2	—	—	—	—	—	—	1		1	—	—	—	—	4
25–29	—	2	1	—	—	1	1	—	2	—	2	—	—	—	—	9
30–34	—	2	1	2	2	1	—	2	3	1	1	—	—	—	—	15
35–39	—	1	—	1	2	2	2	4	4	3	1	3	2	—	—	25
40–44	—	1	—	2	2	3	2	5	6	3	5	1	—	—	—	30
45–49	—	2	1	—	1	1	2	4	4	2	2	2	—	—	1	22
50–54	—	—	—	1	2	2	2	2	2	5	7	1	3	3	—	33
55–59	—	—	1	2	3	2	3	3	5	9	8	2	1	1	2	42
60–64	—	—	3	3	1	3	8	9	9	12	8	3	2	—	—	61
65–69	1	1	2	4	1	2	1	8	13	14	13	11	3	1	—	78
70–74	—	—	1	4	4	3	6	13	10	14	16	12	6	—	—	89
75–79	—	—	3	1	7	6	4	14	9	16	21	16	9	2	1	109
80–84	1	1	1	2	6	1	1	5	13	16	17	15	6	3	1	89
85–89		—		1	1	1	1	3	5	5	5	12	5	3	—	42
90–94	—	—	—	—	—	—	3	—	3	—	2	2	1	1	—	12
95–99	—	—	—	—	—	—			—	1	1				—	2
Totals	2	12	14	23	32	28	39	72	89	101	110	83	38	14	5	662

TABLE IV.

Correlation of Husband's Duration of Life, and his Seniority. Wensleydale and District.

Husband's Age at Death.

Husband's Seniority.

Years	25—29	30—34	35—39	40—44	45—49	50—54	55—59	60—64	65—69	70—74	75—79	80—84	85—89	90—94	95—99	Totals
−20	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
−19	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
−16	—	—	1	—	—	—	—	—	—	—	—	1	1	—	—	3
−15	—	—	1	—	—	—	—	—	—	—	1	—	—	—	—	2
−13	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
−12	—	—	—	1	—	—	1	—	—	3	—	—	—	—	—	5
−11	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
−10	—	—	—	—	1	—	2	1	—	—	—	—	—	—	—	4
−9	—	—	—	1	—	1	1	1	1	3	—	—	—	—	—	8
−8	—	—	—	2	—	—	—	4	—	—	1	3	1	—	—	11
−7	—	—	—	—	—	—	—	1	1	1	1	—	1	—	—	5
−6	—	1	—	1	—	3	—	1	1	3	—	2	—	—	—	12
−5	—	—	—	1	1	—	—	4	4	2	1	2	—	—	—	15
−4	—	1	—	2	1	—	2	2	3	7	2	3	—	—	—	23
−3	—	1	—	2	3	1	1	2	3	2	4	—	1	—	—	20
−2	—	—	—	1	1	1	2	3	8	5	5	4	3	—	—	33
−1	1	2	3	1	4	1	2	1	3	5	7	4	1	—	—	35
0	—	3	1	1	1	2	2	9	7	9	10	6	1	2	—	54
+1	1	—	1	—	1	2	6	6	7	6	6	6	2	—	—	41
+2	—	1	2	3	2	4	2	4	8	7	10	8	4	—	—	55
+3	—	—	1	2	3	2	4	2	5	4	5	4	2	1	—	35
+4	—	2	2	2	2	3	1	6	6	6	9	5	—	2	—	46
+5	—	1	1	1	1	1	3	6	4	9	3	6	4	2	—	42
+6	—	—	—	1	1	—	1	2	4	4	8	2	4	1	—	28
+7	—	—	—	—	3	2	4	3	3	4	4	2	1	—	—	26
+8	—	—	—	1	1	1	—	3	7	2	4	6	—	3	—	28
+9	—	—	1	—	1	1	1	3	1	4	1	2	2	1	—	18
+10	—	—	—	—	1	—	2	—	1	1	3	3	—	—	1	12
+11	—	—	—	—	—	—	—	1	2	1	3	—	1	—	—	8
+12	—	—	—	—	—	2	—	1	1	2	1	2	1	—	—	10
+13	—	—	—	—	1	—	1	1	3	—	3	2	2	—	—	13
+14	—	—	—	—	—	—	—	—	1	1	—	—	—	1	1	4
+15	—	—	—	—	1	—	—	—	—	2	—	1	—	1	—	5
+16	—	—	—	—	—	—	—	—	1	—	5	1	—	—	1	8
+17	—	—	—	—	—	—	—	1	—	1	1	—	—	—	—	3
+18	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
+19	—	—	—	—	—	1	1	—	1	3	1	—	—	—	1	8
+20	—	—	—	—	—	—	—	1	—	—	1	—	2	—	—	4
+21	—	—	—	—	1	—	—	—	—	—	—	1	1	—	—	3
+22	—	—	—	—	—	—	—	1	—	—	2	1	1	—	—	5
+23	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	2
+24	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
+25	—	—	—	—	—	—	—	—	—	2	1	1	1	—	—	5
+26	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	2
+27	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
+28	—	—	—	—	—	—	—	1	—	—	1	1	—	—	—	3
+29	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
+30	—	—	—	—	—	—	—	—	—	—	1	1	1	—	—	3
+31	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
+32	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
+35	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
+52	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
+53	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	1
Totals	2	12	14	23	32	28	39	72	89	101	110	83	38	14	5	662

TABLE V.

Correlation of Wife's Duration of Life and Husband's Seniority. Wensleydale and District.

Wife's Age at Death.

Husband's Seniority.

Years	20—24	25—29	30—34	35—39	40—44	45—49	50—54	55—59	60—64	65—69	70—74	75—79	80—84	85—89	90—94	95—99	Totals
−20	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
−19	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
−16	—	—	—	—	—	—	1	—	—	1	—	1	—	—	—	—	3
−15	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	2
−13	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
−12	—	—	—	—	—	—	—	1	—	2	—	—	1	1	—	—	5
11	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	1
−10	—	—	—	—	—	—	—	1	—	1	1	1	—	—	—	—	4
−9	—	—	—	—	—	—	—	1	—	2	1	2	2	—	—	—	8
−8	—	—	—	—	—	1	—	—	1	—	3	2	2	1	—	1	11
−7	—	—	—	—	—	—	—	1	—	1	—	1	1	1	—	—	5
−6	—	—	—	1	1	—	—	—	—	1	3	4	2	—	—	—	12
−5	—	—	—	1	—	1	—	1	1	1	3	4	2	1	—	—	15
−4	—	—	—	—	2	—	1	1	3	3	2	6	4	1	—	—	23
−3	—	—	—	1	—	—	—	—	3	1	5	4	2	3	1	—	20
−2	—	—	—	—	1	2	1	3	5	2	5	4	9	—	1	—	33
−1	—	—	—	—	—	4	—	3	3	6	3	9	4	3	—	—	35
0	—	2	1	—	3	—	4	5	3	8	3	8	7	7	2	1	54
+1	—	1	1	3	4	—	1	—	5	12	4	8	3	1	1	—	44
+2	1	—	2	5	3	1	4	3	1	4	10	9	9	1	2	—	55
+3	—	1	—	2	1	2	1	2	1	5	3	5	6	2	1	—	35
+4	1	—	3	—	2	—	4	6	3	3	8	9	2	4	1	—	46
+5	1	—	1	2	1	2	2	1	5	3	8	4	7	5	—	—	42
+6	1	2	—	—	1	—	1	1	2	—	4	6	8	1	1	—	28
+7	—	—	1	6	—	2	3	1	1	3	2	4	2	1	—	—	26
+8	—	2	1	—	2	—	2	4	4	—	3	4	1	5	—	—	28
+9	—	1	—	1	2	—	—	1	4	1	1	2	1	—	1	—	18
+10	—	—	1	—	1	—	—	—	1	1	4	1	2	1	—	—	12
+11	—	—	1	1	—	1	—	—	—	2	2	1	—	—	—	—	8
+12	—	—	—	1	—	1	—	1	1	1	1	2	2	—	—	—	10
+13	—	—	—	—	—	—	1	1	3	1	4	—	3	—	—	—	13
+14	—	—	—	—	1	1	—	—	—	—	—	2	—	—	—	—	4
+15	—	—	1	—	—	—	—	—	1	1	—	—	2	—	—	—	5
+16	—	—	—	—	—	—	—	2	—	1	2	1	1	1	—	—	8
+17	—	—	—	—	—	—	—	—	2	—	—	1	—	—	—	—	3
+18	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
+19	—	—	—	—	2	1	—	1	—	2	2	—	—	—	—	—	8
+20	—	—	—	—	1	—	1	—	—	1	—	—	—	1	—	—	4
+21	—	—	—	—	—	—	—	—	—	2	—	—	1	—	—	—	3
+22	—	—	—	1	—	1	2	—	—	—	—	1	—	—	—	—	5
+23	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	2
+24	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
+25	—	—	—	—	—	—	2	1	—	—	—	1	—	—	1	—	5
+26	—	—	—	—	1	—	—	—	—	—	1	—	—	—	—	—	2
+27	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
+28	—	—	—	—	—	—	—	—	1	—	—	—	1	1	—	—	3
+29	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
+30	—	—	—	—	—	—	—	—	2	—	—	1	—	—	—	—	3
+31	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
+32	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	1
+35	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
+52	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
+58	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
Totals	4	9	15	25	30	22	33	42	61	78	89	109	89	42	12	2	662

TABLE VI.

Villages round Oxford. Correlations of Lengths of Life of Husband and Wife.

Age of Husbands.

Age of Wives.	*20—24*	*25—29*	*30—34*	*35—39*	*40—44*	*45—49*	*50—54*	*55—59*	*60—64*	*65—69*	*70—74*	*75—79*	*80—84*	*85—89*	*90—94*	*95—99*	Totals
15—19			—		—	—		—	—	—		—	—	—	—	—	
20—24						1	1										2
25—29		—	—	1	—	—	—	—	1	2	—	—	1	—			5
30—34			1	3	3		7		2	1	2	2			1		22
35—39	—	—	1	3	1	—	1	1	5	2	5	2	2				23
40—44				1	3	3	1	7	3	1	7	3	4				36
45—49	—	—	—	1	2	3	8		5	3	7	3	2	1	1		39
50—54					5	1	5	5	10	6	10	8	1				57
55—59	—	—		1	1	3	6	6	5	8	7	6	1	5	1		53
60—64		—		1	—	4	3	8	8	12	13	12	8	3	2	—	74
65—69				1	3	6	5	8	10	19	17	22	13	4	1		109
70—74	—	1	3	—	4	5	12	11	21	22	31	23	16	1	5		158
75—79		1	—	1	—	3	4	7	14	19	22	27	15	7	3		123
80—84		—	—	—	3	2	4	8	10	17	20	16	19	14	4	1	118
85—89	—	1	—	—	—	2	3	6	3	9	7	11	10	2			54
90—94	—	—	—	—	—	1	2	1	—	4	2	1	3	1	—	—	15
95—99	—					—	—				1	1				—	2
Totals	—	3	5	13	25	37	62	68	97	128	151	137	101	44	18	1	890

TABLE VII.

Villages round Oxford. Random pairs, Man with same seniority over Woman as in the case of the Man's actual Wife.

Age of Man.

Age of Woman.	*25—29*	*30—34*	*35—39*	*40—44*	*45—49*	*50—54*	*55—59*	*60—64*	*65—69*	*70—74*	*75—79*	*80—84*	*85—89*	*90—94*	*95—99*	Totals
20—24	—	—	—	—	—	—		1	1	—	—		—		—	2
25—29	—	—	—	—	1	—	—	1	1	—	—	1	1	—	—	5
30—34	—	—	—	2	1	—	1	5	2	2	5	2	1	1	—	22
35—39	1	1	—	—	—	3	1	3	1	4	4	2	—	—	—	23
40—44	—	—	2	—	3	3	4	5	3	5	4	6	—	—	1	36
45—49	—	—	—	1	2	3	5	2	10	7	4	4	1	—	—	39
50—54	—	—	—	1	—	7	3	7	10	7	10	7	4	1	—	57
55—59	—	1	1	3	—	2	4	2	4	13	9	8	5	1	—	53
60—64	—	—	2	3	6	5	9	9	7	7	15	7	2	2	—	74
65—69	—	1	1	4	4	7	11	10	22	19	13	10	3	4	—	109
70—74	2	1	1	2	6	13	6	20	24	35	21	13	9	5	—	158
75—79	—	1	4	5	4	10	9	12	14	17	21	18	8	—	—	123
80—84	—	—	2	2	6	4	10	15	16	23	18	13	8	1	—	118
85—89	—	—	—	2	4	4	2	2	8	9	13	8	2	—	—	51
90—94	—	—	—		—	1	3	2	1	3	—	2	—	3	—	15
95—99	—	—	—	—	—	—	—	1	1	—	—	—		—	—	2
Totals	3	5	13	25	37	62	68	97	128	151	137	101	44	18	1	890

TABLE VIII.

Oxfordshire.

Husband's Age.

Husband's Seniority.

Years	25—29	30—34	35—39	40—44	45—49	50—54	55—59	60—64	65—69	70—74	75—79	80—84	85—89	90—94	95—99	Totals
−25										1						1
−21	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	2
−20			—			—	1			—	—	1		—	—	2
−19	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
−15	—	—	—				—	1		—	—	—	—	—	—	1
−14		—	—	—	—	—	—	—	—	—	2	1	—	—	—	3
−13		1	—	—	1	—	—	—	1	—	—	—	—	—	—	3
−12	—		—	—	1	—	1	—	—	1	2	—	—	—	—	5
−11	—	—	—	—		1	—	1	1	1	—	1	—	—	—	5
−10	—	—	1	—	1	1	1	1	1	—	2	—	1	1	—	10
−9	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	2
−8	—	—	—	1	2	—	2	3	1	1	—	—	—	—	—	10
−7		—	—	—	—	1	2	2	—	2	2	3	—	—	—	12
−6	—	—	—	—	—	—	2	1	3	2	3	—	1	—	—	12
−5	—	1	1	1		3	4	1	3	3	3	2	1	1	—	27
−4	—	—	—	—	2	1	—	3	3	5	3	5	1	—	—	23
−3	—	—	1	2	5	3	—	7	3	5	—	4	—	1	—	31
−2	—	—	2	1	3	3	8	6	8	9	7	6	2	—	—	55
−1	1	—	—	1	2	4	4	8	16	7	10	3	2	4	—	62
0	—	—	1	3		3	4	7	11	12	11	7	3	—	—	65
+1	—	—	1	1	5	8	3	6	7	13	5	6	1	—	—	56
+2	1	1	2	3	3	2	5	5	9	13	13	9	1	3	—	70
+3	1	1	1	3	5	5	4	9	12	7	8	7	5	—	—	68
+4	—	—	—	1		5	7	4	5	11	9	11	3	—	1	57
+5	—	1	2	2	1	2	3	1	8	9	11	3	2	1	—	46
+6	—	—	—	—	1	3	4	5	6	6	5	5	5	1	—	41
+7	—	—	1	—	—	2	3	7	5	3	3	6	3	1	—	31
+8		—	—	1	1	2	1	2	3	8	7	4	2	2	—	33
+9	—	—	—	2	1	1	2	3	2	4	5	4	1	1	—	26
+10	—	—	—	2	—	1	5	4	4	2	7	1	—	1	—	27
+11	—	—	—	1	—	1	—	2	1	5	2	—	1	—	—	13
+12	—	—	—	—	1	1	1	1	2	3	4	2	—	—	—	15
+13	—	—	—	—	—	4	—	—	3	2	1	4	2	—	—	16
+14	—	—	—	—	—	—	—	1	2	1	—	2	1	—	—	7
+15	—	—	—	—	—	—	—	1	1	4	—	2	3	—	—	11
+16	—	—	—	—	1	1	—	—	2	1	3	—	1	—	—	9
+17	—	—	—	—	—	1	—	—	—	—	—	—	1	—	—	2
+18	—	—	—	—	—	—	—	1	—	—	2	—	1	—	—	4
+19	—	—	—	—	—	—	—	—	—	1	1	1	—	—	—	3
+20	—	—	—	—	—	—	—	—	—	1	2	1	—	—	—	4
+21	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	2
+22	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	2
+23	—	—	—	—	—	1	—	—	1	1	—	—	—	—	—	3
+24	—	—	—	—	—	—	—	1	—	2	—	—	—	—	—	3
+25	—	—	—	—	1	—	—		—	—	—	—	—	—	—	1
+26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
+27	—	—	—	—	—	—	—	—	—	1	1	—	—	1	—	3
+28	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
+29	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
+30	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
+31	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
+32	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
+33	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
Totals	3	5	13	25	37	62	68	97	128	151	137	101	44	18	1	890

TABLE IX.

Oxfordshire.

Wife's Age.

Husband's Seniority.	15—19	20—24	25—29	30—34	35—39	40—44	45—49	50—54	55—59	60—64	65—69	70—74	75—79	80—84	85—89	90—94	95—99	Totals
−25	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
−21	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	2
−20	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	2
−19	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	1
−15	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
−14	—	—	—	—	—	—	—	—	—	—	—	—	2	—	—	—	1	3
−13	—	—	—	—	—	—	—	—	—	—	—	1	2	—	—	—	—	3
−12	—	—	—	—	—	—	—	—	—	1	1	1	1	1	—	—	—	5
−11	—	—	—	—	—	—	—	—	1	—	—	—	2	1	1	—	—	5
−10	—	—	—	—	—	—	1	1	1	1	1	3	2	—	—	—	—	10
−9	—	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	2
−8	—	—	—	—	—	—	—	1	—	1	1	1	3	—	2	1	—	10
−7	—	—	—	—	—	—	—	—	—	1	3	2	2	2	1	1	—	12
−6	—	—	—	—	—	—	—	1	—	—	2	3	4	1	—	1	—	12
−5	—	—	—	—	—	1	1	1	1	3	2	8	4	4	2	—	—	27
−4	—	—	—	1	—	2	1	1	2	4	1	3	1	5	2	—	—	23
−3	—	—	—	2	—	—	2	3	1	4	5	5	3	2	3	1	—	31
−2	—	—	—	2	2	2	3	1	4	5	6	10	7	8	3	2	—	55
−1	—	—	—	—	1	2	4	2	3	3	9	14	10	7	5	2	—	62
0	—	—	—	1	2	1	3	4	7	4	8	16	7	6	5	1	—	65
+1	—	—	—	2	—	2	2	5	2	4	9	11	9	8	1	1	—	56
+2	—	—	—	—	2	7	3	1	4	5	11	12	10	11	3	1	—	70
+3	—	—	—	3	2	2	3	6	3	5	10	11	13	4	6	—	—	68
+4	—	—	—	1	4	2	3	4	3	2	8	8	3	15	3	1	—	57
+5	—	—	—	—	4	2	3	4	1	6	5	8	5	5	2	1	—	46
+6	—	—	1	2	1	—	1	1	2	4	5	13	6	4	1	—	—	41
+7	—	—	1	1	—	2	1	3	2	4	1	5	5	7	2	—	—	34
+8	—	—	—	2	1	1	1	3	1	2	2	9	4	6	1	—	—	33
+9	—	—	1	1	—	3	—	2	2	4	3	2	—	5	3	—	—	26
+10	—	—	—	—	—	1	2	3	4	5	2	—	2	4	2	2	—	27
+11	—	—	—	1	1	—	1	—	1	1	3	2	2	—	1	—	—	13
+12	—	—	1	—	—	1	—	1	3	—	2	4	2	—	1	—	—	15
+13	—	—	—	—	1	1	1	—	—	3	3	2	3	—	1	—	1	16
+14	—	—	—	—	—	1	1	1	1	—	—	—	1	2	—	—	—	7
+15	—	—	1	—	—	—	1	—	1	—	2	1	2	2	1	—	—	11
+16	—	—	—	2	—	1	—	1	1	1	1	—	1	1	—	—	—	9
+17	—	—	—	1	—	—	—	—	—	—	—	—	—	1	—	—	—	2
+18	—	—	—	—	—	—	—	1	—	1	—	1	1	—	—	—	—	4
+19	—	—	—	—	—	—	—	2	—	—	—	—	1	—	—	—	—	3
+20	—	—	—	—	1	1	—	—	—	—	1	—	1	—	—	—	—	4
+21	—	—	—	—	—	—	—	1	—	—	—	1	—	—	—	—	—	2
+22	—	—	—	—	—	—	—	1	—	—	—	—	—	1	—	—	—	2
+23	—	1	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	3
+24	—	—	—	—	1	—	1	—	—	—	—	—	—	1	—	—	—	3
+25	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
+26	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
+27	—	—	—	—	—	—	—	—	—	—	2	—	—	1	—	—	—	3
+28	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
+29	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
+30	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
+31	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
+32	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
+33	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	1
Totals	—	2	5	22	23	36	39	57	53	74	109	158	123	118	54	15	2	890

TABLE X.

Records of Society of Friends.

Age of Husband.

Age of Wife.	*24—28*	*29—33*	*34—38*	*39—43*	*44—48*	*49—53*	*54—58*	*59—63*	*64—68*	*69—73*	*74—78*	*79—83*	*84—88*	*89—93*	*94—98*	*99—103*	Totals
18—22	2	1	1	—	—	1	2	1	1	—	1	2					12
23—27	5	4	2	—	3	4	1	1	2	2	4	—	1	—	—	1	30
28—32	1	1	2	3	3	6	3	4	4	3	5	1	2		—		38
33—37	1	1	3	2	9	10	3	8	8	4	12	3	6	2		—	72
38—42	1	1	4	4	3	6	5	3	4	4	8	2	2		—		47
43—47	1	1	3	1	3	3	4	3	4	5	7	4	2		1	—	42
48—52	—	—	1	—	3	6	4	7	6	4	5	3	4	—	—	—	43
53—57			—	2	4	—	2	5	3	6	8	5	6	1		—	42
58—62	—	—	2	3	1	4	7	9	8	9	13	2	7	—	—	—	65
63—67	1	—	2	3	1	5	8	8	9	17	13	11	4	1	2	—	85
68—72	—	2	1	1	11	4	6	13	15	13	20	10	7	1	1		105
73—77	—		2		7	3	3	10	17	11	17	20	10	4	1		105
78—82	—	1	1	4	5	5	7	5	13	11	24	19	16	1	1	—	113
83—87	1	—		2	3	1	9	4	7	4	13	13	18	3	1	—	79
88—92	1	—	2	—	1	1	1	4	5	7	9	6	2	2	—	1	42
93—97				1	1	2		1	1	3	1	1	—		1	—	12
98—102	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	2
103—107		—		—	—	—	1									—	1
Totals	14	12	26	26	58	62	66	87	107	103	160	102	87	15	8	2	935

TABLE XI.

London Cemeteries.

Age of Husband.

Age of Wife.	*24—28*	*29—33*	*34—38*	*39—43*	*44—48*	*49—53*	*54—58*	*59—63*	*64—68*	*69—73*	*74—78*	*79—83*	*84—88*	*89—93*	*94—98*	Totals
20—24	1		—	—	2	—	—	—		—	—		—	—	—	3
25—29	1	1	1	—	1	1	—	1	—	—	—	—	—	—	—	6
30—34		2	4	2	1	—	—	1		—	2	—	1	—	—	13
35—39	1	2	4	3	3	3	5	4	1	5	5	—	—	—	—	36
40—44	—	1	3	3	3	6	4	6	2	2	3	2	—	—	—	35
45—49	—	1	3	3	7	5	6	3	—	5	4	3	2	—	—	42
50—54		1	2	2	3	9	10	13	11	13	4	6	1	—	—	75
55—59	—	—	—	3	3	12	11	10	15	12	9	4	3	—	—	82
60—64	—		—	1	2	9	16	20	17	22	20	13	2	1	—	123
65—69	1	1	1	2	1	6	8	26	23	23	24	9	8	1	1	135
70—74		—	2		6	3	7	16	16	34	29	22	7	3		145
75—79		—	2	1	5	2	5	15	21	34	25	25	7	5	2	149
80—84	1	—	—	—	3	6	4	11	10	10	17	27	11		1	101
85—89	—	—	—	2	1	3	1	3	6	6	3	5	8	3	—	41
90—94	—	—	—	—	—	—	1	2	—	2	—	—	2	—	—	7
95—99	—	—	—	—		1	—	1	—	2	2	—	1	—	—	7
Totals	5	9	22	22	41	66	78	132	122	170	147	116	53	13	4	1000

MISCELLANEA.

I. **Inheritance in** *Phaseolus vulgaris.*

Professor F. Johannsen has just published a summary of his recent experiments on inheritance. His work *Ueber Erblichkeit in Populationen und in reinen Linien* (Fischer, Jena, 1903) is dedicated to Francis Galton, as the "Schöpfer der exakten Erblichkeitslehre"; it shows that the author has realised the importance of adequate statistical methods in any attempt to deal with the problem of inheritance, and we wish to express our gratitude to him for the courteous tone adopted in speaking of "Biometriker," and for the patient effort he has made to understand their work.

Professor Johannsen's material is collected from three generations of beans: he has (1) a series of individual seeds, chosen out of a sample of about 16,000, which had been harvested and mixed together; these afford his evidence concerning the character of a grandmaternal generation; (2) each grandmaternal bean, when sown, yielded a crop of maternal beans, the mean character (weight or length-breadth index) of those borne by one plant being taken as the measure of the maternal character of the line of ancestry to which the plant belongs; (3) from the series of seeds borne by each mother plant, a certain number (from two to seven) were sown, the mean characters of the seeds borne by the resulting plants being taken as a measure of the characters studied in the filial generation of each line.

From these data three principal sets of tables are constructed; on p. 25, *Uebersichtstabelle* I. gives material for measuring the regression of individual filial beans on the mean weight of the maternal beans; a series of tables on pp. 21—24 gives the relation between the weights of the individual beans of the maternal plants, which were sown, and the mean weight of the seed produced by every resultant plant; finally on pp. 36—37 a table is given which Professor Johannsen believes to show that the progeny of every seed, within a particular line of ancestry, exhibits a complete "Rückschlag" to the type of its line, the coefficients of correlation and regression between parent and offspring within the line being each = 0.

On the basis of these tables Professor Johannsen attempts to explain the apparent discrepancy between Galton's law of regression and the results obtained by de Vries and others: but his view of the consequences supposed by "Biometriker" to follow from Galton's results shows that he has not fully realised what those consequences are.

It is fully realised by "Biometriker" that the general regression observed when we compare a filial generation with a parental generation is compounded of a series of sub-regressions, the members of each line of ancestry regressing to the "type" of their line; the effect of selecting definite ancestry for a small number of generations is also recognised; these points were fully dealt with in 1898, although few biologists seem to have realised the fact; it was then said:

"We now see that with the law of ancestral heredity......a race with six generations of "selection will breed within 1·2 per cent. of truth ever afterwards,"* or in other words the fixity of a line of ancestry is asserted when the ancestral purity is far less than that involved in Professor Johannsen's "reine Linie," which he defines as consisting of "Individuen, welche "von einem einzelnen selbst befruchtenden Individuum abstammen" (p. 9).

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* K. Pearson: "On the Law of Ancestral Heredity," *Roy. Soc. Proc.* Vol. 62, 1898. Cf. pp. 397—402, "On the Variability and Stability of Selected Stock."

Again, "with a view to reducing the *absolute* variability of a species it is idle to select beyond "grandparents, and hardly profitable to select beyond parents. The ratio of the variability of "pedigree stock to the general population decreases 10 per cent. on the selection of parents, and "only 11 per cent. on the additional selection of grandparents. Beyond this no sensible change "is made."[*]

In these two passages the fixity of the type and the high variability of individuals about the type are asserted as absolutely as they are asserted by de Vries in the passage of the *Mutationstheorie* quoted by Professor Johannsen. With our present knowledge of the coefficients of inheritance in man, horse, and dog, it would seem that from 2 to 4 generations of selection suffice to form a line varying greatly about its type, yet remaining true to that type. When we are told that a bean breeds true to its line we are told something which has been shown to be a necessary consequence of the law of ancestral heredity; if it were not true, the whole law would be upset.

The difference between the view put forward by Professor Johannsen, and that expressed in 1898 in the paper "On the Law of Ancestral Inheritance" is therefore not a difference which concerns the focus of regression in the offspring of selected ancestors; it is simply a difference as to the relation between successive generations of individuals within the line of ancestry. Professor Johannsen believes the tables on pp. 21—24 of his work to show that "*die persönliche* "*Beschaffenheit der Eltern, Grosseltern, oder irgend eines Ahnen hat*—soweit meine Erfahrung "reicht—*keinen Einfluss auf den durchschnittlichen Charakter der Nachkommen.* Es ist aber "*der Typus der Linie*, welche den durchschnittlichen Character der Individuen bestimmt......" (pp. 61—62). Within the same line of ancestry, whatever individuals of a generation be chosen as parents, the character of the resultant filial generation will be the same according to Professor Johannsen, or the coefficients of correlation and regression between parents and offspring, within the same line of ancestry, will each = 0.

The experimental results do not seem to us consistent with this view. If the offspring of every generation, within a given line of ancestry, breed true to the type of their line, subject to such seasonal and climatic influences as affect the whole generation of their year, then when the whole filial generation is compared with the whole parental generation, correlation between the two must necessarily be perfect, and the coefficient of regression must be simply the ratio between the standard deviations of the two generations. Professor Johannsen should, we think, first have shown that perfect correlation does in fact exist between parents and offspring in two successive generations of plants; and this he has failed to do; in the case of his maternal and filial generations he has, however, published data which enable us to determine the required correlation, and the table below gives the result. With such data, the only method available is the first method used for Shirley Poppies[†]. It consists in determining the correlation between *every individual bean* of the filial generation, and the *mean character of its parent*. The absolute value of the correlation so obtained will not be significant, but the coefficient of regression will closely represent the true parental correlation,—being a relation between mean filial character and mean parental character. Taking Professor Johannsen's *Uebersichtstabelle* I. (p. 25), and the maternal means given on pp. 21—24, the following table has been constructed, giving a coefficient of correlation = 0·531 and of regression = 0·591 ± 0·125. This latter value represents the coefficient of correlation, so far as the data allow it to be determined, between filial and maternal plants; and considering the paucity of maternal plants (only 19) the result is not in bad accord with previous results for parental correlation[‡]. The value 0·591 ± 0·125 is not very divergent from 0·5, but it cannot be held to approximate to unity!

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[*] Pearson: *loc. cit.*

[†] *Biometrika*, Vol. II. Part I. p. 69.

[‡] *Biometrika*, Vol. II. p. 379.

TABLE A.

Correlation between Individual Filial Beans and Mean Maternal Beans.

Filial Beans.

Mean Maternal Beans

Weights in milligrams	*10—15*	*15—20*	*20—25*	*25—30*	*30—35*	*35—40*	*40—45*	*45—50*	*50—55*	*55—60*	*60—65*	*65—70*	*70—75*	*75—80*	*80—85*	*85—90*	Totals
A & D 600	—	—	—	5	2	11	26	47	82	98	84	46	26	17	6	2	452
C 570	—	—	—	—	—	5	14	50	76	58	44	29	5	1	—	—	282
B 520	—	—	—	1	6	19	32	66	88	100	90	50	19	1	3	—	475
E 512	—	—	—	4	1	12	29	62	65	57	19	6	—	—	—	—	255
K 510	—	—	1	2	6	31	55	55	28	6	4	—	—	—	—	—	188
Q 440	—	—	1	2	7	16	44	93	80	52	10	—	—	—	—	—	305
R 410	—	—	—	2	3	12	17	27	19	3	—	—	—	—	—	—	83
S 405	—	—	1	2	3	8	27	47	37	30	4	—	—	—	—	—	159
G & J 400	—	1	5	23	66	155	283	353	241	97	15	—	1	5	—	—	1245
F & T 395	—	—	—	2	9	27	66	111	85	58	22	1	1	—	—	—	382
P 390	—	—	—	3	1	18	35	27	13	3	4	2	—	—	—	—	106
H 380	—	—	1	6	20	60	106	114	75	33	3	—	—	—	—	—	418
L 360	—	—	1	5	15	37	88	76	33	13	4	1	—	—	—	—	273
M 340	—	—	4	9	26	56	82	76	32	9	1	—	—	—	—	—	295
N 312	1	3	11	22	29	72	120	69	23	5	2	—	—	—	—	—	357
O 310	4	4	5	19	69	69	44	5	—	—	—	—	—	—	—	—	219
Totals	5	8	30	107	263	608	1068	1278	977	622	306	135	52	24	9	2	5494

The letters *A* to *T* are used by Professor Johannsen to denote his nineteen "pure lines."

If Professor Johannsen's hypothesis were true, the only way in which he could account for the regression here observed,—a regression whose existence he himself admits,—would be by assuming that the characters, by which he has described the plants of his pure lines, are imperfectly correlated with the actual mean characters of his plants which he supposes to represent the types of his lines; but this assumption, while leaving his hypothesis logically unshaken, would destroy the whole value of his experiments as evidence in its favour, by destroying the value of the measure by which he has determined the characters studied.

Some of the difficulties we have felt in following Professor Johannsen are undoubtedly due to the imperfect way in which he has measured the characters of ancestral plants*. The *grand-maternal plants*, for example, are determined each by the character of a single bean; the small value of such a determination may be judged from the variability among the beans of a single plant produced by the last generation; the mean number of such beans was 84·5, and the mean standard deviation of all arrays due each to a single plant was 75·37 milligrams, so that the mean character of a mother-plant, inferred from a single bean chosen at random among the offspring, would be equally likely to lie inside or outside the limits (true maternal mean + 50 mgrm.) and (true maternal mean − 50 mgrm.). There is thus, we venture to say, no strong probability that the numbers by which Professor Johannsen describes his grandparental beans represent the mean character of the seeds of the corresponding plants within ± 100 mgrm.

Again, the whole evidence, that the coefficient of filial regression within the line is zero, rests on the tables on pp. 21—24; but in these tables we are only told (1) the mean weight of seeds produced by a mother-plant, representing the maternal character in a line; (2) the weights of individual seeds, taken from this mother-plant and sown; and (3) the mean weight of the seeds borne by each resultant plant. The regression coefficient, which Professor Johannsen regards (without any very adequate proof) as equal to 0, is between the deviation of the *individual beans* (2) and that of the mean character of the series of plants (3) resulting from them. We have therefore (1) the maternal generation defined by the mean character of all the beans borne by a plant; (2) a second generation, children of the maternal plant, each defined in the same way, by the mean character of the beans it bears; we have no third generation at all, and the regression which Professor Johannsen has observed seems to have little bearing on the question at issue, which could only be determined by growing a third generation from representative offspring of the filial plants, describing each plant of this generation as those of the two previous generations were described, in terms of the mean character of its seeds, and then determining the correlation between the characters so described in the two successive generations, the children and grandchildren of the single plant originally used to determine the line.

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* A preliminary study of homotyposis in the bean must of necessity precede any attempt to measure plant character by a single seed. Professor Johannsen would have to show that the homotypic correlation was perfect to justify his measure. This is very far from the fact not only in *Phaseolus vulgaris*, but in all beans hitherto examined from this standpoint.

The question, which the tables given do to some extent answer, is the question what relation exists between the character of two seeds from the same mother-plant, and the character of the plants produced when those seeds are sown. Now it seems clear that if we take small beans out of a general harvest of seed, we shall be to some extent selecting the seeds of plants which bore on an average small beans; but what reason is there for supposing that the small and the large seeds from one and the same plant will lead to groups of plants bearing respectively small and large seeds? The hypothesis involved in this supposition seems somewhat analogous to the view that out of two eggs of a clutch, the smaller will produce a hen laying smaller eggs than that produced from the larger; this may well be quite fallacious, and it may yet be true that out of large masses of eggs, small eggs produce on the whole hens which lay small eggs. The absence of relationship of marked kind between the weight of seed sown, and mean weight of seed produced by the resulting plant, seems to have no bearing on the problem whether selection within the line can produce a change of character.

One further point we must notice; the *Uebersichtstabelle* 4 (pp. 36—37) has been treated in a quite illegitimate way, which would make the coefficients of correlation and regression = 0 between any two variables whatever.

If, as Professor Johannsen believes, the individual differences between members of any one generation were mere fluctuations, having no hereditary value, then a given generation ought as he says to be as well determined by selection of its grandparents as by selection of its parents. We cannot determine whether this is true of the average character of plants in Professor Johannsen's experiments, because the necessary data are wanting; but we can determine roughly the relation between three successive generations of *individual beans*. The material has been selected in such a way that the standard deviations of the successive generations have clearly quite artificial values, so that the correlations obtained are not very trustworthy; further, the exact weights of beans are not always given, so that we have been obliged to place a bean recorded as lying between say 400 and 450 mgrm. in the middle of its category. With these qualifications, we find

Correlation of Mother Bean and Offspring Bean $r_{01} = 0{\cdot}3481 \pm 0{\cdot}0080$,

Correlation of Grandmaternal Bean and Offspring Bean $r_{02} = 0{\cdot}2428 \pm 0{\cdot}0086$.

If we wish to predict the weight of a given bean of the filial generation from the known weight of its maternal bean, we must form a regression equation, which becomes

Probable weight of Offspring bean = 538·31 + 0·2691 × weight of Maternal Bean ....(i).

If we wish to predict the weight of a filial bean from knowledge of the grandmaternal bean, we obtain the regression equation

Probable weight of Offspring bean = 417·41 + 0·1074 × weight of Grandmaternal Bean...(ii).

This shows us at once that from Professor Johannsen's own data the maternal bean is more than twice as influential as the grandmaternal bean in settling the weight of the filial generation.

If we correlate the selected maternal and grandmaternal beans, we find the correlation, $r_{12} = 0{\cdot}2532$; and from this and the preceding correlations we find the double regression formula

Probable weight of Offspring bean

= 330·71 + 0·2373 × weight of Maternal Bean + 0·0731 × weight of Grandmaternal Bean...(iii),

showing again the predominant influence of the maternal bean.

If we were to calculate the mean weights of each array of offspring means from (i), (ii) and (iii), we should expect the mean errors of the results to be in the ratio of

$$\sqrt{1 - r^2_{01}},\ \sqrt{1 - r^2_{02}} \text{ and } \sqrt{\frac{1 - r^2_{01} - r^2_{02} - r^2_{12} + 2r_{01}r_{02}r_{12}}{1 - r^2_{12}}},$$

or in this case as

$$1{\cdot}014 : 1{\cdot}050 : 1.$$

We have applied (i) (ii) and (iii) to the 65 arrays of offspring given by Professor Johannsen, and the mean errors are

$$44{\cdot}3;\ 45{\cdot}5, \text{ and } 42{\cdot}8$$

or in the ratio of

$$1{\cdot}035 : 1{\cdot}060 : 1.$$

These numbers are, perhaps, as close as we could expect, and they show that we do in fact get better results from a knowledge of maternal bean than from knowledge of grandmaternal bean, and better results from a knowledge of both together than from a knowledge of either alone.

We hold therefore that Professor Johannsen's results prove:

(1) That there is a regression from parent to offspring, leading to the inference that parental correlation has for *Phaseolus vulgaris* a value closely identical with that found for other animals and plants, when we compare mean parental and mean filial characters;

(2) That when we compare the characters of individual seeds in successive generations the correlation between a seed and its parental seed is so much greater than that between a seed and its grandparental seed (both belonging to the same pure ancestral line) as to give strong evidence that characters arising in one generation within the line are inherited, and do therefore afford a basis on which selection may act.

W. F. R. W. and K. P.

II. **Addendum to "Graduation and Analysis of a Sickness Table"** (*Biometrika*, Vol. II. p. 260). By W. Palin Elderton.

On p. 261 it is stated that Gompertz' hypothesis may be viewed as the quotient of two normal curves, and it will be interesting to see how Makeham's useful modification of Gompertz' theory may be stated from the same point of view. Makeham's hypothesis is that the force of mortality may be represented as $A + Bc^x$, which, from our point of view, means that if we take a normal curve as the exposed we get $(A + Bc^x)\, y_0 e^{-x^2/2\sigma^2}$ for the deaths, and this can be thrown into the form of two normal curves of the same standard deviation, viz.,

$$\frac{N_1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2} + \frac{N_2}{\sigma\sqrt{2\pi}} e^{-(x-h)^2/2\sigma^2}.$$

If we wish to apply Makeham's hypothesis to a table we can take a normal curve, multiply by q_x (the probability of dying in a year), and then fit the expression just given to the figures we obtain. The best way to do this will be by moments about the origin of the normal curve chosen for the exposed and, writing μ_1 and μ_2 for these, we have

$$h=\frac{\mu_2-\sigma^2}{\mu_1} \quad \text{and} \quad N_2=\frac{(N_1+N_2)\,\mu_1}{h},$$

which is the complete solution. As Makeham's hypothesis only holds good for part of a mortality table we must choose our exposed curve so that the deaths resulting therefrom will be insignificant below say 25 in youth and above 85 in old age. Using $\sigma=8{\cdot}3$ and 50·5 as origin I made a trial with the H^M Mortality Table (*Institute of Actuaries, Healthy Male Lives*). I grouped the deaths corresponding to this assumed normal curve for every five years of age and using Sheppard's adjustment for the second moment, found $h=6{\cdot}4703$ years, and hence $\log_{10}c=\frac{h}{\sigma^2}\log_{10}e={\cdot}040464$. The graduation in the Text Book for actuaries uses ·039657 and Professor Pearson gave ·0406405 in *Biometrika*, Vol. I, Part 3. We thus reach an extremely easy method of satisfactorily finding c.

Using the same normal curve and process with the graduated sickness rates I found $\log_{10}c={\cdot}0465901$ and the resulting rates gave a good agreement from age 30 to age 80. An attempt to fit the whole sickness table was made by taking $\sigma=12{\cdot}5$ and 47·5 as origin and the value of $\log_{10}c$ was found to be ·040133 but the result was unsatisfactory as regards graduation.

The fact that Makeham's hypothesis fits Sutton's sickness table to the same extent as it does a mortality table is further evidence of the close relationship existing between the two classes of tables.

III. **Craniological Notes.**

(iv) *Homogeneity and Heterogeneity in Crania.*

By Charles S. Myers.

Professor Pearson's vigorous denunciation* of my criticism† of Miss Fawcett's memoir‡ on the Naqada skulls gives me a welcome opportunity of strengthening the position I took up.

(i) It may be remembered that, in order to determine whether or not the Naqada material belonged to a homogeneous race, Miss Fawcett compared the variability of skull-lengths (and of other measurements, which for brevity's sake I have here to neglect) in the Naqada series with the variabilities in four series, which were considered to be of an admittedly homogeneous character, viz., in a Bavarian, an Aino, a French and an English series. The variabilities of skull-lengths in these four series were found to be 6·088, 5·936, 7·202 and 6·446 respectively. Now, taking the skull-lengths of a mixed series of Australian, Guanche, Eskimo and Chinese skulls, I obtained a variability of 8·389; whereupon I concluded that if a small series, which was as heterogeneous as could be conceived, showed a skull-length variability of 8·389, such material as contained variabilities ranging between 5·722 (in the Naqada series) and 7·202 (in the French series) could not reasonably be termed homogeneous. That inference, I thought and still think, was one which could be safely drawn, even by a mind so devoid of mathematical and statistical training as Professor Pearson imagines mine to be.

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* *Biometrika*, Vol. II., pp. 345—347.

† *Man*, February, 1903, pp. 28—32.

‡ *Biometrika*, Vol. I., pp. 408—467.

(ii) Professor Pearson has met my criticism by two different lines of argument. In the first place, he pretends to be able to determine the degree of probability that a series of five members will contain a sixth of a larger value; and he asserts that the odds are 42,552 to 1 against the appearance of the variability of my mixed skull series (= 8·389) in a series composed of variabilities of the so-called homogeneous series. Surely common sense repudiates the legitimacy of such a calculation. But if mathematics allows it, what must also be the odds against the appearance of the figure 7·202 in the same series! Yet this figure actually occurs there, being the measure of variability of the 860 French skulls from the Paris catacombs.

(iii) Hereupon Professor Pearson changes front and casts a doubt upon the homogeneity of the French series; instead of the figure 7·202 he proceeds to introduce the figure 5·942, obtained from some 77 Parisian skulls which suit his purpose better. Moreover he substitutes the figure 6·085 for the figure 6·446 which Miss Fawcett had used without mention of the fact that it was the variability of a measurement which differed from the rest. With these modifications and with the addition of other suitable data Professor Pearson obtains still greater odds against the appearance of my figure 8·389 in a "homogeneous" series.

(iv) My criticism, however, is as valid now as it was when delivered. It was directed not against Professor Pearson's later revocations, but against Miss Fawcett's paper (for "the editing and arrangement" of which he definitely admits himself "responsible"), in which the variability 7·202 is unquestionably included in a table of variabilities of material, the whole of which is styled "admittedly homogeneous." Surely Professor Pearson must see that, by neglecting the inconvenient values 7·202 and 6·446, the former of which approaches so nearly the measure of variability of my heterogeneous series, he evades the question at issue. What I had in mind to ask was this—where, in the opinion of the biometric workers of University College, does homogeneity end and heterogeneity begin, if variabilities of 5·7, 6·4 and 7·2 appear in homogeneous series and a variability of 8·4 appears in a series which has as composite a character as can be imagined? My criticism, if it has served no further purpose, has at least elicited an answer to this question. In his defence of Miss Fawcett's memoir, Professor Pearson fixes the higher limit of homogeneity at or about 6·5. Consequently the French catacomb skulls ought never to have been included in a table purporting to contain only homogeneous material.

Passing from the mathematical to the biological side of the problem, I will admit the right of the mathematician to distinguish relative homogeneity and heterogeneity; but he must remember that the difference is purely a relative one. The Naqada material, if styled homogeneous today, would become heterogeneous tomorrow, should a series of still earlier and far more homogeneous crania come to hand. But the mathematician has no right to conclude, as in point of fact Miss Fawcett concludes, that therefore "we are justified...in speaking of a Naqada *race* and not merely of the Naqada crania." Professor Pearson may be able to define homogeneity to his satisfaction, but in our present ignorance none can define the exact meaning of racial purity. It may be that the variability of the ancient Egyptians is relatively small; but their population undoubtedly included well-nigh as many specimens of "races" as it does at the present day. I have examined "prehistoric" Egyptian skulls in sufficient number, to be convinced that the Naqada series contains skulls which have markedly negroid characters, between which and the more delicately chiselled features of Mediterranean and allied peoples there is every gradation, just as occurs among the *fellahin* of modern times. So also, to speak of a *race* of Englishmen disinterred from Whitechapel, or to speak of a *race* of French peasants, as Professor Pearson would have us do, is to ignore every lesson which physical anthropology, philology and history can teach us. The truth is that at present we have no evidence of an isolated race, which has never been contaminated by admixture with other races. We are ignorant, therefore, of the characters of such a race, of its variability, for instance, in head-, nose- or hair-form. And until appropriate material comes to hand, statistics and anatomy are alike powerless to help us.

Remarks on Dr C. S. Myers' Note.

Dr Myers seems to have entirely mistaken, not only the nature of Miss Fawcett's original memoir but of my defence of that memoir. A reply to him must have much of the nature of the traditional fight between the horse and the whale. I have not much hope of convincing him of the necessity for using exact methods in statistical enquiry, but I have hope that a new school of anthropologists will arise, who will be able to choose between the old and the new paths.

(i) Miss Fawcett started her work on the Naqada crania, without I am sure any prejudice, as to their heterogeneity or homogeneity. The test of the category into which they were to be placed was not as Dr Myers suggests a mere comparison of these skulls with "four series which were considered to be of an admittedly homogeneous character." On the contrary we were fully aware of the view taken by certain Egyptologists, that the Naqada crania differed widely from those of the historic Egyptians, and further that a whole series of races were supposed to have occupied Egypt in prehistoric times, and further that these races were represented in this series of crania. For this reason the crania were classified into groups according to the antiquity of the fragments found in the graves. This matter is dealt with in pp. 422–3 of the memoir. On the basis of this investigation no significant chronological differences could definitely be predicted of the Naqada crania. This was the first stage in Miss Fawcett's test. The second stage was to compare the variability of the Naqada crania for length and breadth with the known values for other series. Judged by both length and breadth they are less variable than the "Altbayerisch" crania; judged by length only than the Aino crania, both undoubtedly very homogeneous series. If the word "race" is to be used at all, it may certainly be used of the Ainos and Altbaiern, where for years close intermarriage must have gone on*. Here, I think, is possibly the source of some of my difference with Dr Myers. In my view any race may originally have arisen from a mixture of races, but such a mixed race is wholly different from a mixture of races, which have not interbred. It is with a mixture of races which have not interbred, Australians, Guanches, Eskimos and Chinese, that Dr Myers proposed to compare our material. I said and I repeat now that our Naqada series belongs to a totally different order of variability. Our English crania belong to one district and one period, our French catacomb crania to one district, but to a succession of periods, our Bavarian and Aino to very limited districts with probably close inbreeding. The Naqada is as little variable as the least variable of these series. There appears accordingly no legitimate foundation for treating it as a series of crania composed of individuals from such different races as the Egyptologists have suggested. Dr Myers without having examined the Naqada crania, says that he has examined sufficient "prehistoric" Egyptian skulls† to be convinced that the Naqada series contains skulls which have markedly negroid characters "between which and the more delicately chiselled features of the Mediterranean and allied peoples there is every gradation." There may be Naqada skulls with negroid characters; it is easy to assert such to exist, and there is no finality to mere matters of opinion, which cannot be put to a quantitative test. But the series has been examined by anatomical craniologists of equal judgment with Dr Myers. Professor Sergi, who has seen the crania, classed them definitely as belonging to his "Mediterranean race." Dr Myers who has not seen the crania finds that they contain skulls of negroid character which differ from those of the Mediterranean peoples. On the whole I think the biometrician can afford to smile at the judgments of the non-biometrical craniologists who presumedly alone pay attention to the "lessons which physical anthropology, philology and history teach us"!

(ii) Dr Myers asserts that I met his criticism by *two* lines of argument. I don't think so at all. I merely showed him how he ought scientifically to have determined whether his mixture of crania belonged or not to the same category as the series dealt with on p. 424 of Miss Fawcett's paper. Having shown that the absurdity of comparing it with these series was measured by odds of 42,552 to 1, I said that the whole problem deserved consideration from a wider standpoint. In other words I took up *de novo* the question of biometrically determining the degree of homogeneity in cranial series on the basis of length and breadth measurements, and obtained results which I hoped might be helpful in future. I did this because mere controversy is very distasteful to me, even when it is striving to pull a great branch of science out of the discreditable rut it has been brought into by the use of hopelessly unscientific statistical methods. I fail to find any change of front in this, for the catacomb data had already been dealt with as suspect on account of the remarkable proportion of the two sexes. Dr Myers' mixed series was dismissed on the odds of 42,552 to 1; it is hopelessly more heterogeneous than the worst of the four series quoted by Miss Fawcett. I then proceeded to get a *more refined* test, by replacing the French data,—which I hold not to be a race mixture, but a mixture of individuals (and possibly sexes) from the same race at different periods,—by a series of Parisian crania from one period, provided for me by the doyen of French craniologists as a homogeneous series. There is no change of front, merely an endeavour to reach a still better criterion*.

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* There is strong feeling throughout the Tirol and surrounding districts against marriage with a stranger,—often leading to the mobbing of a bridegroom from another locality.

† I shall on another occasion have a good deal to say about the conclusions Dr Myers has published on Egyptian skulls, and on the methods he has used in drawing those conclusions.

(iii) Lastly, I turn to the saddest part of Dr Myers' letter, for it shows how little hope there is that he can ever be converted to the more modern view of statistics as an exact science. Instead of in the least replying to my chief contention *that no number is absolutely great or small*, he writes as follows about my calculation of the odds against his mixture of 42,552 to 1:

> "Surely common sense repudiates the legitimacy of such a calculation. But if mathematics allows it, what must also be the odds against the appearance of the figure 7·202 in the same series! Yet this figure actually occurs there, being the measure of variability of the 860 French skulls from the Paris catacombs."

All mathematics are merely common sense in symbols and the divergence between the two can only arise from a slip in the mathematics, or because a writer labels as "common sense" what is not sense but what he wants to believe. Let me suggest to Dr Myers the following experiment: Let him take 12 pennies and toss them 100,000 times. He will find that only about 24 times in 100,000 trials, or once in 4167 trials, he will obtain all twelve heads. He would obtain nine heads in about 5371 cases out of his 100,000 trials, or about once in 18 trials. This is mathematical, but I know from experience that it is very closely indeed "common sense." Now would Dr Myers be justified in saying that $12-9=3$ is very small and arguing as follows?

> "Surely common sense repudiates the legitimacy of a calculation giving 4166 to 1 against 12 heads. But if mathematics allows it what must also be the odds against the appearance of the figure 9 in the same series of penny tossings! Yet this figure actually occurs in penny tossings."

I hope Dr Myers will not think me flippant, but the arguments are really quite parallel, except that his 8·389 is *more than ten times as improbable as the twelve heads occurring*. The standard deviation in this case of penny tossings is 1·732 and the mean number of heads of course, six. Thus, nine heads differ by not quite twice, and 12 heads by not quite four times, the standard deviation from the mean. Now in the case of the crania the mean was 6·279, the standard deviation ·5185, and the French skulls' variability 7·202, or it differs 1·780 times the standard deviation from the mean. A deviation greater than this will occur once in 25 trials, as the odds are 24 to 1 against it. In other words, in every eight tables like Miss Fawcett's of like material, we should expect to find a skull-series of length variability equal to the French *once*. There is therefore nothing whatever improbable about it on the data of Miss Fawcett's table. But Dr Myers' mixture has a variability which differs 4·07 times standard deviation from the mean, and this will occur between 2 and 3 times only in 100,000 trials. In other words in 10,000 tables like Miss Fawcett's, we might expect to find one case like Dr Myers' mixture! Shortly the odds in the two cases are not comparable. I am sorry if this is unintelligible to Dr Myers, but he asks in astonishment what must be the odds against the French series with a variability of 7·202, thus again only emphasising his failure to appreciate what are 'large' and what are 'small' differences. Such differences can only be appreciated by a study of the elements of the theory of probability, and I would venture to suggest a little experimental coin-tossing and possibly a study of Westergaard to all craniologists who imagine that nothing but the elements of arithmetic are necessary in statistical investigations.

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* Dr Myers' mixture is absurd in comparison with the French catacomb crania. Why is it a change of front to show it is infinitely more absurd when we use still more homogeneous material? His proper reply would be to show that as we improve our material the Naqada series becomes very improbable, but this is not the case.

K. P.

(v) *Note on Cranial Types.*

By Professor Dr AUREL VON TÖRÖK.

Da im Part III. Vol. II. 1903, S. 339 der *Biometrika* ("I. Prof. Aurel v. Török's attack on the Arithmetical Mean") behauptet wird, dass ich den Begriff eines Typus so auffasse: als würde mittels desselben das Individuelle von irgend einer Bevölkerung von dem Individuellen einer anderen Bevölkerung differenziert ("which differentiates an *individual* of one population from an *individual* of a second population") welche Behauptung aus den Erörterungen (meiner und meines Mitarbeiters Abhandlung "Über das gegenseitige Verhalten etc.") mit Berechtigung nicht aufgestellt werden kann, weil hiergegen auch schon der Grundbegriff des von mir definierten Typus spricht die auch Herr Prof. Karl Pearson zitiert: "So kann...die Bestimmung eines Typus in gar nichts anderem bestehen, als dass man unter dem Mehrerlei der Einzelmerkmale dasjenige herausgreift, was in der grossen Überzahl an zu treffen ist" (aus der Überzahl der Einzelfälle kann doch niemals das Individuelle abgeleitet werden, weil umgekehrt aus den einzelnen Individuellen die Allgemeinheit bez. Überzahl abgeleitet wird); so bitte ich um die Aufnahme folgender Berichtigung.

Mein Angriff (attack) gegen die arithmetische Mittelzahl beschränkt sich einzig allein auf den bisher nur zu oft benützten Unfug: um schon mittels alleiniger Mitteilung der arithm. Mittelzahlen den Typus für eine Menschengruppe (Rasse) auf zu stellen. Wenn also Herr Prof. Karl Pearson in der Kritik gegen mich aussagt: "A knowledge of the arithmetic means only would never enable one to say of an individual skull that it belonged to one race and not to another"; so kann gerade ich hierin eine Bestätigung meiner Ansicht, über den bisherigen allgemeinen Gebrauch der arithm. Mittelzahlen mit Genugtuung auffinden.

Ferner bitte ich um die Erlaubnis noch folgender Bemerkungen.

Ich halte den Begriff "Rasse" und "Typus" streng auseinander. Der Begriff "Rasse" ist ein aus der Zoologie übernommener und ist mit der Abstammungsfrage unzertrennlich verbunden; hingegen habe ich seit je her unter dem Begriffe "Typus" nur die in Überzahl der Einzelfälle innerhalb jedweder Menschengruppe (Familie, Geschlecht, Sippe, Stamm, Volk, Rasse) vorkommenden Merkmale verstanden. Es verhält sich der von mir interpretierte Typus zur Rasse, wie ein Unterbegriff zum Oberbegriff. (Der Begriff Rasse vereinigt in sich mehrere Typen.)

Nur der Typus oder die Typen innerhalb irgend einer Rasse können mittels der mathematischen Methode bestimmt werden; hingegen die Frage ob die betreffende Menschengruppe

(Rasse) "blutrein" oder "blutvermischt" ist d. h. der Nachweis, dass die betreffende Menschengruppe niemals einer irgendwelchen Blutmischung unterworfen war—kann wenigstens nach meiner Überzeugung niemals mittels der bisher bekannt gemachten mathematischen Methoden nachgewiesen werden. Wäre dies möglich, so könnte auch die ganze Abstammungsfrage der Menschheit (die sog. Rassenlehre) auf rein mathematischem Wege endgültig geschlichtet werden. —Ich muss wenigstens für die bisher angewendeten mathematischen Methoden—und so auch für die von Herrn Prof. Karl Pearson eingeführte Methode dies aufrecht halten.

Es wäre doch im Interesse der "Biometrika" wenn Herr Prof. Karl Pearson sich die Mühe nehmen würde, um die für uns so höchst wichtige Rassenfrage (d. h. die Abstammung und Begrenzung der einzelnen Rassen) auf rein mathematischem Wege behandeln und zum endgültigen Abschluss führen würde.

Endlich hätte ich die Bitte, wenn Herr Prof. Karl Pearson sich die Mühe nehmen würde, mein neues Verfahren: die Einzelmaasse nach den drei Vergleichungsgruppen (kleine, mittelgrosse, grosse Maasswerte) zu bestimmen, um den korrelativen Typus aufstellen zu können—einer strengen ausführlichen Kritik unter zu ziehen: da der wesentliche Teil der von ihm kritisirten Arbeit eben sich auf dieses Moment bezieht.

Prof. Dr AUREL VON TÖRÖK,
Anthropologiai Múzeum, Budapest, 1903. vii. 10.

Remarks on Professor Aurel von Török's Note.

I feel precisely the same difficulty about controversy with Professor von Török as with Dr Myers. We do not think in the same plane. Professor von Török has criticised the arithmetic mean because: (i) it does not give what occurs in the great majority of individuals, (ii) it does not give the modal value, (iii) it does not give the median value of a character. My defence of the arithmetic mean was that nobody expected it to fulfil (i), but that within the limits of random sampling it did in the material actually used in Professor von Török's memoir satisfy (ii) and (iii), although that material was cited to prove the exact *opposite*. To all this Professor von Török makes no reply whatever.

I am sorry if I have misunderstood Professor von Török in supposing him to consider that the type differentiates an *individual* of one population from an *individual* of a second population. What he actually wrote was:

> Wenn unter dem Wortausdrucke "Typus" dasjenige verstanden werden muss, was für die Gesammtheit irgendwelcher Dinge charakteristisch ist, wodurch man also dieselben von anderen sofort präcis zu unterscheiden im Stande ist: so kann—wie dies schon in dem voraufgehenden Capitel berührt wurde—die Bestimmung eines Typus in gar nichts anderem bestehen, als dass man unter dem Mehrerlei der Einzelmerkmale dasjenige herausgreift, was in der grossen Ueberzahl anzutreffen ist*.

I can only consider that an over-emphasised sense of grammatic purity led me to hold that *dieselben* referred to the *Dinge* in the *Gesammtheit* and not to the *Gesammtheit* itself and *anderen* to *Dinge* in other *Gesammtheiten* and not to other *Gesammtheiten* themselves. But if the *type* be as I suppose, and as I am glad to find Professor von Török agrees it should be, something characteristic of the population or *Gesammtheit*, why is it at all needful that it be present in "the great majority" of the individuals of the population? If the type be not that which distinguishes an individual of one race from an individual of another, but only a feature of the population as a *whole*, why should Professor von Török attack the arithmetic mean, because it is not a characteristic of the majority of a population? The standard deviations and the correlations of characters have absolutely no meaning relative to any individual of a population, but only to the population as a whole. Yet they help to fix its type in precisely the same way as the arithmetic mean does. Why should we demand of the latter, any more than of the two former statistical "population-constants" that they should be characteristic of the great majority of a population? It appears to me that there is still lingering in Professor von Török's mind something of the old confused notion of type, when he demands that any statistical constant which helps to describe the type shall be characteristic of the great majority of a population.

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* *Zeitschrift für Morphologie und Anthropologie*, Bd. IV. p. 511.

For the biometrician the type of any group (or "population" in the biometric sense) is fixed by the whole complex of statistical constants—means, standard deviations, correlations, skewnesses, etc., which suffice to differentiate it sensibly from other groups or populations. Very frequently the arithmetic means of a number of characters will suffice, if so they fix the type. Often we have to use a number of other constants—correlations or what not—and the question of whether two groups are different in type becomes an extremely delicate one, only solved by a careful consideration of the probabilities connoted by the probable errors of differences.

I am aware that the conception of 'type' thus given is not that of many anthropologists, but biometry is essentially a science of exact quantitative definition, and if it is to be of service in rendering anthropology an exact branch of science, it must replace vague ideas by numerically definite conceptions. If any anthropologist replies that: "'Type' is something to be judged by general appreciation," then I personally have no reply to make. He speaks like the anatomical craniologist in the language of an esoteric science and I do not pretend to criticise his conclusions. All I demand is that he should not use, what are to the mathematician hopelessly inadequate and faulty statistical methods to justify his "type" appreciations. If on the other hand he says: "There is a definite quantitative character I understand by type, which is not involved in your definition above," then biometricians are perfectly prepared to try and understand what it is, and will endeavour to give it a significant biometric definition and quantitative measure. When Professor von Török tells me that the type is something which is characteristic of the great majority of a population, group or *Gesammtheit*, I am at a loss, for although I have examined the characters of many groups, I have never found this something, to which I could attach the name type, except where the smallness of the variability rendered it possible on mere inspection to differentiate one population from a second,—as a black man from a white, a dark-haired from a light-haired people and so forth. Here, however, the unreasoned differentiation really depends on the biometric conceptions of different means and small variabilities. But because these conceptions are not the conscious source of the classification, the anthropologist appears to hold that the biometrician applies a wholly different process when he uses means and variabilities to distinguish the types of groups which in part overlap, as far as mere individuals are concerned. I am inclined to think that it is these cases of widely divergent means with small variabilities where differentiation is a matter of inspection only, that really lead to the confused notion of many anthropologists that the "type" is something characteristic of the great majority of a group.

Almost equally vague with the conception of type is the current conception of *race* in man. I am wholly in accordance with Professor von Török when he says that the problem of race is inseparably united with the question of descent, but if he thinks that biometric methods cannot apply to this problem I must venture to differ, and refer him for the sort of lines on which I think the problem will one day be solved to my memoir on the *Influence of Natural Selection on Variation and Correlation*[*]. Turning to the special problem as concerns man, I hold that the whole evolutionary evidence is in favour of descent from very few groups, perhaps a single group. Hence Professor von Török's statement that it is impossible to settle by mathematical methods the problem of whether a given "Menschengruppe (Rasse) 'blutrein' oder 'blutvermischt' ist," understanding by "blutrein" that "die betreffende Menschengruppe niemals einer irgendwelchen Blutmischung unterworfen war" has no meaning at all for me, because his definition of "blutrein" seems to involve an eternal past for humanity or a separate creation of each so-called race. In other words his phrase: "*niemals* einer irgendwelchen Blutmischung unterworfen" seems to me to exclude entirely the evolution of humanity from a single original group or at least very few groups. It is not a practical definition of race at all, for under it we must say that race in man does not exist. Again granted differentiation into subgroups from an original stock, and that these subgroups have been acted upon by selection throughout long ages, again I hold that a definition of race as a subgroup which has remained without *Blutmischung* since that original differentiation is hopelessly idle. In all probability such a race does not exist in man. Probably every group of men is in this sense *blutvermischt*. If my view be a correct one, race in man is merely a *relative* term. I understand by a race of men, what I understand by a race of snails or birds or fox-terriers, i.e. a group which has intermixed freely with itself but not with other groups for a number of generations, and during this process has been equally freely subjected to the action of natural or artificial selection. Sensible isolation is generally needed for the first condition, probably fixity of locality for the second condition. The "perfect race" would be that which had for many generations been isolated in one locality and had freely "intrabred." As a result we should have a distinct "type," more or less stable to its environment. Of course in man we only get more or less close approximations to these conditions, and from such approximations we have every shade of imperfect mixture down to the mixed population of a European settlement in Asia, with but small *Blutmischung* among its different castes or subgroups. This pure relativity of race which is all I can conceive of in man as a practical definition, diverges probably very widely from the conception of Professor von Török, but it enables us at once to set up a standard, a relative one it is true, but none the less perfectly sound for biometric purposes—of what may or may not be considered as homogeneous material. To apply our definition we must start from human groups which we know have been for a number of generations in the bulk isolated and freely interbreeding. These will give us a standard by which to judge of the comparative value of other groups. In this sense I think we are justified in speaking of the Ainos or the Altbaiern as races; we may speak of the French of a given rural locality as a race for there is comparatively little immigration, and so forth. But if we mix data for the same group at different periods, we shall not get a race because progressive changes are taking place. Above all we must never forget that our whole conception is one of *relative* degree. I shall no doubt be told that this conception of race is quite invalid, that races can be sorted out by types, long after blood-mixture, by the expert anthropologist. This may be so, but it throws us back on "appreciation" and the undefined, if not undefinable "type," a field quite outside biometric investigations. The instances of Eurasians, Indo-Spaniards, and half-castes in general, seem to me to show a continuous blending in man as the *Blutmischung* is continued. Stop the immigration into North America and let its multiracial elements interbreed for a number of generations, and be environmentally selected, and the complete blend of any locality will be as much a "race" as any district of Europe can now show 1000 years after the *Völkerwanderung*. With the extreme mobility of man and his ready fertility with any group of his own species, I do not believe that a more practical conception of race in man can be formed. The "purest" race is for me the one which has been isolated, intrabred, and selected for the longest period. It may well in the dim past have been a blend of the most diverse elements. Starting from this conception of race biometric investigations can enable us to form some rough measure even at present, of the position of groups on a racial scale. At any rate they enable us to assert that Ainos, Naqadas and Altbaiern, are indefinitely far removed on such a scale from mere cranial mixtures*. I do not suppose this conception of race will satisfy Professor von Török, any more than Dr Myers or other craniologists or anthropologists of the old school. But I would ask them to express concisely and definitely what they understand by "type" and by "race," and then we shall be able to see whether a quantitative definition can be found for the ideas which appeal to them. To the biometrician their ideas appear to be confused notions, for they turn either on "appreciation," which is out of his plane altogether, or on statistical methods which in nine cases out of ten are unsound.

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[*] *Phil. Trans.* Vol. 200, pp. 1–66. See especially p. 22.

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* It would be quite possible to make out of Australian, Guanch, Chinese and European crania by an adroit selection a mixture which for one character was as little variable as that character in a "race" like the Aino, but I believe the biometrician would detect the mixture by simple analysis of other little correlated characters.

In conclusion I would only reply to Professor von Török's request that I will submit to *einer strengen ausführlichen Kritik* his method of dealing with correlation, that I have already stated my opinion of his method*. It may be summed up as follows: In the present state of statistical science it is an extremely cumbersome method of dealing with multiple correlation. Four correlation coefficients and a multiple regression equation (such as those given in *Biometrika*, Vol. II., p. 349) should replace some 70 pages of Professor von Török's memoir. This replacement would not only give increased brevity, but increased accuracy, as all such cranial data demand graduation. Further in the present state of our craniological knowledge, it is fallacious to extend any result of intraracial correlation from one race to a second †, hence I do not think any conclusion from Hungarian data to the Neanderthal skull or the Anthropopithecus Dubois, can amount to aught but the vaguest suggestion.

The great value of Professor von Török's paper lies in the fact that a craniologist *von Fache* emphasises the importance of dealing with correlation, when cranial results are considered. This is an immense gain, and will have its influence on future continental craniological publications. At the same time the mathematical theory of correlation is quite simple, and one can only hope that in future continental writers will adopt the practice of publishing coefficients of correlation (such as have been used in biometric cranial memoirs since 1895) to indicate at a glance the relative degree of association between characters. The "multiple regression equations" and "partial correlation coefficients" then give concisely that relationship between the arrays, which Professor von Török is rightly but most laboriously seeking to express.

K. P.

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* *Biometrika*, Vol. II. p. 353, footnote.

† *Biometrika*, Vol. II. p. 353, text.

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